



**A NOTE ON ESTIMATING REALIGNMENT PROBABILITIES –
A FIRST-PASSAGE-TIME APPROACH¹**

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Abstract

This paper proposes a path-dependent approach for estimating realignment probabilities of targeted exchange rates based on first-passage-time distributions instead of the commonly used path-independent approach. We consider that path dependency is an intrinsic characteristic of realignment risk because a realignment of an exchange rate can occur whenever a committed band by a central bank is breached. A mean-reverting lognormal process is considered in the first-passage-time approach. Based on market data of the British pound and mark during the ERM crisis of 1992, the realignment probabilities of the pound estimated under the proposed approach show that path dependency is quantitatively significant, compared with the path-independent approach.

JEL Classification: F31, G13

Keywords: realignment risk, mean-reversion, first-passage-time probability

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The views and analysis expressed in this paper are those of the authors, and do not necessarily represent the views of the Hong Kong Monetary Authority.

¹ The paper will be published in the Journal of International Money & Finance. The authors gratefully acknowledge useful comments from Hans Genberg.

Executive Summary:

- *This paper proposes a path-dependent approach for estimating realignment probabilities of targeted exchange rates based on first-passage-time distributions instead of the commonly used path-independent approach. We consider that path dependency is an intrinsic characteristic of realignment risk because a realignment of a currency can be triggered by an important economic-political event during a given time horizon. A mean-reverting lognormal process is considered in the first-passage-time approach.*
- *Based on the ERM crisis of 1992, the realignment probabilities of the British pound estimated under the proposed path-dependent approach show that boundaries are quantitatively significant, compared with the path-independent approach. A central bank which adopts a targeted or managed-floating exchange-rate regime could interpret implied realignment probabilities based on the proposed approach as an indicator to assess the realignment risk.*

I. INTRODUCTION

Option markets have the desirable property of being forward-looking in nature and thus are a useful source of information for gauging market sentiment about future values of financial assets. Options, whose payoff depends on a limited range of the expected exchange rate, offer broader information about market expectations than the forward exchange rate. The entire risk neutral probability density function of the exchange rate can be inferred from option prices. Several studies have used option prices to estimate realignment risk of targeted exchange rates during the ERM crisis in 1992. Malz (1996), Mizrach (1996) and Söderlind (2000) estimated the realignment risk of the British pound exchange rate based upon its risk neutral probability density functions for the crisis period. Campa and Chang (1996) calculated a minimum realignment size for the exchange rate using arbitrage arguments only.

The approach used by Malz (1996) to infer the probability density function is based on the assumption that the pound exchange rate follows a jump-diffusion process. This choice appears relevant to assess the credibility of targeted exchange rate regimes, because the expected jump of the underlying exchange rate as well as its probability can then be recovered from such a specification (Bates, 1991; Bates, 1996). Specifically, the Bernoulli version of the jump diffusion postulates either one jump until option maturity or no jump at all (Ball and Torous, 1983; Ball and Torous, 1985). Then the valuation formula turns out to be a mixture of two lognormal distributions. Implicitly, the estimated realignment probabilities are defined as the likelihood that the spot pound exchange rate is below the lower band (i.e. the lower fluctuation limit) at the end of a specified time horizon. Therefore, the estimation is independent of the path of the exchange rate within the time horizon. Such a path-independent approach has also been used in estimating realignment risk in Campa and Chang (1996), Mizrach (1996) and Söderlind (2000).

This paper proposes an approach for estimating realignment probabilities of targeted exchange rates based upon a first-passage-time (FPT) approach, which is path-dependent, instead of the commonly used path-independent approach. It demonstrates that there is a significant difference between realignment probabilities measured by the path-independent approach and by the path-dependent approach. It is because the risk measurement of the path-independent approach depends on the exchange rate only at the end of some time interval, and not on the particular path. This means that the path-independent realignment probability does not take into account the risk of the exchange rate passing through the target zone boundary during some time interval. It also implicitly assumes that a realignment can only occur at the end of the time horizon of risk assessment, and that the exchange rate is otherwise free to move to any level relative to the boundary. The path-independent approach therefore underestimates the realignment risk by an amount equal to the probability of breaching

the boundary during some time interval. This paper however considers that path dependency is an intrinsic characteristic of realignment risk because a realignment of an exchange rate can occur whenever a committed band by a central bank is breached, that could be triggered by an important economic-political event. The FPT approach explicitly recognizes the consequences of a realignment whenever the exchange rate falls below (or rises above) the boundaries. Consequences like a readjustment of the boundaries will also move the target exchange rate of a currency.

An important prediction of the theoretical literature on targeted exchange rates is that mean reversion of the exchange rate is expected when the central banks engage in intramarginal intervention. This mean-reverting property is widely referred to in the literature (Krugman, 1991; Svensson, 1992; Rose and Svensson, 1994; Anthony and MacDonald, 1998). Several recent studies have attempted to investigate empirically this theoretical prediction by examining the time-series properties of the currencies participating in the European Monetary System (Ball and Roma, 1993; Ball and Roma, 1994; Svensson, 1993; Rose and Svensson, 1994; Nieuwland *et al.*, 1994; Anthony and MacDonald, 1999; Kanas, 1998). While their investigations are with mixed results, the empirical results suggest that mean reversion is present. In view of this evidence, the exchange rate within a target zone is assumed to follow a mean-reverting lognormal process for estimating realignment probabilities in this paper.²

In the following section, we derive realignment probabilities under the mean-reverting lognormal process using the FPT approach. In section 3, the realignment probabilities of the pound during the ERM crisis of 1992 are calculated from the FPT approach and the results are compared with those obtained from the path-independent approach. The final section summarizes the findings.

II. FRAMEWORK UNDER MEAN-REVERTING LOGNORMAL PROCESS

A mean-reverting process for the exchange rate is a useful approximation for a target zone situation when the central banks engage in intramarginal interventions. Such interventions could be aimed at preventing or smoothing sharp and disruptive exchange rate fluctuations from time to time. Therefore, the interventions may occur not only at the band limits of the target zone but also at some levels near the central parity. Another feature of the interventions is apparently that the authorities try to bring the exchange rate back to some mean level or the central parity. In addition to central bank interventions, market participants might expect the exchange rate band to be fully

² Ball and Roma (1998) indicate that the mean-reverting behaviour of the exchange rates in the European Monetary System may be due to the effect of reflecting barriers where the intervention boundaries are credible. However, this paper uses an absorbing barrier, where the intervention boundary is not necessarily credible, to calculate the FPT probabilities.

credible and engage in stabilizing speculation such that the resulting tendency of the exchange rate is to return to a mean level.

It is assumed that the spot exchange rate S (the foreign currency value of a unit of domestic currency) is the only state variable and mean-reverting. The risk-free interest rates of the underlying exchange rate are constant and the interest rate parity relationship holds. The Kolmogorov's backward equation governing the transition probability $P(S, t)$ of S , that may breach a boundary H at the backward time t , under a mean-reverting process is:³

$$\frac{\partial P(S, t)}{\partial t} = \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 P(S, t)}{\partial S^2} + [\kappa (\ln S_0 - \ln S) + (r - r^*)] S \frac{\partial P(S, t)}{\partial S}, \quad (1)$$

where S_0 is the conditional mean exchange rate, κ is the parameter measuring the speed of reversion to this mean, r and r^* are the risk-free interest rates of the foreign and domestic currencies respectively, and σ is the volatility of the exchange rate.⁴ The corresponding FPT distribution $P_{fp}(S, t)$ of Eq. (1) is then given by $1 - P(S, t)$. As shown in the Appendix, the corresponding FPT distribution, which is the realignment probability of S at an upper band $h(t) = H \exp\{-e^{\kappa t} [c_2(t) + \beta c_1(t)]\}$, is found to be

$$P_{fp}(S, t) = N(\tilde{b}_1) + \left(\frac{H}{\tilde{S}}\right)^{-\beta} e^{\beta(\beta-1)c_1} N(-\tilde{d}_1), \quad (2)$$

where

$$\tilde{S} = \left(\frac{S}{H}\right)^{\exp(-\kappa t)-1} e^{c_1+c_2} S, \quad (3)$$

$$c_1 = \frac{\sigma^2}{4\kappa} (1 - e^{-2\kappa t}), \quad (4)$$

$$c_2 = [\ln(S_0 / H) - \sigma^2 / 2\kappa + (r - r^*) / \kappa] (1 - e^{-\kappa t}), \quad (5)$$

$$\tilde{b}_1 = \frac{\ln(\tilde{S} / H) - c_1}{\sqrt{2c_1}}, \quad (6)$$

$$\tilde{d}_1 = \frac{\ln(H / \tilde{S}) - c_1 - 2(\beta - 1)c_1}{\sqrt{2c_1}}, \quad (7)$$

β is a real number and $N(\cdot)$ is the cumulative normal distribution function.

³ It is an Ornstein-Uhlenbeck process for the log of the exchange rate.

⁴ S_0 can be interpreted as the historical mean instantaneous exchange rate.

Similarly, the corresponding risk-neutral realignment probability at a lower band of $h(t)$ under the mean-reverting process is

$$P_{fp}(S, t) = N\left(-\tilde{b}_1\right) + \left(\frac{H}{\tilde{S}}\right)^{-\beta} e^{\beta(\beta-1)c_1} N\left(\tilde{d}_1\right). \quad (8)$$

To approximate the FPT distribution through a fixed boundary at H , an optimal value of the adjustable parameter β is chosen in such a way that the integral

$$\int_0^t [h(t')]^2 dt'$$

is minimum. In other words, it is to minimize the deviation of the moving boundary from a fixed boundary by varying the parameter β . Simple algebraic manipulations then yield the optimal value of β as follows:

$$\beta_{opt} = -\frac{\int_0^t c_1(t')c_2(t')e^{2\kappa t'} dt'}{2\int_0^t c_1^2(t')e^{2\kappa t'} dt'}$$

The parameter β can therefore be adjusted such that the solutions in Eq. (2) and Eq. (8) provide the best approximation by using a simple method developed by Lo and Hui (2006) for computing the FPT distribution of a time-dependent Ornstein-Uhlenbeck process to a moving absorbing boundary. The method is based upon simulating the fixed boundary as a slowly fluctuating barrier with a small oscillating amplitude by tuning the parameter β . The upper and lower bounds (in closed form) provided by the method are also very tight for the exact values. Since the bounds and estimates of the exact results appear in closed form, they can be computed very efficiently. Furthermore, the bounds can be improved systematically and these improved bounds are again expressed (in closed form) in terms of the multivariate normal distribution functions.

The resemblance between Eq. (2) and Eq. (8) of the FPT distribution under the mean-reverting process derived above and the corresponding formulas with the exchange rate under the lognormal process is obvious. The random variable $\tilde{S}$ specified in Eq. (3) above is lognormal, resulting in similar formulas as those under the lognormal process. The $\tilde{\sigma}^2$, which is the variance of the logarithm of the price $\tilde{S}$, replaces σ^2 of the lognormal model, which has the same meaning. In other words, $\tilde{S}$ is lognormal with $\text{var}[\ln \tilde{S}] = \tilde{\sigma}^2 / t$ where

$$\tilde{\sigma} = \sqrt{2c_1 / t} = \frac{\sigma}{\sqrt{2\kappa t}} \sqrt{1 - e^{-2\kappa t}}. \quad (9)$$

The resemblance is equivalent to the change of numeraire of S by $\tilde{S}$. Different magnitudes of the speed κ of reversion to this mean give some intuitive interpretations of $\tilde{S}$. When the speed is very strong (i.e. $\kappa \gg 1$), $\tilde{S}$ converges to S_0 . This means that the dynamical process of the exchange rate is almost deterministic such that it will stick to $\tilde{S}$ with the “effective volatility” $\tilde{\sigma} \rightarrow 0$. This illustrates that the presence of mean reversion makes the associated FPT distribution quite different from that in the lognormal process. The difference will be shown numerically in the following section. Conversely, when the speed is very weak (i.e. $\kappa \rightarrow 0$), the mean-reverting process specified in the model converges to a lognormal process where $\tilde{S} \cong S$ and $\tilde{\sigma} \cong \sigma$ such that the corresponding FPT distribution is that based on the lognormal process with a boundary of $h(t) \cong H$. Eq. (9) and the limits of $\tilde{\sigma}$ with different κ show that $\tilde{\sigma}^2$ is a decreasing function with κ and is less than σ^2 . The proof of $\tilde{\sigma}^2$ being a decreasing function with κ is given in the Appendix.

III. ESTIMATION OF REALIGNMENT PROBABILITIES

The model parameters used to estimate realignment probabilities are those used by Malz (1996) based upon combining the information from several options with different strike prices covering the period from 31 March to 16 September 1992. On 16 September 1992, the pound left the ERM after a period of turbulence. The realignment probabilities of the pound with a one-month time horizon based on the FPT distribution under the lognormal process and the mean-reverting process respectively are shown in Figure 1. They are calculated for each day by using σ presented in Malz (1996) under the jump-diffusion model, which are estimated from market option price data as well as market data of S , r , and r^* .⁵ The same exchange rate volatility σ is used for both the path-dependent approach (with the lognormal and mean-reverting processes) and the path-independent approach (with the jump-diffusion process). Given a market option value, the corresponding implied exchange rate volatility (i.e. the diffusion part) under the jump-diffusion process is smaller than that under both the lognormal and mean-reverting processes because part of the market option value is attributable to the jump risk.⁶ Therefore, the values of σ used for the path-dependent approach are actually smaller than the market implied volatility based on the underlying processes. While this may downwardly bias the realignment probabilities estimated by the path-dependent approach, the following numerical results show that such bias does not affect the analysis of the estimated realignment probabilities based on the two approaches.

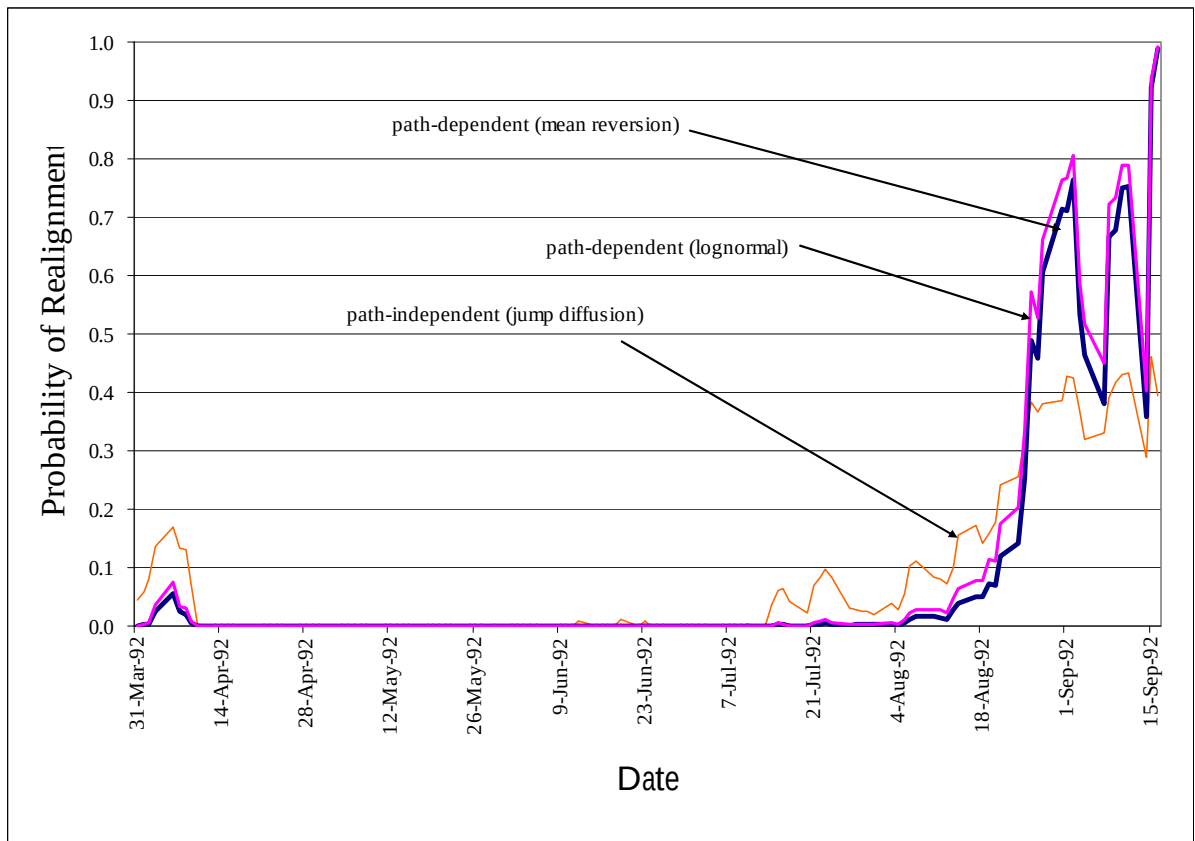
⁵ Other market data are from Bloomberg.

⁶ It is noted that the volatilities presented in Malz (1996) are smaller than the market volatilities implied from the Black-Scholes model. The differences between the market volatilities implied from the Black-Scholes model and the mean-reverting process are immaterial (the volatility estimations are available from the author upon request).

The lower band limit H is DEM2.778 per GBP. For the mean-reverting process, the conditional mean exchange rate S_0 is set at DEM2.95 which is the central parity during the period. The speed of reversion κ is set to be 0.478.⁷ The realignment probabilities estimated by the two path-dependent models based on the FPT approach, where Eq. (8) is used, are compared with the realignment probabilities based on the path-independent jump-diffusion approach in Malz (1996).

Figure 1 shows that the probabilities estimated by the three models are almost zero during most of time from April to early July of 1992. During mid-July and early August, their realignment probabilities start to increase but are below 0.3 which is relatively low. The probabilities estimated by the path-independent approach are higher than those estimated by the FPT distribution with the lognormal and mean-reverting processes. This shows that the jump risk is predominant during this period of time.

Figure 1. Probability of realignment of the pound in next month from 31 March 1992 to 16 September 1992.



⁷ κ is estimated by applying the maximum likelihood technique to the mean-reverting specification of S using 394 weekly observations of the DEM/GBP exchange rate in the period from 6 March 1985 to 15 September 1992. The corresponding S_0 is DEM2.8515. It is not the objective of this paper to investigate the mean-reverting behaviour of the exchange rates in the ERM. The estimation is just for the purpose of numerical illustrations of the mean-reverting model.

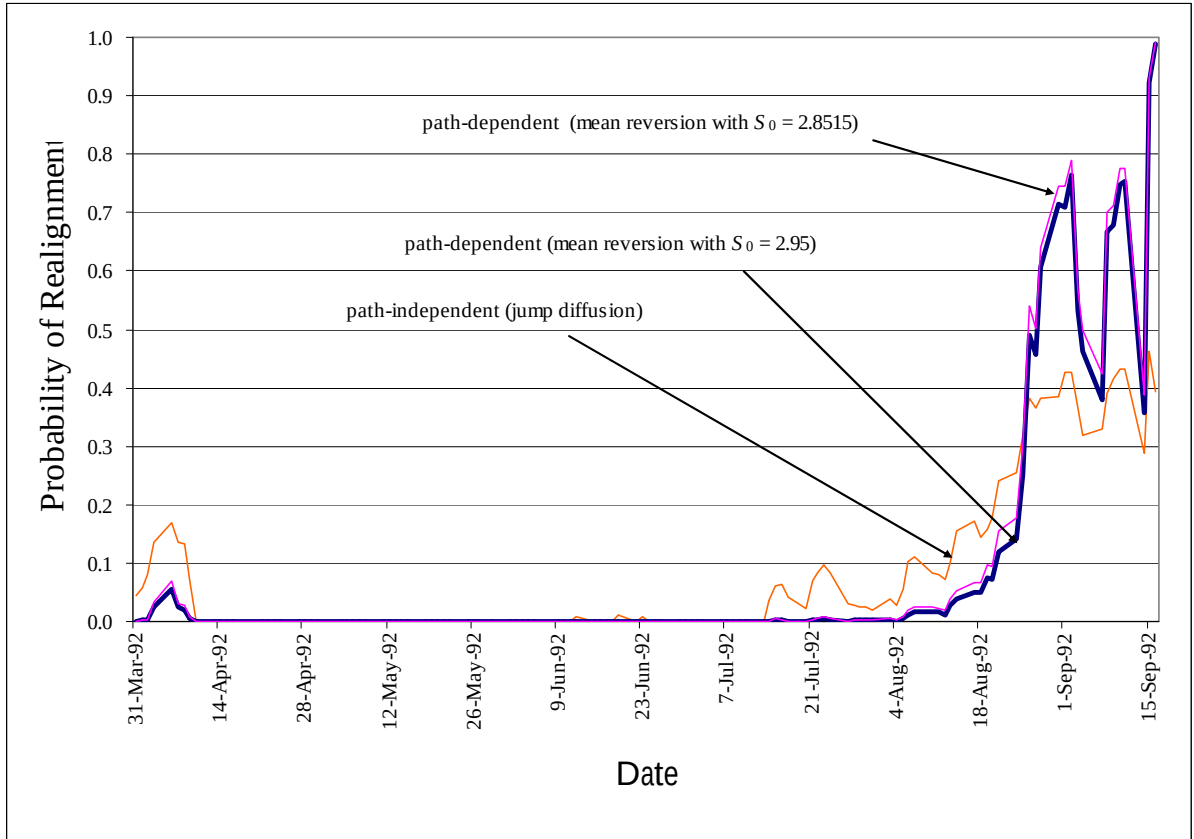
The realignment probabilities then rise sharply in the second half of August, peaking on September 16. The probabilities estimated by the FPT approach with both the lognormal and mean-reverting processes, which are above 0.5 during most of time of the period, are substantially higher than those (about 0.4) estimated by the path-independent approach. In particular, the probabilities estimated by the FPT approach are higher than 0.7 several times in September, while the corresponding probabilities estimated by the path-independent approach range only between 0.39 and 0.46. Even for the pound's two brief respites from pressure, following the UK's announcement of plans to borrow ECU 10 billion to defend the pound on September 3, and following the lira devaluation on September 13, the probabilities estimated by the FPT approach with the lognormal process are still about 0.5, while those estimated by the path-independent approach are below 0.4. On 16 September 1992 when the pound left the ERM, the probabilities estimated by the FPT approach with both the lognormal and mean-reverting processes reach almost 1, while those estimated by the path-independent approach are only around 0.4. The results show that the path dependency is an important characteristic of estimating realignment probabilities during the period of turbulence. In terms of quantitative estimations, the FPT approach gives much clearer signals of realignment risk than those given by the path-independent approach.

It is noted that under the FPT approach the probabilities estimated with the mean-reverting process is lower than those estimated with the lognormal process. It is because the mean-reverting process will push the exchange rate back towards the central parity and thus reduces the realignment risk, when the exchange rate is between the central parity and the lower band limit during the period from 31 March to 16 September 1992. This means that higher the speed of reversion, lower the realignment probabilities will be under the mean-reverting process.⁸ However, the surges of the probabilities estimated under the mean-reverting process after the second half of August are similar to those under the lognormal process. The results show that the realignment risk during the period before the pound left the ERM is mainly attributed to the existence of the boundary which captures any realignment within the one-month time horizon.

The realignment probabilities estimated by the mean-reverting process with a lower conditional mean exchange rate of $S_0 = \text{DEM}2.8515$ are shown in Figure 2. As the conditional mean is closer to the lower band limit at DEM2.778, the probabilities are higher than those estimated using $S_0 = \text{DEM}2.95$, but the differences are small. This shows that the realignment probabilities are not very sensitive to the level of the conditional mean unless there is a drastic change of the level relative to the lower band limit.

⁸ A strong mean reversion could come about due to interventions in the foreign exchange market by the central banks. Such interventions have been shown in many studies (see MacDonald (1988)).

Figure 2. Probability of realignment of the pound in next month from 31 March 1992 to 16 September 1992, where $S_0 = \text{DEM}2.8515$ and 2.95 per GBP respectively under the mean-reverting process.



IV. CONCLUSION

This paper proposes a path-dependent approach based on first-passage-time distributions to estimating realignment probabilities of currencies in target zones and considers that path dependency is an intrinsic characteristic of realignment risk. The path-independent-options approach adapted by Malz (1996) ignores the possibility of realignment prior to option maturity. In essence, this perspective assumes that the realignment risk remains zero regardless of the changes in the exchange rate during a time horizon. However, if the exchange rate breaches a pre-specified boundary (i.e. one of the bands), realignment can occur.

Based on the ERM crisis of 1992, the realignment probabilities of the pound estimated under the proposed path-dependent approach show that boundaries are quantitatively significant, compared with the path-independent approach. A central bank which adopts a targeted or managed-floating exchange-rate regime could interpret implied realignment probabilities based on the proposed approach as an indicator to assess the realignment risk.

APPENDIX

The corresponding FPT distribution $P_{fp}(S, t)$ is given by $1 - P(S, t)$ where $P(S, t)$ the transition probability of S . By defining $x = \ln(S/H)$ and $\theta = [\ln(S_0/H) - \sigma^2/2\kappa + (r - r^*)/\kappa]$, where H is a level to determine the boundary of realignment, Eq. (1) becomes

$$\frac{\partial P(x, t)}{\partial t} = \frac{1}{2} \sigma^2 \frac{\partial^2 P(x, t)}{\partial x^2} + [\kappa(\theta - x)] \frac{\partial P(x, t)}{\partial x}. \quad (A1)$$

It is noted that the evolution of the stochastic variable x is characterized by an Ornstein-Uhlenbeck process. With further changing variables:

$$\tilde{P}(xe^{-\kappa t}, t) = P(x, t), \quad (A2)$$

$\tilde{P}(x, t)$ satisfies the following equation:

$$\frac{\partial \tilde{P}(x, t)}{\partial t} = \frac{1}{2} (e^{-\kappa t} \sigma)^2 \frac{\partial^2 \tilde{P}(x, t)}{\partial x^2} + e^{-\kappa t} \kappa \theta \frac{\partial \tilde{P}(x, t)}{\partial x}. \quad (A3)$$

As noted by Alili *et al.* (2005) and Lo and Hui (2006), there are four representations of analytical nature to approximate the FPT distribution of an Ornstein-Uhlenbeck process. Using the method of images proposed by Lo and Hui (2006), the solution of Eq. (A3), which vanishes at the boundary $h(t) = H \exp\{-e^{\kappa t}[c_2(t) + \beta c_1(t)]\}$ (i.e. an upper band) at any backward time $t \geq 0$,⁹ is given by

$$\tilde{P}(x, t) = \int_{-\infty}^0 dx' [G(x, t; x', t=0) - G(x, t; -x', t=0) e^{-\beta x'}] \tilde{P}(x', t=0), \quad (A4)$$

where

$$G(x, t; x', t=0) = \frac{1}{\sqrt{4\pi c_1(t)}} \exp\left\{-\frac{[x' - x - c_2(t)]^2}{4c_1(t)}\right\}, \quad (A5)$$

⁹ The other three representations are: (i) based upon an eigenfunction expansion involving zeros of the parabolic cylinder functions; (ii) an integral representation involving some special functions; and (iii) in terms of a functional of a three-dimensional Bessel bridge.

with $c_1(t)$ in Eq. (4) and $c_2(t)$ in Eq. (5). Here β is a real number. Eq. (A4) denotes a parametric class of closed-form solutions of Eq. (1) with a moving absorbing boundary whose movement is controlled by the parameter β . As a result, the corresponding FPT distribution conditional to $\tilde{P}(x, t=0)=1$, which is the realignment probability of S at an upper band $h(t)$, is Eq. (2). Similarly, the corresponding risk-neutral realignment probability at a lower band of $h(t)$ under the mean-reverting process is Eq. (8).

As specified in Eq. (9), $\tilde{\sigma}$ is

$$\tilde{\sigma} = \frac{\sigma}{\sqrt{2\kappa t}} \sqrt{1 - e^{-2\kappa t}}. \quad (\text{A6})$$

By putting $y = 2\kappa t$ and square of Eq. (A6), it becomes

$$\tilde{\sigma}^2 = \frac{\sigma^2}{y} (1 - e^{-y}). \quad (\text{A7})$$

The derivative of $\tilde{\sigma}^2$ with respect to y is

$$\frac{d\tilde{\sigma}^2}{dy} = -\frac{\sigma^2}{y^2} (1 - e^{-y}) + \frac{\sigma^2}{y} e^{-y}. \quad (\text{A8})$$

It is noted that

$$\frac{1}{y^2} (1 - e^{-y}) = \frac{e^{-y}}{y} \times \frac{1}{y} (e^y - 1). \quad (\text{A9})$$

By using Taylor expansion for e^y , Eq. (A9) becomes

$$\begin{aligned} \frac{1}{y^2} (1 - e^{-y}) &= \frac{e^{-y}}{y} \times \frac{1}{y} \left(1 + y + \frac{y^2}{2!} + \frac{y^3}{3!} + \dots - 1 \right) \\ &= \frac{e^{-y}}{y} \left(1 + \frac{y}{2!} + \frac{y^2}{3!} + \dots \right), \quad y > 0. \\ &> \frac{e^{-y}}{y} \end{aligned} \quad (\text{A10})$$

From Eq. (A8), because of the inequality in Eq. (A10), we have

$$\frac{d\tilde{\sigma}^2}{dy} < 0. \tag{A11}$$

This shows that $\tilde{\sigma}^2$ is a decreasing function with y and thus with κ .

REFERENCES

- Alili L., Patie P., Pedersen J. L. 2005. Representations of the First Hitting Time Density of an Ornstein-Uhlenbeck Process. *Stochastic Models* **21**: 967-980.
- Anthony M., MacDonald R. 1999. The Width of the Band and Exchange Rate Mean Reversion: Some Further ERM-Based Results. *Journal of International Money and Finance* **18**: 411-28.
- Anthony M., MacDonald R. 1998. On the Mean Reverting Properties of Target Zone Exchange Rates: Some Evidence from the ERM. *European Economic Review* **42**: 1493-523.
- Ball C., Roma A. 1998. Detecting Mean Reversion within reflecting barriers: application to the European Exchange Rate Mechanism. *Applied Mathematical Finance* **5**: 1-15.
- Ball C., Roma A. 1994. Target Zone Modelling and Estimation for European Monetary System Exchange Rates. *Journal of Empirical Finance* **1**: 385-420.
- Ball C., Roma A. 1993. A Jump Diffusion Model for the European Monetary System. *Journal of International Money and Finance* **12**: 475-492.
- Ball C., Torous W. N. 1985. On Jumps in Common Stock Prices and Their Impact on Call Option Pricing. *Journal of Finance* **50**: 155-73.
- Ball C., Torous W. N. 1983. A Simplified Jump Process for Common Stock Returns. *Journal of Financial and Quantitative Analysis* **18**: 53-65.
- Bates D. S. 1996. Dollar Jump Fears, 1984-1992: Distributional Abnormalities Implicit in Currency Futures Options. *Journal of International Money and Finance* **15**: 65-93.
- Bates D. S. 1991. The Crash of '87; Was It Expected? The Evidence from Options Markets. *Journal of Finance* **46**: 1009-1044.
- Campa J. M., Chang P. H. K. 1996. Arbitrage-Based Tests of Target-Zone Credibility: Evidence from ERM Cross-Rate Options. *American Economic Review* **86**: 726-740.
- Kanas A. 1998. Testing for a Unit Root in ERM Exchange Rates in the Presence of Structural Breaks: Evidence from the Boot-Strap. *Applied Economics Letters* **5**: 407-10.
- Krugman P. 1991. Target Zones and Exchange Rate Dynamics. *Quarterly Journal of Economics* **106**: 669-82.
- Lo C. F., Hui C. H. 2006. Computing the First Passage Time Density of a Time-Dependent Ornstein-Uhlenbeck Process to a Moving Boundary. *Applied Mathematics Letters* **19**: 1399-1405.

- MacDonald R. 1988. *Floating Exchange Rates*, Unwin Hyman, London.
- Malz A. M. 1996. Using Option Prices to Estimate Realignment Probabilities in the European Monetary System: the Case of Sterling-Mark. *Journal of International Money and Finance* **15**: 717-48.
- Mizrach B. 1996. Did Option Prices Predict the ERM Crisis? Working Paper no. 96-10, Rutgers University.
- Nieuwland F. G. M. C., Verschoor W. F. C., Wolff C. C. P. 1994. Stochastic Jumps in EMS Exchange Rates. *Journal of International Money and Finance* **13**: 699-727.
- Rose A. K., Svensson L. E. O. 1994. European Exchange Rate Credibility Before the Fall. *European Economics Review* **38**: 1185-216.
- Söderlind P. 2000. Market Expectations in the UK Before and After the ERM Crisis. *Economica* **67**: 1-18.
- Svensson L. E. O. 1993. Assessing Target Zone Credibility. *European Economic Review* **37**: 763-802.
- Svensson L. E. O. 1992. An Interpretation of Recent Research on Exchange Rate Target Zones. *Journal of Economic Perspectives* **6**: 119-44.