ASSESSING THE CREDIBILITY OF THE CONVERTIBILITY ZONE OF THE HONG KONG DOLLAR

Prepared by Laurence Fung and Ip-wing Yu
Research Department

Abstract

The features under the two-sided Convertibility Zone of the Hong Kong dollar resemble in many ways the target zone exchange rate regime in the literature. Following Tronzano et al. (2000), this paper utilises a Bayesian extension of the Svensson (1991) test, which takes into account the exchange rate movement and the differential between domestic and foreign interest rates, to assess the credibility of the Convertibility Zone since it was introduced in May 2005. The empirical evidence suggests that the Hong Kong Monetary Authority has been successful in building a high degree of credibility in maintaining the Convertibility Zone.

JEL Classification Numbers: C11; C15; C22; F31
Keywords: Bayesian Analysis; Credibility; Convertibility Zone; Markov Chain Monte Carlo
Author’s E-Mail Address: 
Laurence_KP_Fung@hkma.gov.hk; Ip-wing_Yu@hkma.gov.hk

The views and analysis expressed in this note are those of the authors, and do not necessarily represent the views of the Hong Kong Monetary Authority.

1 The authors acknowledge the comments from Hans Genberg and Cho-hoi Hui.
Executive Summary:

- The introduction of the three refinements of the Currency Board in May 2005 by the Hong Kong Monetary Authority (HKMA) essentially made the Linked Exchange Rate system a target zone.

- It is essential to have timely assessment of the credibility on the new arrangement. In this paper, we adopt Tronzano et al. (2000) Bayesian framework in the Svensson (1991) test of target zones to measure the time-varying degrees of credibility of the Convertibility Zone of the Hong Kong dollar. The framework takes into account the exchange rate movement and the interest rate differential between the Hong Kong dollar and the US dollar.

- In the Bayesian framework, given the current macroeconomic situation and the past actions of the central bank on maintaining an exchange rate regime, market participants continue to update their prior beliefs on the distribution of the exchange rate and assess to what extent the current exchange rate conforms to a particular set of restrictions associated with a credible target zone.

- The evolution of this probability largely reflects market participants’ perception on the reputation of the HKMA in fulfilling its obligation under the Convertibility Zone and how market participants, based on the past history of the Government’s commitment to the Linked Exchange Rate system, judge the consistency of the exchange rate policy.

- The empirical results show that notwithstanding some declines in credibility when there was repeated speculative inflow to test the strong-side Convertibility Undertaking during the early days of the introduction of the Convertibility Zone, the HKMA manages to attain a high degree of overall credibility regarding its maintenance of the Convertibility Zone. By the end of October 2007, the overall credibility stood at a level of 91% for the 1-month horizon and 62% for the 3-month horizon.

- It should be emphasised that the credibility estimates derived here are based on a particular type of model, which is one of many models in the literature developed to assess the credibility of a target zone. Even though the methodology deployed on the model has the advantage of providing a time-varying measure of the credibility, given the assumptions of the model and the Bayesian nature of the estimation technique, the credibility thus derived should be regarded as an indicative measure only. Furthermore, given that the data do not always satisfy the technical property as required in the estimation, the credibility estimates should be interpreted with caution.
I. INTRODUCTION

The introduction of the three refinements of the Currency Board on 18 May 2005 by the Hong Kong Monetary Authority (HKMA) made some fundamental changes to the operation of the Hong Kong dollar Linked Exchange Rate system. The most important one is the definition of the two-sided Convertibility Zone.\textsuperscript{2} The Convertibility Zone resembles in many ways the target zone exchange rate regime, in which the central bank commits itself to keep the exchange rate of the currency to move within a specified range (the target exchange rate levels).\textsuperscript{3} The strong- and weak-side Convertibility Undertakings of the Convertibility Zone are the upper and lower bounds (the band) of the exchange rate. Within the zone, the HKMA may choose to conduct market operations consistent with Currency Board principles to remove any market anomalies that may arise from time to time.

The success of the Convertibility Zone depends on the reputation of the HKMA in fulfilling its obligation under the Convertibility Undertakings and also whether market participants, based on the past history of the Government’s commitment to the Linked Exchange Rate system, believe the exchange rate band is credible. The credibility reflects the degree of confidence that market participants have on the band given the economy’s current fundamentals and its future development. In the target zone literature, one of the conventional methodologies in assessing the credibility of the band is the test proposed by Svensson (1991).\textsuperscript{4} The specification of the Svensson test takes into account the exchange rate movement and the differential between domestic and foreign interest rates. In the test, the credibility of a target zone is measured by checking whether or not the domestic interest rate is inside the rate-of-return band associated with the lower and upper limits of the exchange rate target zone. The target zone is completely credible if the domestic interest rate is inside the corresponding rate-of-return band or vice versa.\textsuperscript{5}

\textsuperscript{2} The three refinements announced on 18 May 2005 are:

i. the introduction of a strong-side Convertibility Undertaking by the HKMA to buy US dollars from licensed banks at HKD 7.75 per USD;

ii. the shifting of the weak-side Convertibility Undertaking by the HKMA to sell US dollars to licensed banks from HKD 7.80 per USD to HKD 7.85 per USD, so as to achieve symmetry around the Linked Rate of HKD 7.80 per USD. The shift is achieved in a gradual manner over five weeks; and

iii. within the zone defined by the levels of the Convertibility Undertakings (the “Convertibility Zone”), the HKMA may choose to conduct market operations consistent with Currency Board principles.


\textsuperscript{3} The seminal paper by Krugman (1991) explains the exchange rate behaviour of a currency with central parity rate and the upper and lower exchange rate boundaries, the so-called target zone model in the exchange rate literature. The Exchange Rate Mechanism (ERM) of the European Monetary System is one of the most prominent examples of a target zone exchange rate system.

\textsuperscript{4} Other methodologies include the drift-adjustment method proposed by Bertola and Svensson (1993), and the less popular methods of the models of discrete choices in Edin and Vredin (1993) and marginal credibility in Weber (1991). See Ledesma-Rodriguez et al. (2000) for a discussion of these alternative credibility indicators.

\textsuperscript{5} It should be noted that the test by Svensson (1991) can only assess whether a given exchange rate band (target zone) is fully credible or non-credible. See Section II for details of the Svensson test.
Building on the Svensson test, Tronzano et al. (2000) introduced the Bayesian framework to assess the credibility of a target zone. This framework allows market participants to update their information on the band credibility as time evolves. In this paper, we adopt the Bayesian framework and apply it to measure the degree of credibility of the Convertibility Zone of the Hong Kong dollar.

There are several empirical studies to examine the Linked Exchange Rate system before the introduction of the Convertibility Zone in May 2005. For example, Tsang (1999) and Tsang et al. (1999) perform the Svensson test over the period from 1984 to 1996 and from 1984 to 1999 respectively. Tsang (1999) concludes that the Linked Exchange Rate system has been very credible except for the summer of 1984 and late 1987. With a longer sample period, Tsang et al. (1999) find that the Linked Exchange Rate system is not perfect and it may fall victim to various shocks and speculative attacks. Using the variance ratio test and the Augmented Dickey-Fuller test, Tsang et al. (1999) also observe that there is a mean reversion in the Hong Kong dollar exchange rate over the period from 1984 to 1999. They conclude that despite there are signs of non-credibility in the Linked Exchange Rate system at different times, the mean-reverting property of the exchange rate is strong enough to maintain the peg. An alternative work to examine the credibility of the Linked Exchange Rate system is by Kwan et al. (1999). Using data from 1984 to 1999, Kwan et al. (1999) apply the drift-adjustment method and assess the credibility of the system by estimating the devaluation risk (and the implicit probability of devaluation) of the Hong Kong dollar. In some episodes of the financial turmoil during 1997 – 1998, they estimate that the Hong Kong dollar would devalue by 5% within three months with the probability of 60%.

In a more recent work, Genberg et al. (2007) presents an analytical framework to examine the movements of the Hong Kong dollar spot and forward exchange rates, as well as the interest rate spreads between the Hong Kong dollar and the US dollar since the introduction of the Convertibility Zone. They observe that (i) when the spot exchange rate was only several pips away from the strong-side Convertibility Undertaking during the second half of 2005 and the first quarter of 2006, the spot exchange rate immediately depreciated away even though the strong-side Convertibility Undertaking

---

6 The work by Tronzano et al. (2000) extends the classic Svensson’s (1991) test in two important ways. First, rather than assessing the target zone is either fully credible or fully non-credible, the Bayesian framework can provide a measure of the degree of credibility of a given exchange rate band. Second, this measure is time-varying and thus allows the policymakers to have a more timely understanding of the market’s assessment on the credibility of the target zone.

7 It should be emphasised that the credibility measure derived here is based on a particular type of model, which is one of many models in the literature developed to assess the credibility of a target zone. Even though the methodology deployed on the model has the advantage of providing a time-varying measure of the credibility, given the assumptions of the model and the Bayesian nature of the estimation technique, the credibility thus derived should be regarded as an indicative measure only and interpreted with caution.
Undertaking was never triggered; (ii) since January 2007, the forward exchange rates have been either inside the target zone or stayed close to the band; and (iii) the Hong Kong dollar interest rate spread against the US dollar have declined from 150 basis points in late 2006 to around 100 basis points in mid-July 2007. Judged against these observations from the framework, they conclude that the Convertibility Zone has become more credible.

Compared to previous works, the Bayesian framework in this study has the advantage of quantifying a time-varying measure of the credibility of the Convertibility Zone, rather than simply concluding that the zone is either fully credible or non-credible in the Svensson test. Furthermore, this study focuses on the sample period after the introduction of the Convertibility Zone which is, technically, more like the target zone regime than the sample periods used by other empirical studies.

The remainder of this paper is organised as follows. Section II outlines the Svensson test and the incorporation of the Bayesian framework into the test. Section III presents the empirical results based on the Hong Kong dollar exchange rate from 18 May 2005 to 14 June 2007. Concluding remarks are provided in the final section.

II. BAYESIAN ANALYSIS OF CONVERTIBILITY ZONE CREDIBILITY

In the Svensson test, the credibility of a target zone is measured by checking whether or not the domestic interest rate is inside the rate-of-return band associated with the lower and upper limits of the exchange rate target zone. Let $S_t$ be the spot nominal domestic currency per US dollar exchange rate at time $t$, $i_t$ be the annualised domestic interest rate at time $t$ for a period of $\tau$ months, and $i_t^\ast$ be the corresponding US dollar interest rate. Assuming that the interest rates $i_t$ and $i_t^\ast$ are

---

8 This observation corresponds to a bounded exchange rate process examined in Hui and Fong (2007), which argues that the Hong Kong dollar spot exchange rate possesses both the random walk and bounded properties, and together they are consistent with a credible target zone. A bounded process contains a reversion effect that only takes place when the exchange rate is close to the boundaries of the target zone, but not in or around the central parity.

9 Before January 2007, the Hong Kong dollar forward exchange rate was persistently stronger than the strong-side Convertibility Undertaking of 7.75. The authors offer two plausible theories (large market friction and transaction costs) to explain why the 12-month Hong Kong dollar forward exchange rate was persistently stronger than the strong-side Convertibility Undertaking of 7.75 from late 2005 to 2006. (For detailed discussion, see Genberg et al. (2007).)

10 Under the covered interest rate parity condition, the width of 1,000 pips of the Convertibility Zone can at most accommodate a negative interest rate spread of 127 basis points, assuming zero transaction cost.

11 In their studies, Tsang (1999) and Tsang et al. (1999) set a 1% limit on either sides of 7.80 as the range of the target zone. However, the HKMA never defined any upper and lower bounds of the exchange rate before the introduction of the two-sided Convertibility Undertakings.
annualised effective interest rates, the annualised effective rate of return \( R^\tau_t \) on holding one US dollar for a period of \( \tau \) months at time \( t \), in terms of the domestic currency, can be expressed as:\(^{12}\)

\[
R^\tau_t = (1 + i^\tau_t)(S_{t+\tau} / S_t)^{12/\tau} - 1
\]  

(1)

In a credible target zone, the nominal exchange rate is bounded within an exchange rate band, such that:

\[
S \leq S_t \leq \bar{S}
\]

(2)

where \( S \) and \( \bar{S} \) are the lower and upper limits of the band, respectively. Given \( S_t, S_{t+\tau} \) and the exchange rate band, Equation (2) can be translated into a corresponding rate-of-return band as follows:

\[
R^\tau_t \leq R^\tau_t \leq \bar{R}^\tau_t
\]

(3)

where the lower (\( R^\tau_t \)) and the upper (\( \bar{R}^\tau_t \)) rates of return can be derived by replacing \( S_{t+\tau} \) in Equation (1) with \( S \) and \( \bar{S} \), respectively. Under perfect capital mobility, a credible exchange rate band implies that the domestic interest rate (\( i^\tau_t \)) must lie inside the rate-of-return band, i.e., \( R^\tau_t \leq i^\tau_t \leq \bar{R}^\tau_t \). As Svensson (1991) notes, if \( i^\tau_t \), the rate of return of holding domestic currency for a period of \( \tau \) months at time \( t \), is outside the rate-of-return band, the target zone is not credible.\(^{13}\)

\(^{12}\) Note that Equation (1) is based on the annual effective interest rate. The annual interest rates usually quoted in the market are slightly different from the annual effective interest rates. The relationship between the quoted annual interest rate \( r_t \) with tenor \( \tau \) and the annual effective rate \( i^\tau_t \) is:

\[
(1 + i^\tau_t) = (1 + r_t / (12/\tau))^{12/\tau}
\]

In this paper, we use this relationship to obtain the annual effective interest rate from the annual interest rate.

\(^{13}\) Tsang (1999) also performs a variant of the Svensson test to examine the credibility. Under the assumption of uncovered interest parity, Tsang (1999) sets up the following equation:

\[
E_t(S_{t+\tau}) = S_t[(1+i^\tau_t)/(1+i^\tau_{t+\tau})]^{1/\tau}
\]

where \( E_t(S_{t+\tau}) \) is the expectation at time \( t \) of the future value of the spot exchange rate at time \( t+\tau \).

The right-hand-side of the equation is the interest rate differential adjusted for the maturity period \( \tau \). In this case, Tsang defines a credible Linked Exchange Rate system when \( E_t(S_{t+\tau}) \) moves between \( S \) and \( \bar{S} \), the lower and upper limits of the exchange rate band, respectively.
To assess the degree of credibility of an exchange rate band as time evolves, Tronzano et al. (2000) incorporate the Bayesian framework into the Svensson test. The setup of the Bayesian framework is as follows. Suppose market participants have some subjective prior beliefs regarding the existing exchange rate given some policy variables and the past actions of the central bank on maintaining an exchange rate regime. They would use these prior beliefs to form the posterior probability about these policy variables and assess to what extent these policy variables conform to a particular set of restrictions associated with a credible exchange rate policy. The relevant set of policy restrictions associated with a credible target zone regime, following the Svensson test, requires the domestic rate of return \( \hat{R}_t^i - i_t^e \) to fall within the rate-of-return band of \( R_t^i \) and \( \bar{R}_t^i \) as follows:

\[
\begin{align*}
\bar{R}_t^i - i_t^e &\geq 0, \quad (4) \\
R_t^i - i_t^e &\leq 0 \quad (5)
\end{align*}
\]

Equations (4) and (5) represent the conditions for the domestic rate of return (in connection with the exchange rate) to be satisfied if the target zone is credible.

To compute the likelihood for the restrictions in Equations (4) and (5) to be satisfied at each point in time, we have to specify some stochastic processes governing the evolutions of \( \bar{R}_t^i - i_t^e \) and \( R_t^i - i_t^e \). For illustration purpose, if these two processes are characterised as a constant plus a random disturbance term, then the random variables in the set of restrictions in Equations (4) and (5) can be written as:\(^{14}\)

\[
\begin{align*}
z_{u,t} &= \mu_u + \varepsilon_u, \quad \text{(6)} \\
z_{l,t} &= \mu_l + \varepsilon_l, \quad \text{(7)}
\end{align*}
\]

where (i) \( z_{u,t} = \bar{R}_t^i - i_t^e \) is the excess rate of return (excess return) from holding one US dollar at time \( t \) and should be positive when the exchange rate is at the upper limit of the exchange rate band, (ii) \( z_{l,t} = R_t^i - i_t^e \) is the excess return from holding one US dollar at time \( t \) and should be negative when the exchange rate is at the lower side of the exchange rate band, (iii) \( \mu_u \) and \( \mu_l \) are the mean of \( z_{u,t} \) and \( z_{l,t} \) respectively. Under a Bayesian framework, both \( \mu_u \) and \( \mu_l \) can be regarded as random variables on which

\(^{14}\) Amisano and Tronzano (2005) use a Bayesian framework to investigate the inflation target credibility of the European Central Bank. Their setup of restrictions regarding anti-inflationary and anti-deflationary targets is similar to that imposed on the exchange rate target zone.
market participants form their own beliefs given the past information of $z_{u,t}$ and $z_{l,t}$, and (iv) $\epsilon_t$ is an error term which is independently and identically distributed as $N(0, \sigma^2)$.

In these cases, the degree of exchange rate regime credibility with regard to both weak- and strong-side limits at time $t$ can be given as follows:

Weak-side credibility: $\text{Prob}\{ \mu > 0 \mid z_{u,t}, z_{u,t-1}, \ldots, z_{u,1} \} = 1 - \int_{-\infty}^{0} f_u(\mu \mid z_{u,t}, z_{u,t-1}, \ldots, z_{u,1})d\mu_u$ (8)

Strong-side credibility: $\text{Prob}\{ \mu < 0 \mid z_{l,t}, z_{l,t-1}, \ldots, z_{l,1} \} = 1 - \int_{0}^{\infty} f_l(\mu \mid z_{l,t}, z_{l,t-1}, \ldots, z_{l,1})d\mu_l$ (9)

where $f_u(\mu \mid z_{u,t})$ and $f_l(\mu \mid z_{l,t})$ are the beliefs of the market on the distributions of $\mu_u$ and $\mu_l$ respectively.\(^\text{15}\) The distribution of $\mu_u$ (or $\mu_l$) is conditional on the prior beliefs on $\mu_u$ (or $\mu_l$) as well as on the past information of $z_u$ (or $z_l$) at time $t$.

The probabilities given in Equations (8) and (9) rely on the assumption that the time series $z_{u,t}$ and $z_{l,t}$ in Equations (6) and (7) are by themselves serially uncorrelated processes. However, this is not always the case as serial correlation test reveals that the time series $z_{u,t}$ and $z_{l,t}$ can sometimes be characterised by other processes such as the autoregressive process. To take into account the serial correlation property, time series $z_{u,t}$ and $z_{l,t}$ are modified as a first-order autoregressive process and following Tronzano et al (2000), the Equations (6) and (7) can be written as:\(^\text{16}\)

$$z_{u,t} = \theta_u + \rho_u z_{u,t-1} + \epsilon_t, \quad (10)$$
$$z_{l,t} = \theta_l + \rho_l z_{l,t-1} + \epsilon_t, \quad (11)$$

where $\rho_u$ and $\rho_l$ are the autoregressive parameters of $z_{u,t}$ and $z_{l,t}$ respectively.

\(^{15}\) These beliefs are also referred to as the posterior probability density functions in the Bayesian literature.

\(^{16}\) Given the computational complexity in calculating the mean-reversion probability of the process under a higher order autoregressive specification, we model the two series under a first-order autoregressive specification to deal with their persistent behavior.
\( \theta_u/(1-\rho_u) \) and \( \theta_l/(1-\rho_l) \) are the unconditional mean of the corresponding models.\(^{17}\)

Based on the specifications in Equations (10) and (11), the credibility conditions are satisfied whenever \( \theta_u/(1-\rho_u) > 0 \) and \( |\rho_u| < 1 \) for the weak-side case, as well as \( \theta_l/(1-\rho_l) < 0 \) and \( |\rho_l| < 1 \) for the strong-side case. That is, the credibility conditions are satisfied when the unconditional mean of the process \( z_{u,t} \) is greater than zero for the weak-side case (and \( z_{l,t} \) is less than zero for the strong-side case) and the processes are mean-reverting.\(^{18}\) Therefore, the empirical analysis in this case not only focuses on the restrictions of their mean (\( \theta_u/(1-\rho_u) > 0 \) and \( \theta_l/(1-\rho_l) < 0 \)), but also needs to assess on whether the stochastic process governing the evolution of \( z_{u,t} \) and \( z_{l,t} \) is mean-reverting (i.e. \( |\rho_u| < 1 \) and \( |\rho_l| < 1 \)).

In summary, the empirical framework of Tronzano et al. (2000) under the Bayesian framework for assessing the degree of credibility of an exchange rate band requires the computations of the following probabilities regarding different scenarios:

i. **Weak-side credibility:** this credibility is associated with the weak-side excess return \( z_{u,t} \) and is defined as the probability of a positive mean (\( \theta_u/(1-\rho_u) > 0 \)) in Equation (10) at each point in time, which is calculated as:

\[ E(z_{u,t}) = E(\theta_u + \rho_u z_{u,t-1} + \epsilon_t) = E(\theta_u) + \rho_u E(z_{u,t-1}) + E(\epsilon_t) = \theta_u + \rho_u E(z_{u,t-1}) + 0 \]

By assuming stationarity (\( |\rho_u| < 1 \)), as \( E(z_{u,t-1}) = E(z_{u,t}) \), it follows that \( (1-\rho_u)E(z_{u,t}) = \theta_u \) or \( E(z_{u,t}) = \theta_u/(1-\rho_u) \).

\(^{17}\) Taking Equation (10) as an example, we can compute the unconditional mean by taking an expectation on both sides of the equation as follows:

\[ E(z_{u,t}) = E(\theta_u + \rho_u z_{u,t-1} + \epsilon_t) = E(\theta_u) + \rho_u E(z_{u,t-1}) + E(\epsilon_t) \]

\[ = \theta_u + \rho_u E(z_{u,t-1}) + 0 \]

\(^{18}\) If \( |\rho_u| > 1 \) (or \( |\rho_l| > 1 \)), the excess return series will be an explosive process. Consider the weak-side credibility restriction of Equation (10) with \( z_{u,t} > 0 \), if \( z_{u,t} \) is non-stationary or explosive, it may eventually hit the threshold of \( z_{u,t} = 0 \) and go beyond the threshold, implying that the credibility constraint is not satisfied (the weak-side target zone is not fully credible). Therefore, the condition that \( z_{u,t} > 0 \) and \( z_{l,t} < 0 \) is as important as the credibility constraints of \( z_{u,t} > 0 \) and \( z_{l,t} < 0 \).
Prob\{ θ_u / (1 - ρ_u) > 0 \mid z_{u,1}, z_{u,t-1}, ..., z_{u,1} \} =
1 - \int_0^1 f_u (θ_u / (1 - ρ_u) \mid z_{u,1}, z_{u,t-1}, ..., z_{u,1}) dθ_u \quad (12)

where \( f_u (θ_u / (1 - ρ_u) \mid z_{u,1}, ..., z_{u,1}) \) is the probability density function (pdf) for \( θ_u / (1 - ρ_u) \).

ii. **Weak-side mean-reversion probability:** this is associated with the characteristics of the autoregressive process corresponding to the weak-side restriction and is defined as the probability related to the mean-reverting property of the \( z_{u,t} \) series in Equation (10), i.e.

\[
\text{Prob}\{ \rho < 1 \mid z_{u,1}, z_{u,t-1}, ..., z_{u,1} \} = \int \frac{1}{-\infty} \frac{z_{u,1}}{\rho} g_u (\rho, z_{u,1}, z_{u,t-1}, ..., z_{u,1}) d\rho \quad (13)
\]

where \( g_u (\rho, z_{u,1}, ..., z_{u,1}) \) is the pdf for the autoregressive parameter in Equation (10).

iii. **Strong-side credibility:** this credibility is associated with the strong-side excess return \( z_{l,t} \) and is defined as the probability of a negative mean \((θ / (1 - ρ)) < 0\) in Equation (11) at each point in time, which is calculated as:

\[
\text{Prob}\{ θ / (1 - ρ) < 0 \mid z_{l,1}, z_{l,t-1}, ..., z_{l,1} \} =
1 - \int_0^1 f_l (θ / (1 - ρ) \mid z_{l,1}, z_{l,t-1}, ..., z_{l,1}) dθ_l \quad (14)
\]

where \( f_l (θ / (1 - ρ) \mid z_{l,1}, ..., z_{l,1}) \) is the pdf for \( θ / (1 - ρ_l) \).

iv. **Strong-side mean-reversion probability:** this is associated with the characteristics of the autoregressive process corresponding to the strong-side restriction and is defined as the probability related to the mean-reverting property of the \( z_{l,t} \) series in Equation (11), i.e.

\[
\text{Prob}\{ ρ_l < 1 \mid z_{l,1}, z_{l,t-1}, ..., z_{l,1} \} = \int \frac{1}{-\infty} \frac{z_{l,1}}{ρ_l} g_l (ρ_l, z_{l,1}, z_{l,t-1}, ..., z_{l,1}) dρ_l \quad (15)
\]

where \( g_l (ρ_l, z_{l,1}, ..., z_{l,1}) \) is the pdf for the autoregressive parameter in Equation (11).

---

19 The weak-side credibility defined in this study is similar to the overall credibility in Tranzano et al. (2000).

20 The probability density functions (f and g) are the posterior ones in the Bayesian literature. For more technical discussion on these two probabilities, see Tronzano et al. (2000).
For the empirical analysis of the Convertibility Zone in this study, the upper limit (weak-side) of the exchange rate band \( (\bar{S}) \) is the same as the weak-side Convertibility Undertaking, while the lower limit (strong-side) of the band \( (\bar{S}) \) is equivalent to the strong-side Convertibility Undertaking. In the estimations, we follow the Bayesian approach based on the Markov Chain Monte Carlo (MCMC) posterior simulation in Amisano and Tronzano (2005). The MCMC method is useful in performing Bayesian inference as it provides a way to take random samples from a posterior distribution.\(^{21}\) These simulated samples are subsequently used to derive the summary statistics of the posterior distribution of a statistical model, such as posterior interval, means, medians and standard deviations etc, as well as to assess posterior probability scenarios similar to Equations (12) to (15). The MCMC posterior simulation is conducted using the MCMCpack which is a formal R package designed for performing inference via the MCMC simulation from the posterior distributions of various models, including the linear regression model with Gaussian errors similar to Equations (10) and (11).\(^{22}\)

### III. Empirical Results

The framework outlined in section II is employed to assess the credibility of the Convertibility Zone since its introduction on 18 May 2005. Chart 1 presents the nominal Hong Kong dollar per US dollar exchange rate and the two-sided Convertibility Undertakings until end-October 2007. The chart shows that the exchange rate was very close to the strong-side Convertibility Undertaking during the period from late 2005 to May 2006. From mid-January 2007 to early August 2007, the Hong Kong dollar exchange rate stayed at the weak side of the zone. It has shifted to the strong side of the zone since late August 2007.

---

\(^{21}\) The MCMC simulation is based on the Gibbs sampling method. The aim of this method is to draw samples from the distributions of the two excess return series respectively (the Monte Carlo part). The Markov chain is defined in such a way that each sample depends on the previous draw so that after a large number of iterations, the distribution of the samples drawn will converge to the posterior distribution of the respective excess return series. In the empirical analysis, 10,000 MCMC iterations are performed. For more details, see Koop (2003).

\(^{22}\) See Martin and Quinn (2005) and Martin and Quinn (2006) for details. R is a free software environment for statistical computing and graphics. The posterior samples returned by the MCMC simulation can be summarised and manipulated by the CODA (Convergence Diagnostics and Output Analysis) package, which is another R package used for analysing the output of the MCMC simulation. See Plummer et al. (2006) on the CODA package.
The nominal excess returns on holding one US dollar over the domestic rates of return for one month, three months and twelve months are examined. The domestic and foreign interest rates are the 1-month, 3-month and 12-month Hong Kong dollar interbank offered rates (HIBORs) and the corresponding US dollar London interbank offered rates (LIBORs) with the same maturities, respectively. 23 Using Equation (1), all rates of return (including the rate-of-return band) on holding one US dollar in terms of the Hong Kong dollar are expressed on an annualised basis. The excess return series $z_{u,t}$ and $z_{i,t}$ with different maturities are shown in Chart 2.

23 The HIBORs and LIBORs are converted into annual effective interest rates before applying in Equation (1). See Footnote 12 for the relationship between the nominal annual rate and the annual effective rate.
Chart 2. Rate-of-Return Restrictions on the Convertibility Zone

(a) Weak-side Convertibility Zone \( z_{w,t} = \bar{R}_t - \bar{i}_t \geq 0 \)

(b) Strong-side Convertibility Zone \( z_{s,t} = \bar{R}_t - \bar{i}_t \leq 0 \)

Source: HKMA estimates.
From the graphs in Chart 2, the weak-side excess return series \((z_{u,t})\) of different horizons appear to be consistent with the restrictions implied by the Convertibility Zone (i.e. \(z_{u,t} > 0\)). The strong-side excess return series \((z_{l,t})\), however, are not always the case. For example, as shown in Chart 2(b), in some occasions (9 – 24 January 2006, 4 – 15 May 2006 and 22 – 31 October 2007) when the Hong Kong dollar exchange rate was close to the strong-side Convertibility Undertaking of 7.75, the 1-month, 3-month and 12-month strong-side excess return series were all larger than zero. In fact, the 12-month strong-side excess return was consistently larger than zero over a long period of time from mid-October 2005 to mid-April 2007. These are not consistent with the restriction implied by the strong-side Convertibility Zone (i.e. \(z_{l,t} < 0\)). As the exchange rate drifted to the strong side of the zone again in early August 2007, the strong-side excess return moved towards the zero level again.

Following the Bayesian approach discussed in Section II, the posterior probabilities associated with the probability given in Equations (12) to (15) with respect to the credibility of the Convertibility Zone and the mean-reversion probability of the excess return series (weak-side and strong-side) are computed by the recursive MCMC simulation process. Chart 3 illustrates the credibility estimated recursively at each point in time for different horizons.

24 There are some occasions when the excess return is close to or greater than zero even though the exchange rate is not close to the strong-side Convertibility Undertaking. This happened, for example, from early December 2006 to early January 2007 when the excess return of the 3-month horizon was greater than zero.

25 Unlike Tronzano et al. (2000) in which they use the first 20 observations to initialise their recursive process, this study uses the first 60 observations to initialise the recursive MCMC simulation process. This is to avoid the possible confusion of the gradual shift over a five-week period of the weak-side Convertibility Undertaking from 7.80 to 7.85 at the beginning of the study period with a policy change in the lower bound of the Convertibility Zone. The recursive process is performed by expanding the sample size for the MCMC simulation by one observation until the full sample is covered. Each recursive process produces 10,000 simulations. The initial priors for \((\theta_{u,t}, \rho_{u,t})\) are set at (0.0004, 0.996), while those for \((\theta_{l,t}, \rho_{l,t})\) are (-0.0002, 0.998). These priors are the estimated coefficients obtained from a linear regression over the full sample period of the autoregressive processes of \(z_{u,t}\) and \(z_{l,t}\), respectively. The priors on the precision error term are set at 0.9. Precision is defined as the inverse of the prior’s variance. The larger the precision value, the smaller the prior’s variance is. Robustness tests are conducted and it is found that the MCMC simulation results are insensitive to the values of the initial priors and just slightly affected by the setting of the precision error term.
Chart 3. Credibility of Convertibility Zone

(a) Weak-side Convertibility Credibility (Prob{ \( \theta_u / (1 - \rho_u) > 0 \) | \( z_{u,t}, z_{u,t-1}, \ldots, z_{u,1} \)})

(b) Strong-side Convertibility Credibility (Prob{ \( \theta_i / (1 - \rho_i) < 0 \) | \( z_{i,t}, z_{i,t-1}, \ldots, z_{i,1} \)})

Source: HKMA estimates.
Chart 3(a) shows that the weak-side convertibility credibility was found to be almost 100% with the one-month horizon during 2006 as the weak-side Convertibility Undertaking was never challenged over the period. The credibility declines (as indicated by the lower probability) as the horizon lengthens. The weakening of the exchange rate during the period between early January 2007 and early August 2007 provides an opportunity for assessing the weak-side convertibility credibility. As shown in Chart 3(a), the weak-side credibility experienced a decline over that period as the exchange rate weakened and drifted further away from the 7.80 level (see also Chart 1). The credibility fell to a low of 64% for a 1-month horizon, 57% for a 3-month horizon and 60% for a 12-month horizon on 6 August 2007. As the exchange rate strengthened afterwards, the weak-side credibility rose rapidly and reached 76% to 91% for different horizons by the end of October 2007.

The evolution of the strong-side convertibility credibility in Chart 3(b) is comparably more informative for assessing the market perception on the reputation of the HKMA in fulfilling its obligation to the Convertibility Undertakings and the Government’s commitment to the Linked Exchange Rate system, as the Hong Kong dollar exchange rate had been very close to the strong-side Convertibility Undertaking during the period between October 2005 and early June 2006, and recently in September and October 2007.26 Taking the case of the 1-month strong-side excess return series for an example, the strong-side credibility for a 1-month horizon in Chart 3(b) dropped from almost 100% to a level as low as 70% on 12 January 2006. It fell again on 11 May 2006 when the exchange rate approached the strong-side Convertibility Undertaking, but this time the drop was less severe (at 76%) when compared to that on 12 January 2006. Then the credibility rose gradually to above 90% for a 1-month horizon until the end of 2006. This may reflect the increase in market participants’ confidence in the commitment of the Government in maintaining the Convertibility Zone. Nevertheless, similar to the weak-side credibility, the strong-side credibility for a 1-month horizon had also declined since early 2007 and moved between 53% and 75% for most of the first seven months of 2007. In early August 2007, the declining trend of the credibility for a 1-month horizon reversed. It rose to 80% by the end of September 2007 while the exchange rate shifted swiftly towards the strong-side Convertibility Undertaking. However, as the exchange rate continued to strengthen during October and reached the strong-side Convertibility Undertaking, the 1-month credibility dropped gradually and settled at 66% by the end of October 2007.

26 It should be noted that even when the exchange rate was under appreciating pressure and stayed close to the strong-side Convertibility Undertaking during the period between October 2005 and early June 2006, the Convertibility Undertaking was never triggered. However, it was triggered in October 2007 when the exchange rate reached the 7.75 level.
The strong-side credibility for a 3-month horizon exhibits a slightly different pattern compared to the one for the 1-month horizon. The credibility for the 3-month horizon dropped to 44% in mid-January 2006 when the exchange rate was close to the strong-side Convertibility Undertaking. In contrast to the rebound in the 1-month horizon, the credibility for the 3-month horizon fell further to as low as 38% on 11 May 2006 when the exchange rate came close to the strong-side Convertibility Undertaking again. The 3-month credibility has been rising afterwards and staying mostly between 50% and 70% since the final quarter of 2006. Even though the strong-side excess return series was again greater than zero during the period between early December 2006 and early January 2007, the credibility for the 3-month horizon at that time dropped only briefly to about 57%, a level significantly improved from the previous episodes. Similarly, when the exchange rate shifted swiftly towards the strong-side Convertibility Undertaking by the end of October 2007, the credibility for the 3-month horizon was around 63%, a level higher than all the previous episodes. However, the credibility estimates for the 12-month horizon has consistently been at a low level of less than 20% since January 2006. This implies a high degree of uncertainty perceived by market participants regarding the maintenance of the strong-side Convertibility Undertaking at 7.75 over a long horizon.\(^\text{27}\) Even though the HKMA conducted market operation on 23 October and the strong-side Convertibility Undertaking was triggered on 26 and 31 October 2007, the strong-side credibility estimates show that these actions only had minimal impact on the strong-side credibility measures under the various horizons.

The credibility in Chart 3 illustrates two interesting observations about this Bayesian measure. First, the evolution of the credibility over different episodes when the exchange rate was close to the strong-side Convertibility Undertaking highlights how market participants assess the credibility. Based on the past history of the Government’s commitment to the Linked Exchange Rate system, it is shown that market participants appear to build up their confidence on the policy of the Convertibility Zone over time, as reflected by the increase in the 3-month credibility from 38% in mid-May 2006 to 63% in end-October 2007 when the Hong Kong dollar was traded very close to the strong-side

\(^{27}\) As discussed in Footnote 7, the credibility measure is model specific and thus the estimates should be interpreted with caution. In particular, the low level of credibility under the 12-month horizon may not necessarily imply that the Convertibility Zone is not sustainable or under severe threat in one year’s time. In fact, similar phenomena are also observed in the HKD forward market. Even if the forward rates lie outside the Convertibility Zone, this does not imply the Zone is not sustainable because opportunity costs and the inherent risks involved in conducting longer-term arbitrage trades may discourage market participants from arbitraging away the potential profits arising from the mis-pricing of the forward exchange rates.
Convertibility Undertaking.\textsuperscript{28} Second, when the Hong Kong dollar first moved to the weak side of the Convertibility Zone in early January 2007 since the implementation of the three refinements, both the strong- and the weak-side credibility fell. The exchange rate movement within the weak side of the zone under the refined arrangement is totally new to market participants. Without any history to rely on, this may affect the Bayesian measure of credibility as perceived by market participants.\textsuperscript{29} As it may take time for market participants getting used to the exchange rate movement within the weak side of the zone, the credibility measures had relatively large swings in response to the market changes in the following months (up to early August 2007).

As discussed in Section II, the credibility conditions also hinge on whether the excess return series (\(z_{u,t}\) and \(z_{l,t}\)) is a mean-reverting process or not. We assess the mean-reverting property of the time series \(z_{u,t}\) and \(z_{l,t}\) by examining the probabilities of \(|\rho_u|\) and \(|\rho_l|\) being less than one. The graphs in Chart 4 present the posterior probabilities of \(|\rho_u|<1\) and \(|\rho_l|<1\) respectively for different horizons, as calculated by the probability scenarios of Equations (13) and (15).

\textsuperscript{28} Similar observations also appear in Tronzano et al. (2000) when they examine the weak-side credibility of the French franc (against the Deutschemark). The weak-side excess return of the French franc dropped close to zero several times over the period from January 1987 to August 1993. Nevertheless, the fall in weak-side credibility was less severe each time and the credibility even managed to pick up to a higher level after each drop. This demonstrates how market participants, on the basis of their subjective prior beliefs and the past actions taken by the central bank, build up their confidence on the exchange rate regime over time.

\textsuperscript{29} In fact, this is reflected in the fall in the strong-side credibility even though the exchange rate was traded in the weak side of the zone.
Chart 4. Mean-reversion Probability of Excess Return

(a) Weak-side Posterior Probability of $\rho_a < 1$

(b) Strong-side Posterior Probability of $|\rho_i| < 1$

Source: HKMA estimates.
As shown in Chart 4, the mean-reversion probabilities for the weak-side and the strong-side excess return series are very similar. Therefore, our discussion will focus on the excess return for the weak-side case. As illustrated in Chart 4(a), the posterior probability of $|\rho_r| < 1$ is almost 100% for the 1-month horizon up to mid-January 2007 when the probability begins to decline. This suggests that for a long period until January 2007, the weak-side 1-month excess return series $z_{u,t}$ can be characterised as a mean-reverting process. This mean-reverting property has weakened since mid-January 2007 as the probability declines to around 70% in early August 2007. For the 3-month and 12-month horizons, the probabilities of the $z_{u,t}$ being a mean-reverting process are lower when compared to that obtained from the 1-month horizon. Nonetheless, the probability for the 3-month horizon has caught up with the 1-month probability since January 2007 and by the end of October 2007, the mean-reversion probabilities reached 91%, 88% and 79% for the 1-month, 3-month and 12-month horizon respectively.

Finally, the robustness of the MCMC method is checked by initialising the recursive estimation process with different priors and precision error term assumptions. It is found that the credibility and the mean-reversion probability estimates are not sensitive to the setting of the initial priors. On the other hand, the precision error term assumption does have some minor impacts on the results of these two estimates, especially during the early stages of the MCMC simulation. Nonetheless, the impact is not significant unless the precision error term is set at a very extreme level. To illustrate this, Chart 5 presents the posterior probabilities of strong-side convertibility credibility with the one-month horizon, using the same initial priors but different precision error term assumptions. The chart shows that the patterns of the estimated posterior probabilities with different precision assumptions are very similar. Furthermore, it is noted that the estimated posterior probabilities converge gradually at the later part of the simulation. The speed of convergence may be subject to how fast market participants sharpen their uncertainty surrounding the implementation of the Convertibility Zone and evaluate the consistency of the exchange rate policy as time evolves.

30 The probabilities for all the return series being a mean-reverting process are greater than 0.5 during the study period. This suggests that these series are likely to be stationary.
31 Precision is defined as the inverse of the prior’s variance. The larger the precision value, the smaller is the prior’s variance.
IV. CONCLUDING REMARKS

This paper applies the Bayesian framework to the Svensson test for assessing the credibility of the Convertibility Zone of the Hong Kong dollar. Rather than simply addressing whether the Convertibility Zone is either fully credible or non-credible, the Bayesian approach provides time-varying estimates about the evolution of the degree of credibility of the Convertibility Zone at each point in time. The evolution of the credibility of the Convertibility Zone reflects market participants’ perception on the reputation of the HKMA in fulfilling its obligation under the Convertibility Undertakings and how market participants, based on the past history of the Government’s commitment to the Linked Exchange Rate system, judge the consistency of the exchange rate policy.

The empirical evidence shows that the degree of credibility of the Convertibility Zone varies according to different time horizons. For the one-month and three-month horizons the credibility of the Convertibility Zone was quite high during the early days of the implementation, despite several challenges to the strong side of the Convertibility Zone in the latter half of 2005 and in 2006 which led to some declines in the strong-side credibility. Notwithstanding these declines, the credibility has picked up gradually and stayed at a high level of over 70%. From January 2007 to early August 2007 when the exchange rate weakened further from the central parity of 7.8 level, the credibility of the Convertibility Zone experienced another decline but maintained at a
level of 50% to 86% for the two horizons. While the exchange rate strengthened again in mid-August to October 2007, both strong- and weak-side credibilities picked up and remained at a high level of 62% to 91% by the end of October 2007 for the short horizons. For the 12-month horizon, the weak-side credibility had reached a level of 72%, representing an increase in credibility on the weak-side Convertibility Undertaking over a long horizon. On the other hand, the credibility on the strong-side Convertibility Undertaking over a long horizon remains low. Nevertheless, such a low level of credibility under a long horizon may not necessarily imply that the Convertibility Zone is not sustainable or under severe threat. Furthermore, estimates of the mean-reversion probability reveal that the two excess return series associated with the Convertibility Zone are likely to be stationary. As of end-October 2007, the probabilities of them being a mean-reverting process are around 78% to 92% for different horizons. However, as the mean-reversion property is not always satisfied and given that the measures are derived from a particular model, the estimates of the credibility of the Convertibility Zone should be interpreted with caution and thus regarded as an indicative measure only.

Overall, the empirical results show that the HKMA manages to achieve a reasonable degree of credibility on maintaining the Convertibility Zone. The shift of the Hong Kong dollar exchange rate in the first half of 2007 from the strong side of the Convertibility Zone to the weak side provides an opportunity for assessing how confident market participants are on the Government’s exchange rate policy. This experience is essential in depicting a complete picture of the credibility of the Convertibility Zone. As the probability estimates capture market participants’ perception on the credibility of the Convertibility Zone, they provide additional information to the HKMA for monitoring the Hong Kong dollar exchange rate.

\[32\] See Footnote 27 for details.
REFERENCES


