



IS THE HONG KONG DOLLAR EXCHANGE RATE “BOUNDED” IN THE CONVERTIBILITY ZONE?

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Abstract

The empirical results show that after the introduction of the three refinements to the Linked Exchange Rate system in May 2005 the Hong Kong dollar follows a bounded process that is consistent with a fully credible exchange rate band. The bounded process will limit the movements of the exchange rate to between the strong- and weak-side limits because its variance vanishes at the Convertibility Undertakings making it inaccessible to the limits. The Hong Kong dollar does not show any strong tendency to revert towards the centre of the Convertibility Zone. This is perhaps not surprising as there have been no interventions in the foreign exchange market since May 2005. There may be few forces or incentives for market participants to drive the exchange rate towards 7.80.

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Executive Summary

- *Policy literature has argued that for a target zone to be credible, the currency exchange rate should have a specific statistical property – a mean-reversion. There is however no strong empirical evidence to show that the Hong Kong dollar possesses such a property after the introduction of the three refinements in May 2005 – which essentially made the Linked Exchange Rate system a target zone. Instead it follows a bounded process.*
- *A bounded process for the currency in a credible target zone can be attributed to potential central bank intervention and the ability of the central bank to defend the band. As long as the exchange rate is “well within” the band, market participants behave as if they are in a comfort zone and do not feel particularly compelled or encouraged to pull the exchange rate towards the central parity. However, when the exchange rate moves closer to the boundary, the market may anticipate an intervention and push the rate back. Simply put, a bounded process contains a reversion effect that only occurs close to the boundaries of the zone, but not in or around the middle. Statistically, it is therefore possible for the random walk and bounded properties to co-exist, and together they are consistent with a credible system.*
- *To test this hypothesis, we use the maximum-likelihood technique to estimate the parameters of the bounded process using the daily Hong Kong dollar spot rates from 18 May 2005 to 29 August 2007. The results show that the process is adequately fitted and all parameters are statistically significant. The volatility is at the maximum (about HK\$0.048 annually or HK\$0.003 daily) near the central parity of 7.8 and approaches zero at the strong-and weak-side limits.*

I. INTRODUCTION

It has been argued in the policy literature that for an exchange rate target zone, the mean-reverting property of the currency is a sign that the market judges the zone to be credible. The introduction of the three refinements to the currency board in May 2005 has essentially made the Linked Exchange Rate system a target zone. Despite the relatively short history, this gives us an opportunity to study the properties of the Hong Kong dollar under the new arrangement and, in particular, whether the currency follows a mean-reverting process.

The driving force behind the mean-reverting property has been widely debated.² Some attribute it to central bank intervention within the target zone.³ Others argue that credibility induces “stability speculation” by market participants, producing forces to pull the exchange rate back to the central parity whenever it drifts away too much from it. Many empirical studies attempted to investigate this theoretical prediction by examining the time-series properties of the European currencies in the era of the Exchange Rate Mechanism. Unfortunately, the results of these studies were mixed.⁴ Some of the currencies were found to follow a random-walk process.

A school of thought argues that the fact that the exchange rate behaves as a random-walk is not necessarily inconsistent with a credible target zone, provided that it is a bounded process.⁵ There are many reasons for the exchange rate to have such a property, the most obvious one being potential central bank intervention and the ability of the central bank to defend the band. As long as the exchange rate is “well within” the band, market participants behave as if they are in a comfort zone and do not feel particularly compelled or encouraged to pull the exchange rate towards the central parity. However, when the exchange rate moves closer to the boundary, the market may anticipate an intervention and push the rate back suddenly. Simply put, a bounded process contains a reversion effect that only occurs close to the boundaries of the zone but not in or around the middle. Statistically, it is thus possible for the random walk and bounded properties to co-exist, and together they are consistent with a credible system.

² For example, Krugman (1991), Svensson (1993), Rose and Svensson (1994), and Anthony and MacDonald (1998).

³ But the exchange rate may not exhibit such a property if intervention occurs only at the limits of the target zone.

⁴ For example, Ball and Roma (1994), Svensson (1993), Rose and Svensson (1994), Nieuwland et al. (1994), Anthony and MacDonald (1998, 1999), and Kanas (1998).

⁵ See Nicolau (2002).

This paper examines the dynamics of the daily Hong Kong dollar spot rate since the introduction of the three refinements in May 2005 and investigates whether its dynamics follows a mean-reverting process or a bounded process. The dynamics is empirically tested in the following two sections. Section IV presents the analytical analysis of the bounded process. The final section concludes the findings.

II. MEAN-REVERTING PROCESS

We study the dynamics of the daily Hong Kong dollar spot rate from 18 May 2005 to 29 August 2007 based on the augmented Dickey-Fuller (ADF) test and the variance-ratio (VR) test to determine whether the time series exhibit stationary and mean-reverting properties.^{6, 7} Table 1 shows the mean, ADF test and VR test statistics of the spot rates. Under the null hypothesis of the presence of a unit root, the ADF test statistic for the level accepts the null hypothesis at the 5% level of significance. This suggests that the time series is not stationary in level. However, the VR test rejects the hypotheses of random walk and mean aversion only for a 5-day horizon at a 10% level of significance. For other time horizons, the VR test shows that the time series does not exhibit any mean-reverting property in level. As a result, there is limited evidence to support that the time series exhibits a mean-reverting property in level.⁸

The results show that there does not exist a strong driving force in the Hong Kong dollar exchange rate dynamics to pull the rate towards the central parity.

⁶ The basic characteristics of a typical stationary time series are that the series fluctuates around a constant long-term mean, and its variance and covariance are finite and time-invariant. One of the commonly used unit root (stationarity) tests is the ADF test. The ADF test constructs a parametric correction for higher-order correlation by assuming that a time series process (y) follows an AR(p) process and adds p lagged differences of y to the test regression of:

$$\Delta y_t = g + a y_{t-1} + b_1 \Delta y_{t-1} + \dots + b_p \Delta y_{t-p} + e_t$$

We test the null hypothesis (the presence of a unit root) of $a = 0$ against the alternative hypothesis of $a < 0$. The test is evaluated using the conventional t -ratio for a (t_a) such that:

$$t_a = \hat{a} / (se(\hat{a})),$$

where $\hat{a}$ is the estimate of a and $se(\hat{a})$ is the coefficient standard error.

⁷ The VR at lag K is defined as the ratio of the variance of the K -period return to the variance of the one-period return divided by K . A unity VR means that the time series is a random process. The time series is considered to be mean reverting if the VR is significantly smaller than one. If the VR is significantly larger than one, the time series is considered to be under a mean aversion. Further details are in Kim et al. (1991).

⁸ The rejection of stationarity does not necessarily mean the rejection of mean reversion. For instance, a time series process which is a sine-wave signal with white noise may not pass the test of stationarity. But it may pass the test of mean reversion because the time series process always goes back to the mean regularly.

Table 1. Statistics of HKD exchange rate (18 May 2005 – 29 August 2007)

	<u>Spot rate</u>	
	Level	Daily change
Mean	7.7807	1.48 x 10 ⁻⁵
ADF test statistics ¹	-0.9950	-24.9041**
Variance ratio test statistics ²	-1.7491*	-9.2879**
Observations	596	596

Notes: 1. ADF statistics are from the ADF unit root test. The critical ADF value at the 5% significance level is -2.86. ** and * indicate significant at the 5% and 10% levels respectively.

2. The variance ratio test statistic is the difference between the variance ratio at a 5-day horizon and unity over the standard deviation. Asymptotically, it converges to standard normal distribution. At 5% and 10% levels of significance, the critical values are -1.96. and -1.65 respectively.

III. BOUNDED PROCESS

Next we test whether the Hong Kong dollar spot exchange rate behaves like a bounded process in the Linked Exchange Rate system. To do so we hypothesise that the exchange rate follows a random walk most of the time, but that it tends to bounce off the Convertibility Undertaking (CU) levels whenever it comes close to them. Formally, we adopt the following specification of the bounded process:

$$dx_t = k(\ln q - \ln x_t)x_t dt + \tilde{s}_t(-\ln x_t)^g x_t dz_t \quad (1)$$

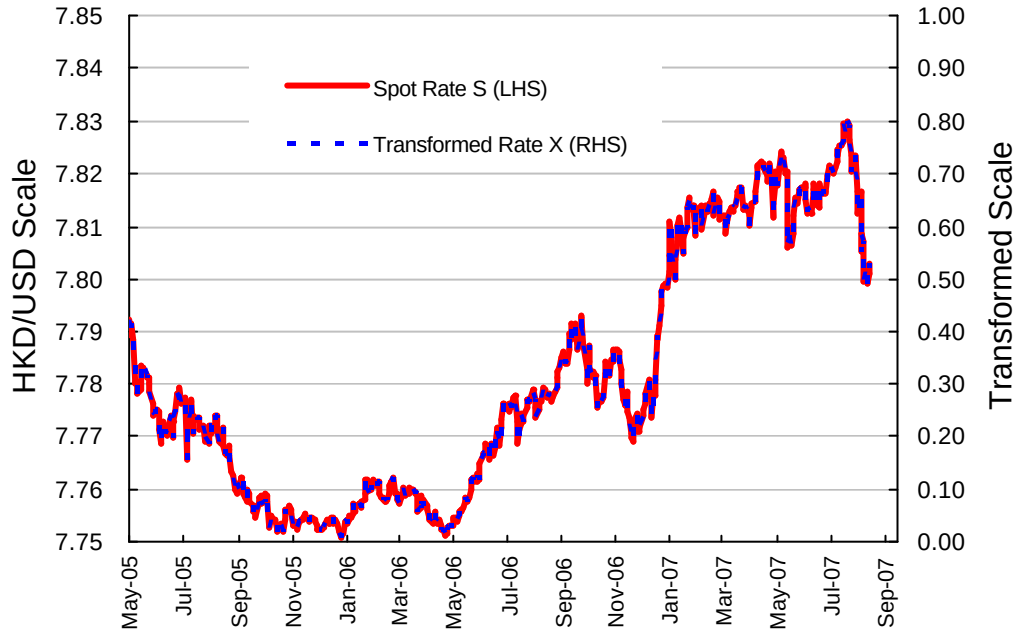
where the Hong Kong dollar exchange rate S is transformed into x with a scale from zero to one:

$$x_t = \frac{S_t - 7.75}{7.85 - 7.75}, \quad (2)$$

$\tilde{s}^2(-\ln x)^{2g} x^2$ is the variance that depends on the level of S , g is a real number, dz is a standard random process and k determines the speed of adjustment towards a

mean of q .⁹ It is noted that x is between zero and one as S is between 7.75 and 7.85 (see Chart 1). The mean-reverting drift in the exchange rate dynamics makes the specification more general.¹⁰ If the coefficient g is positive, the process will be bounded, because when S gets very close to either of the CUs (7.75 and 7.85), the variance of the exchange rate vanishes.¹¹ This in turn implies that the exchange rate is unlikely to touch either CU. The larger is g , the more sensitivity is the variance of the exchange rate process to the distance from the target zone limit.

Chart 1. Spot exchange rate and transformed rate of HKD



To test the above hypothesis, the continuous-time model specified in equation (1) is estimated using a discrete-time econometric specification as:¹²

$$\frac{x_t - x_{t-1}}{x_{t-1}} = a(f - \ln x_{t-1}) + e_t \quad (3)$$

$$E(e_t) = 0, \quad E(e_t^2 | x) = s_t^2 = \tilde{s}^2 (-\ln x_{t-1})^{2g}. \quad (4)$$

⁹ The specification of the bounded process is generalised from the specification used in Lo (2007).

¹⁰ The result of the VR test in the previous section shows that a mean-reverting drift could be present.

¹¹ When S converges to 7.75, $(-\ln x)^y x$ will converge to zero. On the other hand, when S is 7.85, x is 1 and $\ln x$ is zero, hence $(-\ln x)^y x$ is zero.

¹² This discrete-time econometric specification is used for testing the continuous-time models of US interest rates in Bali (1999).

The parameter a and f have the same meaning as k and $\ln q$ in equation (1). In addition, the variance is modelled by the conditional heteroskedasticity through the lagged rate of $(-\ln x_t)^g$. The parameters in equations (3) and (4) of the discrete-time econometric model are estimated using the maximum likelihood technique.

The estimation results are presented in Table 2. The Ljung-Box test statistics suggest that the model is adequately fitted because the standardised residuals are not serial correlated and not heteroskedastic.^{13, 14} The parameters for the drift, a and f , are estimated to be 0.01 and -0.76 respectively, while the parameter for the variance, $\tilde{\sigma}$ and g , are 0.08 and 0.86 respectively. The coefficients are all statistically significant. The annualised volatility in the exchange rate at different levels under the bounded process is shown in Chart 2. The volatility is at the maximum (about HK\$0.048 annually or HK\$0.003 daily) near the central parity of 7.80 and approaches zero at the strong- and weak-side limits.¹⁵

Table 2. Maximum likelihood estimations of discrete-time econometric model

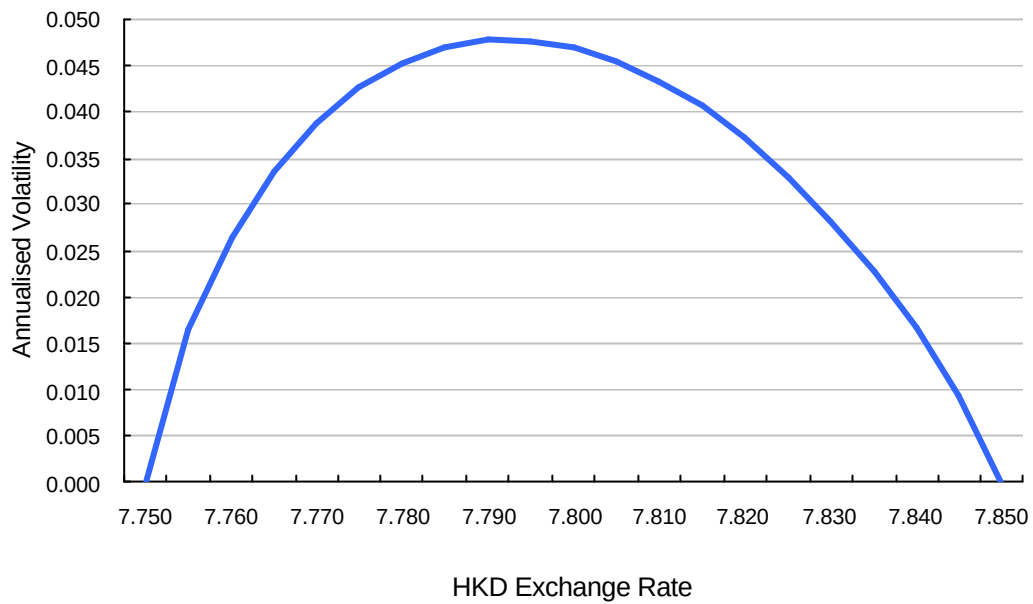
Parameter Variable	Estimated Coefficients	Standard Error	P-value
α	0.0111*	0.0049	0.0240
f	-0.7580**	0.2778	0.0064
σ	0.0814**	0.0015	0.0000
γ	0.8612**	0.0168	0.0000
Log likelihood	536.6340		
Ljung-Box test on standardized residuals up to lag 24	31.0360		0.153
Ljung-Box test on squared standardized residuals up to lag 24	26.0870		0.349
Observations		596	

Notes: ** and * denotes coefficient significant at the 1% and 5% levels respectively.

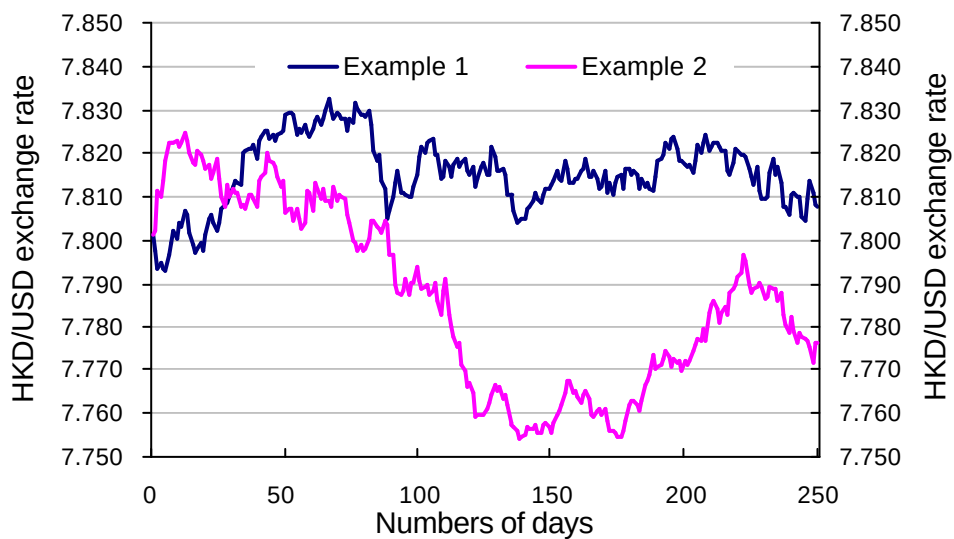
¹³ The Ljung-Box test, similar to the Portmanteau test, identifies whether the autocorrelations among data are jointly zero up to a specified lag. Compared with the Portmanteau test, it is more powerful. Accepting the null hypothesis of the test means that the data is not serial correlated.

¹⁴ The standardised residual is the residual divided by the estimated volatility. If the model fits well, the standardised residuals will be serial uncorrelated and homoskedastic. The homoskedasticity can be checked by the Ljung-Box test on the jointly zero-autocorrelation among the squared standardised residuals. Accepting the null hypothesis of the test suggests that the standardised residuals are homoskedastic. See McLeod and Li (1983) for details.

¹⁵ The changes are not symmetric at the central parity because most of the time the exchange rates had stayed in the strong-side Convertibility Zone since the introduction of the three refinements.

Chart 2. Annualised volatility in HKD exchange rate at different levels

To illustrate the dynamics of the exchange rate, Chart 3 shows two simulated paths of the Hong Kong dollar for 250 days under the bounded process with the estimated parameters. The paths are selected on the basis of the 25% and 75% of the end-points of 10,000 simulations. The movements of the exchange rate are more volatile at the central parity and less volatile near the strong- and weak-side limits.

Chart 3. Two selected simulated paths of HKD exchange rate

IV. ANALYTICAL ANALYSIS OF BOUNDED PROCESS

In this section, the bounded process with a special case of $g = 0.5$ is studied analytically.¹⁶ We let the weak-side limit be S_U and the strong-side limit be S_L . With the variable of $x = (S - S_L)/(S_U - S_L)$, the interest rate differential Δr in relation to the domestic (Hong Kong dollar) and foreign (US dollar) interest rates (i.e. r and r^*) respectively is expressed as

$$\Delta r = \left(1 + \frac{S_L}{(S_U - S_L)x} \right) (r - r^*) \quad (5)$$

Under a no-arbitrage environment, equation (1) becomes

$$dx = [k(\ln q - \ln x) + \Delta r]xdt + sx\sqrt{-\ln x}dz, \quad 0 < x < 1 \quad (6)$$

With changing variables of

$$\begin{aligned} y &= \sqrt{-\ln x} \\ &= \sqrt{-\ln \left(\frac{S - S_L}{S_U - S_L} \right)}, \quad 0 < y < \infty \end{aligned} \quad (7)$$

where $y \rightarrow \infty$ corresponds to $S \rightarrow S_L$ and $y \rightarrow 0$ corresponds to $S \rightarrow S_U$, equation (6) becomes

$$dy = \frac{1}{2} \left(\frac{n}{y} - xy \right) dt - \frac{s}{2} dz \quad (8)$$

where $x = k - \frac{1}{2}s^2$ and $n = -\left[k \ln q + \Delta r + \frac{1}{4}s^2 \right]$. The equation governing the transition probability density function $u(y, t)$ associated with the stochastic process specified in equation (8) is

$$\frac{\partial u(y, t)}{\partial t} = \frac{1}{8}s^2 \frac{\partial^2 u(y, t)}{\partial y^2} - \frac{\partial}{\partial y} \left[\frac{1}{2} \left(\frac{n}{y} - xy \right) u(y, t) \right] \quad (9)$$

and the solution of the equation is given by¹⁷

$$\begin{aligned} u(y, t) &= \frac{1}{c_3(t)} \left\{ y_0^{1-b} y^{1+b} \exp \left[\frac{b+3}{2} c_2(t) \right] \right\}^{1/2} \times \\ &\exp \left[-\frac{y_0^2 + y^2 \exp[c_2(t)]}{2c_3(t)} \right] \times \\ &I_{\frac{b-1}{2}} \left[\frac{y_0 y \exp[c_2(t)/2]}{c_3(t)} \right] \end{aligned} \quad (10)$$

¹⁶ The analytical solution of the transition probability density function is only available for $g = 0.5$.

¹⁷ The derivation can be found in page 238 in Karlin and Taylor (1981).

where

$$b = \frac{4n}{s^2}$$

$$c_2(t) = \left(k - \frac{1}{2}s^2 \right) t$$

$$c_3(t) = \frac{\frac{1}{4}s^2}{k - \frac{1}{2}s^2} \left[e^{\left(k - \frac{1}{2}s^2 \right) t} - 1 \right]$$

I_y is the modified Bessel function

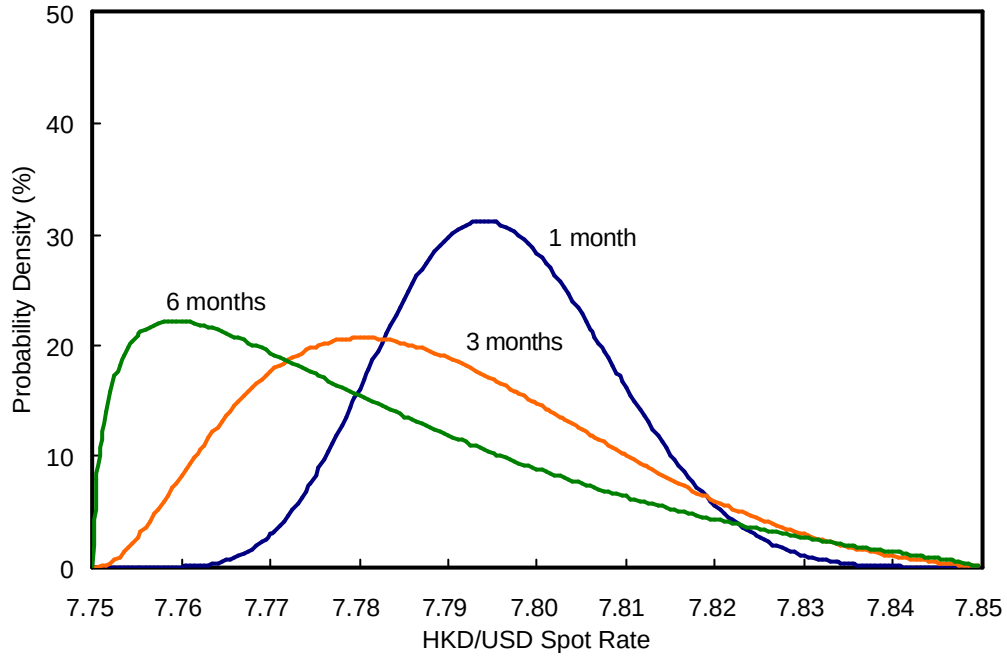
$$I_y(z) = \sum_{k=0}^{\infty} \frac{(z/2)^{2k+y}}{k! \Gamma(k+y+1)}$$

and $\Gamma(\cdot)$ is the gamma function. From equation (10), the transition probability density function of S is

$$P(S, t) = \frac{u(y, t)}{2y \exp(-y^2)(S_u - S_L)}$$

$$= \frac{1}{2(S - S)_L \sqrt{-\ln \frac{S - S_L}{S_u - S_L}}} u\left(\sqrt{-\ln \frac{S - S_L}{S_u - S_L}}, t\right) \quad (11)$$

The probability density function of the Hong Kong dollar exchange with the estimates in Table 2 and $\Delta r = -0.1\%$ in time horizons of one month, three months and six months is plotted in Chart 4. The peak of the probability density function shifts to the strong-side limit over time because of the negative interest rate differential Δr .

Chart 4. Probability Density Function for HKD/USD Spot Rate

The strong-side limit ($S = S_L$) is a natural boundary (as $y = \infty$) for all specifications of b , where the probability density function is zero. The exchanger rate takes infinite time to move towards the boundary, therefore, the strong-side limit is inaccessible. The exchange rate dynamics at the weak-side limit ($S = S_U$; $y = 0$) however depends on the parameter

$$b = \frac{4n}{s^2} \\ = -1 - \frac{2(k \ln q + \Delta r)}{\frac{1}{2}s^2}$$

in equations (10) and (11), where $2(k \ln q + \Delta r)$ is the drift. The dynamics can be classified into the following three conditions:¹⁸

$$(i) \quad b > 1, \text{ i.e. } (k \ln q + \Delta r) < 0 \text{ and } |(k \ln q + \Delta r)| > \frac{1}{2}s^2$$

When the exchange rate is approaching the weak-side limit, the drift is strong enough to push the exchange rate towards the central parity under this condition. The weak-side limit is inaccessible and the probability density function at the limit is zero. The dynamics of the exchange rate is thus completely bounded between the strong- and weak-side limits.

¹⁸ See Chapter 15 in Karlin and Taylor (1981).

$$(ii) \quad -1 < b < 1, \text{ i.e. } (k \ln q + \Delta r) < 0 \text{ and } 0 < |(k \ln q + \Delta r)| < \frac{1}{2} s^2$$

Under this condition, the weak-side limit is accessible as the drift is not strong enough relative to the volatility to restore the exchange rate towards the central parity. The exchange rate will be “sticky” at the limit over time, while the dynamics of the exchange rate is still bounded inside the zone.

$$(iii) \quad b \leq -1, \text{ i.e. } (k \ln q + \Delta r) \geq 0$$

The bounded process specified in equation (6) is not valid under this condition. The exchange rate could exit the zone at the weak-side limit in finite time and the transition probability density function in equation (11) is therefore not normalisable.

In summary, the exchange rate is bounded at the strong-side limit for all specifications of b and at the weak-side limit when $b > -1$ (i.e. $(k \ln q + \Delta r) < 0$). The bounded process is not valid only if $(k \ln q + \Delta r) \geq 0$, where the interest rate differential Δr is positive (i.e. the Hong Kong dollar interest rate is higher than the US dollar interest rate) and larger than $k \ln q$ (which is negative) in absolute terms. Since the introduction of the three refinements to the currency board in May 2005, the interest rate differential has been negative. Therefore, the market condition has been consistent with the bounded process.

V. CONCLUSION

The results show that after the introduction of the three refinements to the Linked Exchange Rate system in May 2005 the Hong Kong dollar does not show any strong tendency to revert towards the centre of the Convertibility Zone. This is perhaps not surprising as there have been no interventions in the foreign exchange market since then.

The results of the empirical tests also suggest, however, that the Hong Kong dollar follows a bounded process, one that is consistent with a fully credible exchange rate band. The bounded process will limit the movements of the exchange rate close to the strong- and weak-side limits because its variance vanishes at the CUs making it almost inaccessible to the limits. Although there may be few forces or incentives for market participants to drive the exchange rate towards 7.80, relatively large movements of the exchange rate towards the central parity are expected when the exchange rates moves close to either side of the band. This occurred when the exchange rate moved from the strong-side Convertibility Zone towards the 7.80 level in January 2007 and the reverse

movement from the weak-side Convertibility Zone towards the central parity was observed in August 2007.

References

- Anthony M, MacDonald R. 1998. "On the Mean Reverting Properties of Target Zone Exchange Rates: some evidence from the ERM". *European Economic Review* **42**: 1493-523.
- Anthony M, MacDonald R. 1999. "The Width of the Band and Exchange Rate Mean Reversion: Some Further ERM-Based Results". *Journal of International Money and Finance* **18**: 411-28.
- Bali T. 1991. "An Empirical Comparison of Continuous Time Models of the Short Term Interest Rate". *Journal of Futures Markets* **19**(7): 777-797.
- Ball C, Roma A. 1994. "Target Zone Modelling and Estimation for European Monetary System Exchange Rates". *Journal of Empirical Finance* **1**: 385-420.
- Kanas A. 1998. "Testing for a Unit Root in ERM Exchange Rates in the Presence of Structural Breaks: Evidence from the Boot-Strap". *Applied Economics Letters* **5**: 407-10.
- Karlin S, Taylor HM. 1981. *A Second Course in Stochastic Processes*. Academic Press, New York.
- Kim MJ, Nelson CR, Startz R. 1991. "Mean Reversion in Stock Prices? A Reappraisal of the Empirical Evidence". *Review of Economic Studies* **58**: 515-528.
- Krugman P. 1991. "Target Zones and Exchange Rate Dynamics". *Quarterly Journal of Economics* **106**: 669-82.
- Lo CF. 2007. "Exact Solution to the Functional Fokker-Planck Equation of a Bounded Stochastic Process and its Application to Modelling Tumour Cell Growth". Working Paper, the Chinese University of Hong Kong.
- McLeod AI, Li WK. 1983. "Diagnostic checking ARMA time series using squared-residual autocorrelations". *Journal of Time Series Analysis* **4**: 269-273.
- Nicolau J. 2002. "Stationary Processes That Look Like Random Walks – The Bounded Random Walk Process in Discrete and Continuous Time". *Econometric Theory* **18**: 99-118.
- Nieuwland FGMC, Verschoor WFC, Wolff CCP. 1994. "Stochastic Jumps in EMS Exchange Rates". *Journal of International Money and Finance* **13**: 699-727.
- Rose AK, Svensson LEO. 1994. "European Exchange Rate Credibility before the Fall". *European Economics Review* **38**: 1185-216.
- Svensson LEO. 1993. "Assessing Target Zone Credibility". *European Economic Review* **37**: 763-802.