MODELLING SOVEREIGN BOND YIELD CURVES
OF THE US, JAPAN AND GERMANY

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Research Department

Abstract

The movement of sovereign yields is important for both investment and risk management. In this paper, we apply a method that was first developed by Diebold et al. (2006b) to model the sovereign bond yield curves of the US, Japan and Germany. By including observable macroeconomic variables and the latent factors of the yield curve, we find evidence of strong interaction between the yield curve and macro variables in the US and Germany but not in Japan. We also estimate the dynamic conditional correlations of the latent factors to reveal cross-country correlations of the bond markets.

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Keywords: Yield curve; Term structure; Interest rate; Kalman filter
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Executive Summary:

- The movement of sovereign bond yields is important for both investment and risk management. This paper applies a method that was first developed by Diebold et al (2006b) to model the sovereign bond yield curves of the US, Japan and Germany.

- By including observable macroeconomic variables and the latent factors of the yield curve, we find evidences of strong interaction between the yield curve and macro variables in the US and Germany but not in Japan.

- We also estimate the dynamic conditional correlations of the latent factors to reveal cross-country correlations of the bond markets. We find evidence that the level factors of the yield curves in US, Germany and Japan are positively correlated, while other factors are not.

- The estimated dynamic correlations suggest that when any particular market is volatile, the correlations between the yield curve latent factors of that market and those of the other markets are likely to be smaller. This may be caused by the phenomenon of “flight to quality”
I. INTRODUCTION

As the term structure of government bond yield is an important indicator in the financial market, its determinants have long been the core interest of investors and central banks. In the literature, the modelling of the bond yield is mainly divided along two lines: (a) the exploring of relationship between the term to maturity and the yield, and (b) the fundamental determinants of the level of bond yields. This difference may reflect the underlying motivation of these studies: financial economists concern the yield more than its linkages to the macroeconomy, while the macroeconomists emphasise the connection between the macroeconomy and the yield. For example, the term structure models of Nelson and Siegel (1987), Dai and Singleton (2000) and Diebold and Li (2006) can be categorised as the “pure-yield model” as they use only yield data to estimate the whole yield curve. In contrast, the models of Estrella and Hardouvelis (1991), Evans and Marshall (2001), Rudebusch and Wu (2004) and Hörahl et al. (2004) can be regarded as “macro-yield model” as they either predict the macro-variables by the term structure or explain the shape of the yield curve by the macro-variables. These models have their own merits and limitations. For example, the pure-yield models are more convenient for yield prediction but have less explanatory power. On the other hand, while the macro-yield models are more illustrative by their linkage with economic theory, they require more data for estimation and are less flexible in describing various shapes of the yield curve.

Most of the macro-yield models assume that the linkages between the macroeconomy and the yield curve are uni-directional, i.e. either the macro-variables determine the yield or the yield affects the macro-variables without any feedback effects. To address these feedback effects, Diebold et al. (2006b), based on the framework of Nelson and Siegel (1987), construct a new class of macro-yield model using a dynamic latent factor approach which combines the unobservable level, slope and curvature factors of the yield curve with three macro-variables (capacity utilisation, policy rate and inflation). This approach has the following advantages over the previous models: (1) it has good flexibility to capture the dynamics of the yield curve due to the changes in the unobserved factors or the changes in the macroeconomy; and (2) it can be conveniently estimated by the Kalmen filtering approach. Applying to the US Treasury bond yield, this latent-factor macro-yield model was found to fit the data very well over the period of January 1972 to December 2000. The estimation results provide strong evidence of the effect of macroeconomy on the yield curve but not vice versa.

Due to the close connection of the interest rate among the G3 (the US, Japan and Germany) economies, it is also important to study how the term structure of interest rates, in particular the macro-yield relationship, is determined in these countries. This paper extends the latent-factor macro-yield model developed by Diebold et al (2006b) to estimate the government bond yield curve of the US, Japan and Germany.
To further understand the bond yield movements in the US, Japan and Germany, this paper also examines how their government bond markets are correlated. The literature on the bond market relationship among different countries focuses mostly on the international linkages of long-term yield or spreads (Hafer et al. (1997); Driessen et al. (2003); Barr and Priestley (2004); Engsted and Tanggaard (2005)). The focus on the level of long-term yield or the spread in previous studies is understandable as the results are easy to interpret. By doing so, however, these studies do not pay much attention to the interaction of the yield curve as a whole. For example, when the long-term yield of one country rises, the long-short spread of other country can widen or narrow, depending on the movement of the short yield relative to the levels. These spillover effects in some cases can be significant and thus should be considered. This study helps us understand more about the correlations of bond markets across countries by looking at the comovement of yield curve factors rather than just the long term yields. Specifically, instead of revealing the long-term relation (in the long bond yield), we examine the short-term dynamics of the whole yield curve between the bond markets, which might be more important for financial analysis.2

We apply the dynamic conditional correlation (DCC-GARCH) model recently proposed by Engle (2002) to estimate the short-term dynamic of the model-implied yield curve factors of the US, Japan and Germany. The DCC-GARCH model has been widely used as it preserves the simplicity of univariate model in a multivariate setting. The estimation of DCC-GARCH model involves less parameters, which thus largely eases the estimation of conditional correlations from a large set of variables.

The remaining of this paper is arranged as follows: Section II discusses the macro-yield model in this study. Empirical estimations and results of individual yield curve models are presented in Section III. The international linkages between the yield curves are studied in Section IV and conclusions are given in Section V.

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2 For instance, bond portfolio management modelling requires the input of correlations between its constituents.
II. THE LATENT FACTOR MODEL WITH MACROECONOMIC FACTORS

As discussed in Diebold and Li (2006), the bond yield of a maturity at any point of time can be expressed by a three-latent-factors model as follows:

\[
y_t(\tau) = L_t + S_t \left( \frac{1 - e^{-\lambda_t}}{\lambda_t} \right) + C_t \left( \frac{1 - e^{-\lambda_t}}{\lambda_t} - e^{-\lambda_t} \right) + \epsilon_t(\tau)
\]  

(1)

where \( y_t(\tau) \) is the bond yield of maturity \( \tau \) at time \( t \); \( L_t, S_t \) and \( C_t \) are the unobservable time-varying level, slope and curvature factors of the yield curve, the factor loading \( 1, \frac{1 - e^{-\lambda_t}}{\lambda_t} \) and \( \frac{1 - e^{-\lambda_t}}{\lambda_t} - e^{-\lambda_t} \) are for the level, slope and curvature factors respectively, and \( \lambda \) is the rate of change of the factor loadings along the horizon of maturity.\(^3\)

For all the bond yields of different maturity at time \( t \), equation (1) can be written in a matrix form as:

\[
\begin{pmatrix}
y_t(\tau_1) \\
y_t(\tau_2) \\
\vdots \\
y_t(\tau_N)
\end{pmatrix} = L_t \begin{pmatrix}
\frac{1 - e^{-\tau_1 \lambda}}{\tau_1 \lambda} & \frac{1 - e^{-\tau_2 \lambda}}{\tau_2 \lambda} & \cdots & \frac{1 - e^{-\tau_N \lambda}}{\tau_N \lambda} - e^{-\tau_N \lambda}
\end{pmatrix} + S_t \begin{pmatrix}
\frac{1 - e^{-\tau_1 \lambda}}{\tau_1 \lambda} - e^{-\tau_1 \lambda} \\
\frac{1 - e^{-\tau_2 \lambda}}{\tau_2 \lambda} - e^{-\tau_2 \lambda} \\
\vdots \\
\frac{1 - e^{-\tau_N \lambda}}{\tau_N \lambda} - e^{-\tau_N \lambda}
\end{pmatrix} + C_t \begin{pmatrix}
\epsilon_t(\tau_1) \\
\epsilon_t(\tau_2) \\
\vdots \\
\epsilon_t(\tau_N)
\end{pmatrix}
\]  

(2)

As discussed in Diebold et al. (2006b), a simple way to describe the macro-finance relationship is to assume both the yield factors and macroeconomic factors to follow a VAR(1) process. In such a process, the movements of the three latent factors of the yield curves and the macroeconomic variables can be written in the matrices as follows\(^4\):

\(^3\) Diebold et al (2006b) shows that the three factors have their own interpretation from the yield curve. While the level factor is given by the longest yield of the whole yield curve, the slope factor corresponds to the difference of the shortest yield minus the longest yield (3-month yield minus 120-month yield in their study), which is the negative of the usual definition of the slope of yield curve (longest minus shortest). The curvature factor corresponds to two times the medium yield minus the sum of the longest and the shortest yield (two times of 24-month yield minus the sum of 120-month and 3-month yield in their study).

\(^4\) We follow Diebold et al. (2006b) to use the VAR(1) setting in this study. This is chosen for its clarity and parsimony.
where \( L, S, C \) are the latent level, slope and curvature factors of the yield curve, \( CAP, PR, INFL \) are the capacity utilisation, the policy rate and the inflation rate at time \( t \) respectively, and \( \mu \) is the expected level of the variable \( X \). \(^5\)

The three macro-economic variables in this study are commonly used to explain the yield movement in many macro-finance models. For example, a central bank may adjust its policy rate in response to inflation pressure and output performance, whereas the current inflation and economic growth may be affected by the interest rate setting in the previous period according to the Taylor rule. The change in the policy rate may also have impact on different latent factors of the whole term structure by the expectations theory. Understanding all these transmission paths will give us important insights on the interaction between the economy and the term structure.

The macro-finance model can be written more conveniently in the matrix notation as:

\[
y_t = \Lambda f_t + \epsilon_t \\
(f_t - \mu) = A(f_{t-1} - \mu) + \eta_t
\]

where \( y_t \) is the column matrix of bond yield at time \( t \), \( \Lambda \) is the matrix of factor loadings, \( f_t \) is a column matrix of latent and macroeconomic factors, \( \mu \) is the column matrix of expected level of factors, \( A \) is the coefficient matrix, and \( \epsilon_t, \eta_t \) are the zero mean white noise processes with distributions given as follows:

\[
\begin{pmatrix} \epsilon_t \\ \eta_t \end{pmatrix} \sim N \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} B & 0 \\ 0 & Q \end{pmatrix}
\]

where the covariance matrix \( B \) is diagonal and the covariance matrix \( Q \) is non-diagonal.

\(^5\) Capacity utilisation is a proxy of output gap, policy rate is an indicator of monetary policy, and inflation is the CPI change over the last 12 months.
Note that the model given by equations (4) and (5) is also considered as a state-space representation and can be then estimated by the Kalmen filer method with (5) as the signal equation and (6) as the state equation.

III. DATA

Month-end zero-coupon bond yields of the US Treasury with maturities of 6, 12, 18, 24, 30, 36, 42, 48, 54, 60, 66, 72, 78, 84, 90, 96, 102, 108, 114 and 120 months from January 1992 to March 2006 are retrieved from the Bank for International Settlement (BIS). The three macroeconomic factors are the total industry capacity utilisation, the Federal funds rate and the change of consumer price index over the past 12 months. They are obtained from the Federal Reserve Economic Database.

For the Japanese data, month-end zero-coupon yields of JGB with maturities of 12, 24, 36, 48, 60, 72, 84, 96, 108 and 120 months from January 1992 to March 2006 are downloaded from Bloomberg. The total industry capacity utilisation, the base rate of the Bank of Japan and the change of consumer price index over the past 12 months are used in the yield curve estimation. These data can be obtained from the Bank of Japan and the Statistic Bureau of Japan.

As for Germany, the month-end zero-coupon yields with maturities of 12, 24, 36, 48, 60, 72, 84, 96, 108 and 120 months from January 1992 to March 2006 are obtained from the BIS. The total industry capacity utilisation, policy rate and the change of consumer price index over the past 12 months are used as the macroeconomic factors and obtained from the Deutsche Bundesbank and the Eurostat.6

IV. ESTIMATION RESULTS

The estimated latent factors and the corresponding level, slope and curvature factors as defined above are plotted in Chart 1, Chart 2 and Chart 3 respectively. From the figures, it is shown that the model fits the yield curve quite well for both the US and German bonds, but not the Japanese bond.

The estimated coefficient matrices of the US, the German and the Japanese models are shown in Table 1, Table 2 and Table 3 respectively. The results of the US model are very similar to those found in Diebold et al. (2006b). For example, all the coefficients of the lagged variables on their own are significant. Among all macro-variables, only the policy rate has impacts on the latent factors. In particular,

6 As capacity utilisation in Germany is only available on quarterly basis, monthly data are obtained by extrapolation. The Germany policy rate was set by the Deutsche Bundesbank before January 1999. After that, the base rate is set by the European Central Bank.
the estimated results show that while the Federal funds rate affect the slope of the yield curve positively, it is insignificant in the level factor equation. This suggests that the Federal funds rate affects the short term yield rather than the long one. All the latent factors of the yield curve have significant impacts on the policy rates, suggesting that the term structure of the bond yield curve has useful information about the future changes in the policy rate. On the other hand, while the bond yield factors do not appear to have any significant effect on other macro variables such as the capacity utilisation and the inflation, they may indirectly affect the macroeconomic performance of the US economy through their impact on the policy rate.

Compared to the US, the estimated factors of the Japanese model do not fit well with the observed counterparts. All except the diagonals in the coefficient matrix are insignificant. These results are not surprising as the relation between the yield curve and the macro variables largely depends on the effectiveness of the monetary policy. During the prolonged depression after the burst of economic bubbles in early 1990s, the Bank of Japan has kept its base rate at the exceptionally low level. Due to many institutional problems in the banking sector, this “super expansionary policy” helped little get the Japanese economy back in track as the inflation rate was already zero or even negative and the economic growth was slow for the last few years. Due to the ineffectiveness of the monetary policy, the linkages between the term structure and the macro-variables in Japan are not obvious. Hence it is not surprise to find that all macro-variables fail to account for the movement of the latent factors of the yield curve.

For the German model, the model-implied factors fit well with the observations. Looking at the estimated coefficient matrix in Table 3, all the diagonal elements are highly significant. When compared to the US, there are more significant off-diagonal coefficients in the German results. Individually, while the curvature factor is affected by the other two yield curve factors and the policy rate, the level factor and the slope factor are not influenced by other yield curve factors and macro-variables. As for the macro-variables, most of the yield and macro factors are significant in explaining their movements.

To examine how the latent factors and the macro variables change with respect to the shocks, the impulse response functions are also studied. The impulse response of the US model and the German model are typical and similar to those of Diebold et al (2006b). The Japanese model, however, further confirms that the Japanese yield curve does not respond to the macroeconomic factors during the sample period, as almost all of the off-diagonal responses are insignificant.
Chart 1.  Level factors

Chart 2.  Slope factors
Chart 3. Curvature factors

Table 1. Coefficient matrix of the US model

<table>
<thead>
<tr>
<th></th>
<th>$L_{t-1} - \mu_L$</th>
<th>$S_{t-1} - \mu_S$</th>
<th>$C_{t-1} - \mu_C$</th>
<th>$CAP_{t-1} - \mu_{CAP}$</th>
<th>$PR_{t-1} - \mu_{PR}$</th>
<th>$INF_{t-1} - \mu_{INF}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_t - \mu_L$</td>
<td><strong>0.99</strong>*</td>
<td>0.05</td>
<td>-0.01</td>
<td>-0.01</td>
<td>-0.01</td>
<td>-0.03</td>
</tr>
<tr>
<td></td>
<td>(0.19)</td>
<td>(0.21)</td>
<td>(0.05)</td>
<td>(0.04)</td>
<td>(0.17)</td>
<td>(0.10)</td>
</tr>
<tr>
<td>$S_t - \mu_S$</td>
<td>-0.07</td>
<td><strong>0.46</strong>*</td>
<td>0.00</td>
<td>0.01</td>
<td><strong>0.24</strong>*</td>
<td>-0.04</td>
</tr>
<tr>
<td></td>
<td>(0.12)</td>
<td>(0.08)</td>
<td>(0.04)</td>
<td>(0.04)</td>
<td>(0.00)</td>
<td>(0.07)</td>
</tr>
<tr>
<td>$C_t - \mu_C$</td>
<td><strong>0.77</strong>*</td>
<td><strong>0.65</strong>*</td>
<td><strong>0.82</strong>*</td>
<td>-0.11</td>
<td>-0.40</td>
<td>-0.16</td>
</tr>
<tr>
<td></td>
<td>(0.39)</td>
<td>(0.35)</td>
<td>(0.08)</td>
<td>(0.10)</td>
<td>(0.30)</td>
<td>(0.18)</td>
</tr>
<tr>
<td>$CAP_t - \mu_{CAP}$</td>
<td>0.36</td>
<td>0.28</td>
<td>0.04</td>
<td><strong>0.98</strong>*</td>
<td>-0.36**</td>
<td>-0.07</td>
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<tr>
<td></td>
<td>(0.22)</td>
<td>(0.21)</td>
<td>(0.05)</td>
<td>(0.05)</td>
<td>(0.17)</td>
<td>(0.09)</td>
</tr>
<tr>
<td>$PR_t - \mu_{PR}$</td>
<td><strong>0.27</strong>*</td>
<td><strong>0.25</strong>*</td>
<td><strong>0.02</strong>*</td>
<td>0.00</td>
<td><strong>0.73</strong>*</td>
<td>-0.03</td>
</tr>
<tr>
<td></td>
<td>(0.05)</td>
<td>(0.04)</td>
<td>(0.01)</td>
<td>(0.02)</td>
<td>(0.04)</td>
<td>(0.03)</td>
</tr>
<tr>
<td>$INF_t - \mu_{INF}$</td>
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<td>-0.03</td>
<td>-0.13</td>
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<tr>
<td></td>
<td>(0.17)</td>
<td>(0.14)</td>
<td>(0.03)</td>
<td>(0.04)</td>
<td>(0.11)</td>
<td>(0.06)</td>
</tr>
</tbody>
</table>

Notes:
1. The zero coefficients and s.e. are actually less than 0.005 and being rounded to 0.00
2. Standard errors in the parentheses
3. *implies coefficient significant at the 5% level
4. ** implies coefficient significant at the 10% level
Table 2. Coefficient matrix of the Japanese model

<table>
<thead>
<tr>
<th></th>
<th>$L_{t-1} - \mu_L$</th>
<th>$S_{t-1} - \mu_S$</th>
<th>$C_{t-1} - \mu_C$</th>
<th>$CAP_{t-1} - \mu_{CAP}$</th>
<th>$PR_{t-1} - \mu_{PR}$</th>
<th>$INF_{t-1} - \mu_{INF}$</th>
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<tr>
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<td>(0.01)</td>
<td>(0.30)</td>
<td>(0.06)</td>
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<td>(0.05)</td>
<td>(0.01)</td>
<td>(0.33)</td>
<td>(0.06)</td>
</tr>
<tr>
<td>$C_{t} - \mu_C$</td>
<td>0.16</td>
<td>0.15</td>
<td><strong>0.89</strong>*</td>
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<td>-0.10</td>
<td>0.03</td>
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<tr>
<td></td>
<td>(0.53)</td>
<td>(0.55)</td>
<td>(0.11)</td>
<td>(0.02)</td>
<td>(0.64)</td>
<td>(0.15)</td>
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<td><strong>0.99</strong>*</td>
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<tr>
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<td>(0.03)</td>
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<td>(0.02)</td>
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<td>(0.01)</td>
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</tr>
</tbody>
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Table 3. Coefficient matrix of the German model

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<td>-0.02</td>
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<tr>
<td></td>
<td>(0.16)</td>
<td>(0.14)</td>
<td>(0.02)</td>
<td>(0.03)</td>
<td>(0.12)</td>
<td>(0.07)</td>
</tr>
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<td>(0.09)</td>
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<td>$C_{t} - \mu_C$</td>
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<td>(0.14)</td>
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<td>$CAP_{t} - \mu_{CAP}$</td>
<td><strong>0.36</strong></td>
<td><strong>0.26</strong></td>
<td>0.05</td>
<td><strong>0.96</strong></td>
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<td>$PR_{t} - \mu_{PR}$</td>
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<td></td>
<td>(0.10)</td>
<td>(0.08)</td>
<td>(0.02)</td>
<td>(0.02)</td>
<td>(0.07)</td>
<td>(0.06)</td>
</tr>
</tbody>
</table>

Notes: 1. The zero coefficients and s.e. are actually less than 0.005 and being rounded to 0.00
2. Standard errors in the parentheses
3. *implies coefficient significant at the 5% level
4. ** implies coefficient significant at the 10% level
V. International Linkage of the Bond Yields

To examine the international linkage of the yield curves of individual countries, the DCC-GARCH model of Engle (2002) is applied to estimate the dynamic conditional correlations of bond markets across countries. As there are nine factors (three from each country) from the yield curves, there are a total of 36 (i.e. $C_9^2$) correlations between these factors and among them, 27 are cross-countries. Cross-factor dynamic correlations are ignored as the results of single-country analyses indicated little cross-factor interaction. Therefore we emphasise the remaining nine cross-country correlation of each factor.

Specifically, the DCC-GARCH model is described as follows:

$$r_t \mid \Omega_t \sim N(0, H_t)$$
$$H_t = D_t R_t D_t$$

where $r_t$ is the $9 \times 1$ matrix of zero mean errors of the univariate GARCH process of the nine yield curve factors conditional on the information set available at time $t-1$, $\Omega_{t-1}$; $D_t$ is a $9 \times 9$ diagonal matrix which contains the univariate GARCH-implied standard deviation ($\sigma_{t_k}$), in which the standard deviation of the $k^{th}$ series is the element in the $k^{th}$ row and $k^{th}$ column ($k = 1,...,9$). The log-likelihood of $r_t$ is given by:

$$-\frac{1}{2} \sum_{i=1}^{T} (k \log(2\pi) + 2 \log(\mid D_t \mid)) - \frac{1}{2} \sum_{i=1}^{T} (\log(\mid R_t \mid + z_t' R_t^{-1} z_t))$$

where $z_t$ is the $9 \times 1$ matrix of standardised $r_t$ (i.e. $z_t = \frac{r_t}{\sigma_{t_k}}$). The first part of (9) contains only terms in $D_t$ while the second part contains only terms in $R_t$. The likelihood can be maximised by a two-step method (Engle (2002)).

The nine estimated cross countries correlations are plotted in Chart 4, Chart 5 and Chart 6. For the sake of convenience in discussion, the correlations are divided into three groups: level-level correlations, slope-slope correlations and curvature-curvature correlations.
Chart 4. Correlations between level factors

Chart 5. Correlations between slope factors
The three DCC series between level factors are persistent (Chart 4). The correlations between the level factors of the US and Germany are over 0.99 during the study period while those related to Japan are also higher than 0.9. There are two episodes (1998-1999 and 2003) in which the DCC fell abruptly. These might due to the abnormal changes in yield during the regional turmoil caused by the Asian Financial Crisis (1998 – 1999) and the outbreak of SARS (2003). The long-end of the Japanese yield curve dropped sharply during these periods but those of the US and Germany were rather stable.

The slope factors correlations are less persistent. The three slope factors DCC series in Chart 5 fluctuate around 0.2 to 0.7 for most of the time. There are two relatively small drops during 1998-1999 and 2003, which are consistent with the pattern of the level factors. There are also two incidents (1992 – 1993, 2000 – 2001) that the slope factors correlations fell abruptly. These may be caused by the mismatch of interest rate cycles among these three countries. For example, the Federal funds rate began to rise in early 1994 while the policy rate of Germany was still going down at that time.

Correlations between curvature factors are even more volatile Chart 6. The three DCC series are ranging from -0.4 to 0.8 with numerous ups and downs.

The estimated correlations above suggest that when any particular market is volatile, the correlations between the yield curve latent factors of that market and the
others are likely smaller. This may be caused by the outflow of funds from the uncertain market to those relatively stable markets, a phenomenon of “flight to quality” that is often observed during financial crisis and turmoil.\(^7\)

Our study is closely related to Diebold et al. (2006a) as both works are built on the framework proposed by Diebold and Li (2006). However, our analysis measures the degree of correlation between the countries’ factors using the DCC analysis rather than based on the existence of some common factors (or global yield factors).

The results of the DCC analysis show that the correlations between the factors of different countries are fairly stable. These results are consistent with the implications of the results found in Diebold et al (2006a).\(^8\)

**VI. CONCLUSIONS**

We have estimated the government bond yield curves of the US, Japan and Germany using a new macro-finance framework similar to Diebold et al (2006b). The yield curve model is a combination of the latent factors (the level, slope and curvature factors of yield curve) and macroeconomic variables (capacity utilisation, policy rate and inflation). The state-space modelling approach is used to estimate the time varying latent yield curve factor. An impulse response analysis is also performed to illustrate the dynamics between the yield curve and the macro-factors.

Estimation results give support to the dynamic interaction between the yield curve latent factors and the macroeconomic variables for the US bond market and the German bond market. The insignificant interaction between the yield curve factors and macro-variables in the Japanese market may be due to the ineffectiveness of monetary policy in a period of prolonged depression.

The correlations of bond markets across countries are also important for financial modelling. Using the DCC-GARCH model of Engle (2002), we find evidence that the level factors of the yield curves in US, Germany and Japan are positively correlated, while other factors do not have a clear pattern. Further analysis is needed in order to explain the reason for the behaviour of co movements of the latent yield factors across countries.

\(^7\) For example, Magnus et al. (2004) also attributes the “flight to quality” phenomenon to the decoupling of correlation between stock and bond returns at times of higher uncertainty.

\(^8\) The Diebold et al. (2006a), they found that the global factors responsible for yields of the US, Germany, Japan and the UK are highly persistent and that the nature of the impact of these factors has changed significantly from period to period.
REFERENCES


