



**VALUING FOREIGN CURRENCY OPTIONS WITH A MEAN-REVERTING
PROCESS: A STUDY OF HONG KONG DOLLAR¹**

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Abstract

The theoretical prediction of target exchange rates expects mean reversion of the exchange rates. This paper presents a model for valuing European foreign exchange options, in which the forward foreign exchange rate follows a mean-reverting lognormal process. The mean-reverting process has material impact on the foreign exchange rate option values and their hedge parameters. The numerical results using the forward exchange rates of the Hong Kong dollar and market data of their options show such impact. As the dynamics of target exchange rates may not follow the standard lognormal process as described by the Black-Scholes model, the mean-reverting option-pricing model may be considered for valuation of options and estimation of associated hedge parameters on target exchange rates.

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Executive Summary:

- *The theoretical prediction on target exchange rates expects mean reversion of the exchange rates. The market data from January 1996 to March 2005 show that the long-term conditional means of the 3-month and 12-month HKD/USD forward rates are generally not deviated far away from the 7.80 peg rate. The estimated speed of reversion is strong. The results provide evidence that the HKD/USD forward rates follow a mean-reverting lognormal (MRL) process. This paper presents a model for valuing European foreign exchange options, in which the forward foreign exchange rate follows such a process.*
- *The numerical results using the forward exchange rates of the Hong Kong dollar show that the MRL process has material impact on the foreign exchange rate option values and their hedge parameters. This tends to decrease the value of a simple put or call. On the other hand, the process also keeps the exchange rate in a small range around the mean level. As this is the region in which an option's intrinsic value is high because of the level of its strike price, there is also a tendency for option values to be enhanced compared with the values under the Black-Scholes model.*
- *Forward exchange rates and foreign exchange option prices are increasingly used to extract market expectations and views about monetary policy. As the dynamics of target exchange rates may not follow the standard lognormal process, monetary authorities who adopt target exchange rates could analyse market information extracted from such derivative instruments in the light of the MRL process.*

I. INTRODUCTION

In recent years, the number and variety of foreign exchange based derivatives have increased in scope and importance, in particular in the over-the-counter (OTC) market.² Free-floating exchange rates, which might well be characterised by the standard lognormal distribution of option pricing theory, are adopted by many countries. However, the central banks of some economies still attempt to hold their exchange rate within a particular range relative to some other currency, usually the US dollar. Such linked exchange rate system includes currency board arrangements which are based on an explicit legislative commitment to exchange domestic currency for a specified foreign currency at a fixed exchange rate, combined with restrictions on the issuing authority to ensure the fulfilment of its legal obligation. This implies that domestic currency will be issued only against foreign exchange and that it remains fully backed by foreign assets. While little scope for discretionary monetary policy is left, some flexibility may still be afforded, depending on how strict the banking rules of the currency board arrangement are.

An example of currency board arrangements is Hong Kong's Linked Exchange Rate system.³ The Hong Kong dollar is linked to the US dollar at the fixed rate of HK\$7.80 to one US dollar, which has been in existence since 17 October 1983. The link is maintained through the operation of a rule-based currency board system, which ensures that Hong Kong's entire monetary base is backed with US dollars kept in Hong Kong's Exchange Fund. In September 1998, a "Convertibility Undertaking" was established whereby the Hong Kong Monetary Authority (HKMA) is obliged to sell US dollars to licensed banks at HK\$7.80 per US dollar, however there was no specification of a Convertibility Undertaking (i.e. to buy US dollars) on the strong side of the link. On 18 May 2005, the HKMA introduced a strong-side Convertibility Undertaking by the HKMA to buy US dollars at 7.75 and shifted the weak-side Convertibility Undertaking by the HKMA to sell US dollars from 7.80 to 7.85, so as to achieve symmetry around 7.80 with bands of $\pm 0.64\%$.⁴

² See the Bank for International Settlements (March 2005), *Triennial Central Bank Survey of Foreign Exchange and Derivatives Market Activity in 2004*.

³ Some countries like Estonia and Bulgaria adopt currency board arrangements.

⁴ Within the zone defined by the levels of the Convertibility Undertakings (the "Convertibility Zone"), the HKMA may choose to conduct market operations consistent with Currency Board principles. These market operations shall be aimed at promoting the smooth functioning of the Linked Exchange Rate system, for example, by removing any market anomalies that may arise from time to time.

Under other conventional fixed peg arrangements, the country (formally or de facto) pegs its currency at a fixed rate to another currency or a basket of currencies, where the basket is formed from the currencies of major trading or financial partners and weights reflect the geographical distribution of trade, services, or capital flows.^{5,6}

Another linked exchange rate system is a pegged exchange rate within horizontal bands. There is a limited degree of monetary policy discretion, depending on the band width. The European Economic Community adopted this system which is a much more tightly controlled system, known as the Exchange Rate Mechanism (ERM) in March 1979, that was replaced with the ERM II on 1 January 1999.⁷

To study the effect of target exchange rates within certain bands on the value of foreign exchange options, Ingersoll (1996) presents a pricing model in which the exchange rate process is strictly bounded where the bands are represented by inaccessible barriers.⁸ The presence of bounds can have a material effect on option values, which are different from those obtained by Garman and Kohlagen (1983) based on the Black-Scholes model developed by Black and Scholes (1973) and Merton (1974). A bounded stochastic process limits the range of the exchange rate at the option's maturity and hence the uncertainty of its payoff. This tends to decrease the value of a simple put or call. On the other hand, the bounds also keep the exchange rate in a small range around the strike price.

It is noted that the bands specified under the above exchange rate pegging systems are fixed by the central banks, but they could be adjusted over time. Such changes make the inaccessible barriers representing the bands too restrictive, which does not allow any change of the bands during option life. In addition, it is observed that the forward exchange rates under these systems are sometimes outside the bands due to market conditions, for example rise (drop) of interest rates of a currency because of its tightened (ample) liquidity in the banking system, or realignments of the bands speculated by market participants. This causes inconsistency to specify the fixed

⁵ An example under a managed-floating regime is the Chinese renminbi.

⁶ Flexibility of monetary policy, though limited, is greater than in the case of exchange arrangements with no separate legal tender and currency boards because traditional central banking functions are still possible, and the monetary authority can adjust the level of the exchange rate, although relatively infrequently.

⁷ Under the ERM of 1979 each member country was required to maintain its exchange rate with the European Currency Unity (ECU) within certain bands. After several realignments of the currencies in the early 1980s, the ERM stabilised with bands of $\pm 2.25\%$ of parity with the ECU for each currency. Later members, Portugal, Spain, and the UK were only required to keep their currencies within approximately 6% of par. In September 1993, Italy and the UK left the ERM because they did not wish to maintain their currencies within the bands. In August 1993, the bands for six member countries (Belgium, Denmark, France, Ireland, Portugal and Spain) were relaxed to $\pm 15\%$.

⁸ Making the barriers accessible and reflecting would appear to be a better choice, but actually admit arbitrage. At an upper (lower) reflecting barrier, the forward price can only drop (rise). This means that a short (long) position in the forward contract, which requires no investment and must make money over the next instant, is an arbitrage.

inaccessible barriers at the bands.

An important prediction of the theoretical literature on target exchange rates is that we might expect mean reversion of the exchange rate when the central banks engage in intramarginal intervention and market participants expect the exchange rate band to be fully credible and engage in stabilising speculation. This mean-reverting property is widely referred to in the literature (see for example, Krugman (1991), Svensson (1992, 1993), Rose and Svensson (1994), and Anthony and MacDonald (1998)). Several recent studies have attempted to investigate empirically this theoretical prediction by examining the time-series properties of the currencies participating in the ERM (see for example, Ball and Roma, (1993, 1994), Svensson (1993), Rose and Svensson (1994), Nieuwland *et al.* (1994), Anthony and MacDonald (1998, 1999), and Kanas (1998)). While their investigations are with mixed results, the empirical results suggest that mean reversion is present.

In view of such a theoretical prediction, we develop an option pricing model in which the forward exchange rate follows a mean-reverting process under the assumption that the central bank will intercede to stabilise the foreign exchange rate within the bands but without any explicit boundary conditions at the bands. The use of the forward exchange rate as the stochastic variable can take the stochastic interest rates of the pair of currencies into account. It is also noted that the behaviour of the spot exchange rate and the two economies' interest rates must be linked in order to prevent arbitrage opportunities under exchange rate pegging systems. The study of the dynamics of forward exchange rates between the Hong Kong dollar and the US dollar in the following section shows that they can be described by a mean-reverting lognormal (MRL) process. The strength of the mean-reverting component of the process measures the robustness and the credibility of the exchange rate bands. While there is no explicit boundary condition specified at the bands, the mean-reverting process reflects the presence of such bands. This paper examines the effects of target exchange rates with MRL dynamics on the value of foreign exchange options. The corresponding closed-form solutions for European options are derived. The resulting pricing solutions resemble the Black-Scholes formulas and have a similar interpretation.

Section II of this paper presents the empirical findings of mean-reverting forward exchange rates based on the data of exchange rates and interest rates of the Hong Kong dollar and the US dollar. Section III illustrates the option pricing models of the MRL process and derives the corresponding option pricing formulas. Section IV presents the effects of the MRL dynamics on the value of foreign exchange options, comparing with those derived from the Black-Scholes model. The purpose is to illustrate how option values under the MRL model will be different from the market values. The hedge parameters based on the lognormal process, that ignores the mean-reverting property present in the dynamics of the forward exchange rates, may

produce erroneous results even though traders consider that the market option values are correct in monetary-value terms. Section V shows hedging performance of options under the Black-Scholes and MRL models. The final section summarises the findings.

II. MEAN-REVERTING FORWARD EXCHANGE RATES

a. Data of HKD/USD forward exchange rates

The Hong Kong dollar (HKD) value of a default-free zero-coupon loan paying one dollar at time t is $B_{HK}(r, t)$. The value of a default-free zero-coupon loan paying one US dollar (USD) at time t is $B_{US}(r^*, t)$. The domestic (HKD) and foreign (USD) continuously compounded rates of interest are r and r^* respectively. These rates are not necessarily assumed to be constant. The HKD/USD spot exchange rate (i.e., the value in HKD of one USD) is S .

The most common foreign exchange derivatives are forward contracts.⁹ To prevent arbitrage the forward rate F for the delivery of USD must be in HKD equal to

$$F = \frac{SB_{US}(r^*, t)}{B_{HK}(r, t)}.$$

Similarly, the forward rate F^* for the delivery of HKD is the reciprocal in USD, i.e.

$$F^* = \frac{B_{HK}(r, t)}{SB_{US}(r^*, t)}.$$

Under the Linked Exchange Rate system, interest rates in Hong Kong have to track closely their US counterparts. The behaviour of the spot HKD/USD exchange rate and their interest rates must be linked in order to prevent arbitrage opportunities. Deviations in interest rates reflect the risk premium (or discount) of the Hong Kong dollar over the US dollar, which is determined by the pressure of depreciation (or appreciation) on the Hong Kong dollar, and rise (drop) of interest rates of the Hong Kong dollar because of its tightened (ample) liquidity in the banking system. Such deviations are priced in the forward exchange rates.

Daily data of 3-month and 12-month HKD/USD forward exchange rates from 2 January 1996 to 4 March 2005 are used for the estimations.¹⁰ Figure 1 illustrates the forward and spot exchange rates during the sample period. The data

⁹ Their exchange-traded counterparts are futures contracts. There are no such instruments being traded in HKD/USD.

¹⁰ According to BIS Triennial Central Bank Survey of Foreign Exchange and Derivatives Market Activity, the daily turnover of currency option transactions in the Hong Kong dollar has become relatively liquid since 1995. Consistent with the use of the liquid market volatility for option pricing, the data series starting from January 1996 for the estimations of the dynamics of the forward rates are used.

series covers the period of (i) the Asian financial crisis and the “double-market play” episode¹¹ between July 1997 and September 1998; (ii) the establishment of the Convertibility Undertaking whereby the HKMA is obliged to sell US dollars to licensed banks at HK\$7.80 per US dollar in September 1998 (HKMA, 1998); and (iii) the increased inflows of funds from the fourth quarter of 2003 to early 2005. In order to compare the stability and the robustness of the estimated parameters, a sub-sample period, starting from November 1998 to 4 March 2005 (i.e. the period after the establishment of the Convertibility Undertaking), is also used for the estimations.¹²

Table 1 shows the means, standard derivations and augmented Dickey-Fuller (ADF) statistics of the levels and the daily changes of 3-month and 12-month forwards. Over the full sample period, the unconditional average level of the 12-month forward rate is HK\$7.83 per USD with a standard deviation of HK\$0.12 per USD, whereas the daily change averaged to almost zero with a standard deviation of HK\$0.02 per USD. During the Asian financial crisis, the Hong Kong dollar experienced depreciation speculations and the 12-month forward rate shot up to a maximum of HK\$8.53 per USD. During 2004, on the other hand, there were increased inflows of funds into Hong Kong, triggered by the weak US dollar and expectations of a revaluation of the Chinese renminbi.¹³ The 12-month forward rate strengthened against the US dollar (to HK\$7.61 per USD at a point) because of expectations of a similar revaluation of the Hong Kong dollar. The ADF test verifies whether the data series are nonstationary, or contain a unit root.¹⁴ Under the null hypothesis of the presence of a unit root, the ADF test statistics reject the null hypothesis in both levels and daily changes of the 3-month and 12-month forward rates at the 5% level of significance.

¹¹ According to the HKMA, a cross-market speculation involving the stock and Hang Seng index derivatives markets, in addition to the foreign exchange market would have destabilised the peg in 1998. The way the speculation worked would have been closely related to the operation of the Hong Kong Currency Board, where an automatic adjustment mechanism links capital inflows and outflows to the overall level of liquidity in the banking system. In case of liquidity shrinkage, the increase in the interest rate should restore exchange rate stability by favouring renewed capital inflows. The HKMA argued that the predictable high sensitivity of interest rate to capital outflows would have been exploited by some speculators who would have engaged in a double-market play, expecting profits from bearish stock markets.

¹² While there have been other measures introduced to the Linked Exchange Rate system, the establishment of the Convertibility Undertaking is considered to be one of the most important refinements to the system during the sample period.

¹³ See Chui *et al.* (2005) for the recent developments of the Hong Kong dollar exchange rate.

¹⁴ The basic characteristics of a typical stationary time series are that it exhibits mean reversion in that the series fluctuates around a constant long-term mean, and its variance and covariances are finite and time-invariant. One of the commonly used unit root (stationarity) tests is the ADF test. The ADF test constructs a parametric correction for higher-order correlation by assuming that a time series process (y) follows an AR(p) process and adds p lagged differences of y to the test regression of:

$$\Delta y_t = \gamma + \alpha y_{t-1} + \beta_1 \Delta y_{t-1} + \dots + \beta_p \Delta y_{t-p} + \varepsilon_t$$

We test the null hypothesis (the presence of a unit root) of $\alpha = 0$ against the alternative hypothesis of $\alpha < 0$. The test is evaluated using the conventional t -ratio for α (t_α) such that:

$$t_\alpha = \hat{\alpha} / (se(\hat{\alpha})),$$

where $\hat{\alpha}$ is the estimate of α and $se(\hat{\alpha})$ is the coefficient standard error.

The ADF test results suggest that the two forward rate series are stationary in both their levels and their daily changes. The implicit mean-reverting (stationary) behaviour of the forward rates thus supports the modelling of the dynamical behaviour of the HKD/USD forward exchange rates under a mean-reverting diffusion model specification.

b. Model specification and estimation results

The dynamics of the 3-month and 12-month HKD/USD forward exchange rates are modelled based on the stationarity property of their first differences as presented by the ADF test in Table 1. A mean-reverting diffusion model is specified as:

$$\frac{dF_t}{F_t} = \kappa(\ln F_{0t} - \ln F_t)dt + \sigma_t dW_t \quad (1)$$

where F_t is the HKD/USD forward rate at time t , F_{0t} is the time t conditional mean forward rate¹⁵, κ is the parameter measuring the speed of reversion to this mean, σ_t is the volatility of the forward rate which is dependent on the level of F_t , and W_t is a standard Wiener process so that dW_t is normally distributed. The diffusion component is the standard lognormal distribution of the Black-Scholes option pricing theory. The continuous-time model specified in equation (1) can be estimated using a discrete-time econometric specification as:¹⁶

$$\frac{F_t - F_{t-1}}{F_{t-1}} = \alpha(\beta - \ln F_{t-1}) + \varepsilon_t \quad (2)$$

$$E[\varepsilon_t] = 0, \quad E[\varepsilon_t^2 | F_{t-1}] = \sigma^2 \quad (3)$$

The parameters α and β have the same meaning as to κ and F_{0t} respectively in equation (1). The estimated parameter of α provides a measure on how fast the mean reversion (the α parameter) of the forward exchange rate is such that the forward exchange rate restores to its mean level (the β parameter). The parameters in equations (2) and (3) of the discrete-time econometric model are estimated using the maximum likelihood technique.¹⁷

The estimation results in Table 2 show that the estimated parameters are statistically significant. The estimated conditional means for the full sample period are 7.7796 (the exponential of 2.0515) and 7.8225 (the exponential of 2.0570) for the 3-month and 12-month HKD/USD forward rates respectively. The conditional mean of the 12-month forward rate is weaker than the 7.80 peg rate, which is attributed to the devaluation expectations of the Hong Kong dollar during the Asian financial crisis.

¹⁵ F_0 can be interpreted as the historical mean instantaneous forward exchange rate.

¹⁶ This discrete-time econometric specification is similar to those discussed in Bali (1999).

¹⁷ Similar techniques have been used by Bali (1999) in empirical tests of continuous-time models of interest rates.

On the other hand, for the sub-period from 2 November 1998 to 4 March 2005, the conditional means are 7.7819 (the exponential of 2.0518) and 7.7788 (the exponential of 2.0514) for the 3-month and 12-month forward rates respectively. They are slightly stronger than the 7.80 peg rate, which may partly reflect the appreciation expectations of the Hong Kong dollar after the third quarter of 2003.¹⁸ Using the Wald coefficient test, the hypothesis that the estimated conditional means (except the 3-month forward rate under the full sample period) are equal to 7.80 (the exponential of 2.0541) cannot be rejected.¹⁹ The results show that the long-term conditional means of the 3-month and 12-month HKD/USD forward rates are generally not deviated far away from the 7.80 peg rate.

The speed of mean reversion is measured by the parameter α . For the 12-month forward rate, the annualised speed for the full sample period is estimated to be 1.94 per unit time of a year, while that for the sub-period is even stronger at 2.11. This reflects that the mean-reverting property (in the long run) of the 12-month forward rate is slightly stronger during the sub-period after the implementation of the Convertibility Undertaking at 7.80. On the other hand, for the 3-month forward rate, the speed of mean reversion is estimated to be 2.11 during the sub-period, which is much smaller than that of 7.03 during the full period. The relatively smaller mean reversion speed probably reflects the heavier short-run speculative pressure in the HKD, which also slows the reversion to its conditional mean. Nonetheless, the mean reversion during the sub-period is still strong as it is comparable to that of the 12-month forward rate. Furthermore, as the weakened mean reversion is reflected in the 3-month forward rate but not in the 12-month forward rate, the effect may be more of a short-term market speculation, rather than a long-term expectation. Thus, even though there was speculation about the strengthening of the Hong Kong dollar, the mean-reversion would eventually bring the 3-month and 12-month forward rates back to their long-term means, which are not deviated far from 7.80.

For the estimated conditional variances of the forward rates, the annualised estimations of the two sample range from 0.56% to 2.97%. It is noted that the volatility during the full sample period is mainly attributed to the Asian financial crisis, otherwise, the volatility of the forwards rates is in general low.

¹⁸ When the estimation is conducted over the period from 2 November 1998 to end-August 2003, the estimated conditional mean is 7.7923 for the 3-month forward rate, and is 7.8137 for the 12-month forward rate.

¹⁹ As shown in Table 1, it is noted that the unconditional mean levels of forward rate may be stronger or weaker than 7.80 during the sample period. Nonetheless, 7.80 is used for the hypothesis tests as it is the central parity of the Convertibility Zone.

III. PRICING EUROPEAN OPTIONS FOR MEAN-REVERTING PROCESS

Options are commonly written with a forward price as the basis. Since the forward price for immediate delivery must equal the spot price, forward- and spot-based European options are identical if the forward contract has the same maturity as the option. Therefore, with no loss of generality, we can assume the basis for the option expiring at time t is a forward contract which expires at t . To value foreign exchange options, Garman and Kohlhagen (1983) assume that the spot exchange rate follow lognormal process and the risk-free interest rates of the underlying exchange rate are constant. The framework is basically the Black-Scholes model. If the interest rates are stochastic, pricing problems are complex. However, the problems can be simplified by using the forward exchange rates as the basis for the option.

In the MRL model it is assumed that the forward exchange rate F evolves according to the diffusion process specified in equation (1). It is assumed that option prices depend on F as the only state variable. Applying the Ito's lemma, the partial differential equation governing the option price $P(F, t)$ (expressed in HKD per USD) with time-to-maturity of t based on the model is

$$\frac{\partial P(F, t)}{\partial t} = \frac{1}{2} \sigma^2 F^2 \frac{\partial^2 P(F, t)}{\partial F^2} + \kappa (\ln F_0 - \ln F) F \frac{\partial P(F, t)}{\partial F} - rP(F, t). \quad (4)$$

By putting $x = \ln F$ and $\theta = (\ln F_0 - \sigma^2 / 2\kappa)$, equation (4) becomes

$$\frac{\partial P(x, t)}{\partial t} = \frac{1}{2} \sigma^2 \frac{\partial^2 P(x, t)}{\partial x^2} + \kappa (\theta - x) \frac{\partial P(x, t)}{\partial x} - rP(x, t). \quad (5)$$

With further changing variables:

$$e^{-rt} \tilde{P}(xe^{-\kappa t}, t) = P(x, t), \quad (6)$$

the solution of equation (5) with an option payoff condition of $\tilde{P}(x, t=0)$ at option maturity is

$$\tilde{P}(x, t) = \int_{-\infty}^{\infty} dx' G(x, t; x', t=0) \tilde{P}(x', t=0), \quad (7)$$

and the distribution function $G(x, t; x', 0)$ is given by

$$G(x, t; x', t=0) = \frac{1}{\sqrt{4\pi c_1(t)}} \exp \left\{ -\frac{[x' - x - c_2(t)]^2}{4c_1(t)} \right\}, \quad (8)$$

where

$$c_1 = \frac{\sigma^2}{4\kappa} (1 - e^{-2\kappa t}), \quad (9)$$

$$c_2 = \theta (1 - e^{-\kappa t}) = (\ln F_0 - \sigma^2 / 2\kappa) (1 - e^{-\kappa t}) \quad (10)$$

The payoff condition of a call option is $\max(F - K, 0)$, where K is the strike price, at option maturity, such that

$$\tilde{P}_{call}(x, t = 0) = \max(e^x - K, 0). \quad (11)$$

The solution of a call option is obtained by solving equation (7) subject to the payoff condition (11). After substituting back the variables, the call option value is

$$P_{call}(F, t) = F^{\exp(-\kappa t)} e^{c_1 + c_2 - rt} N(d_1) - K e^{-rt} N(d_2), \quad (12)$$

where

$$d_1 = \frac{\ln(F^{\exp(-\kappa t)} e^{c_1 + c_2} / K) + c_1}{\sqrt{2c_1}}, \quad (13)$$

$$d_2 = \frac{\ln(F^{\exp(-\kappa t)} e^{c_1 + c_2} / K) - c_1}{\sqrt{2c_1}}, \quad (14)$$

and $N(\cdot)$ is the cumulative normal distribution function. In the solution of the call option, equation (12) entails the change of numeraire of F by $\tilde{F}$ under the stochastic process specified in equation (15), where

$$\tilde{F} = F^{\exp(-\kappa t)} e^{c_1 + c_2}. \quad (15)$$

Moreover, $\tilde{F}$ is lognormal with $\text{var}[\ln \tilde{F}] = \tilde{\sigma}^2 / t$ where

$$\tilde{\sigma} = \sqrt{2c_1 / t} = \frac{\sigma}{\sqrt{2\kappa t}} \sqrt{1 - e^{-2\kappa t}}. \quad (16)$$

It is noted that the price of the European call option equals the discounted expected value of $\max(\tilde{F} - K, 0)$ for some random variable $\tilde{F}$ (independent of K). Different magnitudes of the speed κ of reversion to this mean give some intuitive interpretations of $\tilde{F}$. When the speed is very strong (i.e., $\kappa \gg 1$), $\tilde{F}$ converges to F_0 . This means that the dynamical process of the forward exchange rate is almost deterministic such that it will stick to F_0 with the “effective volatility” $\tilde{\sigma} \rightarrow 0$ and the call option value converges to $\max[(F_0 - K), 0]e^{-rt}$. Conversely, when the speed is very weak (i.e., $\kappa \rightarrow 0$), the dynamical process specified in the model converges to a lognormal process where $\tilde{F} \cong F$ and $\tilde{\sigma} \cong \sigma$ such that the corresponding call option value is that based on the Black-Scholes model. Equation (16) and the limits of $\tilde{\sigma}$ with different κ show that $\tilde{\sigma}$ is a decreasing function with κ and is less than σ . Therefore, options under the RML model are less risky than the corresponding options

under the Black-Scholes model.²⁰

Using equation (7) and the payoff condition of $\max(K - F, 0)$, the corresponding put option value is

$$P_{put}(F, t) = Ke^{-rt} N(-d_2) - F^{\exp(-\kappa t)} e^{c_1 + c_2 - rt} N(-d_1). \quad (17)$$

The resemblance between the option pricing formulas (12) and (17) and the Black-Scholes formulas with the forward exchange rate as the underlying asset is obvious.²¹ In both cases, the random variable $\tilde{F}$ above is lognormal, resulting in similar formulas. The $\tilde{\sigma}^2$, which is the variance of the logarithm of the price $\tilde{F}$ at option expiration, replaces σ^2 of the Black-Scholes model, which has the same meaning. In other words, with these substitutions, the Black-Scholes model and the MRL model produce identical option values if the input parameters are the same. The resemblance implies that the put-call parity under the MRL model is

$$\begin{aligned} P_{call}(F, t) - P_{put}(F, t) &= F^{\exp(-\kappa t)} e^{c_1 + c_2 - rt} - Ke^{-rt} \\ &= \tilde{F}e^{-rt} - Ke^{-rt}. \end{aligned} \quad (18)$$

The put option value in equation (17) can also be obtained from this put-call parity.

When dealers trade any derivative instruments, they often need to hedge their positions. It is important to determine the hedge parameters of the derivative instruments. The delta of the call expressed in equation (12) is

$$\begin{aligned} \Delta &\equiv \frac{\partial P(F, t)}{\partial F} \\ &= \tilde{F}e^{-rt} N(d_1) \cdot \left(\frac{e^{-\kappa t}}{F} \right). \end{aligned} \quad (19)$$

Its gamma is defined as

$$\begin{aligned} \Gamma &\equiv \frac{\partial^2 P(F, t)}{\partial F^2} \\ &= \tilde{F}e^{-rt} \frac{N'(d_1)}{\sqrt{2c_1}} \cdot \left(\frac{e^{-\kappa t}}{F} \right)^2 + \tilde{F}e^{-rt} N(d_1) \cdot \left[\frac{e^{-\kappa t} (e^{-\kappa t} - 1)}{F^2} \right], \end{aligned} \quad (20)$$

where

$$N'(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}.$$

²⁰ The same point can also be made by evaluating the option price elasticity of FP_F / P of the two models.

²¹ The derivation of the formulas under the Black-Scholes model can be found in Garman and Kohlagen (1983).

Its vegas (the effect of volatility on option values) is defined as

$$\frac{\partial P(F, t)}{\partial \sigma} = \tilde{F} e^{-rt} N(d_1) \cdot \left[-\frac{\sigma}{2\kappa} (e^{-\kappa t} - 1)^2 \right] + \tilde{F} e^{-rt} \frac{N'(d_1)}{\sqrt{2c_1}} \left(\frac{2c_1}{\sigma} \right). \quad (21)$$

Similarly, the delta, gamma and vega of a put can be derived according to equation (17).

IV. NUMERICAL RESULTS OF OPTION PRICES

As the option pricing formulas under the MRL model and the Black-Scholes model have similar structures, the differences of their characteristics are determined by the random variable $\tilde{F}$ (the expected value of F at option maturity) and its associated variance $\tilde{\sigma}^2$. The values of these two parameters reflect the effect of the mean-reverting process on the option values. Figure 2 plots $\tilde{F}$ with different values of κ using the 3-month and 12-month HKD/USD forward exchange rates with $F = 7.62$, $F_0 = 7.8$ and $\sigma = 3\%$. The results show that $\tilde{F}$ is higher than F of 7.62 and increases with κ . The increase in κ means that the restoring force towards the mean level F_0 increases. Given an at-the-money call option under the Black-Scholes model, where the strike price K is equal to the current forward rate F of 7.62, the option becomes an in-the-money call option under the MRL model because of $K < \tilde{F}$. Conversely, an at-the-money put option under the Black-Scholes model becomes an out-the-money put option under the MRL model. While the at-the-money option value of a call is equal to that of a put under the Black-Scholes model, their corresponding option values under the MRL model will be different and the differences depend on the values of κ and F_0 .

The HKD/USD exchange rate options are used to illustrate the differences between option values under the MRL and Black-Scholes models. The data used cover the period from 2 January 1996 to 4 March 2005, and consist of daily quotes of OTC European options drawn from the J.P. Morgan's website.²² OTC options are quoted in implied volatilities according to different delta for several maturities.²³ This is a convenient way to express the option values, the volatility being the only unobservable parameter in the Black-Scholes formula. The option values based on the Black-Scholes can therefore be considered as the market option values. As the purpose of the analysis is to compare the two option pricing models, only at-the-money options are considered here. We therefore choose the data of at-the-money options with maturities of three and twelve months in the following numerical illustrations.²⁴

²² www.morganmarkets.com.

²³ These indicative quotes are averages of the bid and ask spread and do not include transaction costs.

²⁴ The call and put values are equal under the Black-Scholes model with the delta about 0.5.

Figure 3 shows the 12-month HKD/USD forward exchange rates F and the corresponding $\tilde{F}$ based on the MRL model in which $\kappa = 1.94$ and $F_0 = 7.8225$ according to the estimations given in Table 2 above.²⁵ It is noted that the value of F_0 is close to the 7.80 peg rate. The results show that the movements of $\tilde{F}$ are less volatile than those of F , which are due to the strong speed κ of reversion of 1.94. As shown in equation (16), when the speed κ is strong, the dynamical process of the forward exchange rate is almost deterministic with small “effective volatility”. Such a strong speed of reversion causes $\tilde{F}$ to be different from F , in particular when F is distant from F_0 . If κ is reduced to 0.5, the movements of $\tilde{F}$ become more volatile and the values of $\tilde{F}$ are closer to F compared with those of $\tilde{F}$ based on $\kappa = 1.94$. The magnitude of the speed of reversion gives different impact on the expected value $\tilde{F}$ of F at option maturity.

The strong speed ($\kappa = 1.94$) of reversion also causes different 12-month option values based on the MRL model from those based on the Black-Scholes model, which are shown in Figure 4. Using at-the-money options under the Black-Scholes model (i.e. the strike price K is set equal to F), Figure 4 shows that the put option values under the MRL model are higher than the call option values and close to those under the Black-Scholes model during the period between mid-1997 and end-1998 when the market values of F are much higher than F_0 . The strong mean-reverting process pushes the forward rate down towards the mean level at 7.8225, that causes the forward rate (which is equal to the spot exchange rate) at the option maturity probably below the strike price and the put option would be exercised. It is also reflected from Figure 3 and the put value formula (17) that the put values under the MRL model are due to their deep in-the-money values where $K = F$ is much higher than $\tilde{F}$. To some extent, the put options under the two different models require similar option premiums during the period. On the other hand, the call option values under the MRL model in which the options are out-of-the-money are much lower than those under the Black-Scholes model during the same period.

During another period between end-2003 and early-2005 when the market values of F are lower than F_0 , Figure 4 shows that the call values under the MRL model, which are in-the-money (where K is lower than $\tilde{F}$, see Figure 3), are higher than those under the Black-Scholes model. The forward rate will move due to mean reversion towards the mean level which is higher than the strike price and thus the call option would be exercised at option maturity. By contrast, the put values under the MRL

²⁵ As the time to maturity of the underlying forward exchange rate and option will be shortened over time, the realistic model parameters should be time dependent. For example, a 12-month option becomes a 3-month option after nine months and F will then follow the dynamics of the 3-month forward rate (instead of the 12-month forward rate).

model in which the options are out-of-the-money are much lower than those under the Black-Scholes model during the same period. Regarding other periods of time, the option values under the Black-Scholes model are generally higher than the call and put values under the MRL model. The results are consistent with the analysis of the option pricing formulas (12) and (17) in the previous section that options under the MRL model are less risky than the corresponding options under the Black-Scholes model because the effective volatility $\tilde{\sigma}$ in equation (16) under the MRL model is less than σ .

The deltas, gammas and vegas (the effect of volatility on option values) of the 12-month options under the MRL and Black-Scholes models are presented in Figures 5, 6, and 7 respectively. The values of gammas are expressed as changes in deltas with a change in 1% of F in Figure 6. Vegas are expressed as changes in option values with an increase in σ of 1% (i.e. $\sigma + 1\%$) in Figure 7. The hedge parameters can also be calculated according to equations (19), (20) (21) or as finite difference approximations to their continuous time equivalents. Figure 5 shows that the deltas of the call options under the Black-Scholes model are around 0.5 as all the options are at-the-money. The deltas of the call and put options under the MRL model, which are sometimes in-the-money or out-of-the-money during the data period, are however much lower than 0.5 (in absolute terms). The results indicate that the sensitivity of option values to the changes in F is relatively small under the MRL model. This is attributed to the mean-reverting process that restores F to the mean level. While the put options under the two different models have similar option premiums as shown in Figure 4, their deltas are quite different as the mean-reverting process restores F towards the mean level F_0 at 7.8225. As the forward rate would stay around the mean level, the changes of the forward rate will not change the option values as much as those under the Black-Scholes model.

The gammas of options under the two models are presented in Figure 6. The gammas of the options under the MRL model are close to zero while those of the options under the Black-Scholes model range from 0.01 to 0.3. This reflects that the mean-reverting process reduces the gamma risk of the options by pushing the forward rate to be around the mean level. Compared with the Black-Scholes model, the delta under the MRL model is relatively stable for effective hedging due to the lower gamma.

Figure 7 shows that the vega risk in the MRL model is smaller than that in the Black-Scholes model. This is consistent with the analysis that the effective volatility $\tilde{\sigma}$ in equation (16) under the MRL model is less than σ . The smaller effective volatility due to the mean-reverting process causes the option values under the MRL model less sensitive to the changes in volatility.

Figure 8 shows the 3-month HKD/USD forward exchange rates F and the corresponding $\tilde{F}$ based on the MRL model in which $\kappa = 7.03$ and $F_0 = 7.7796$ according to the estimations given in Table 2 above. Similar to the results of the 12-month HKD/USD forward exchange rates, the movements of $\tilde{F}$ are less volatile than those of F , which are due to the strong speed κ of reversion of 7.03. The strong mean-reverting process pushes the forward rate to stay around at the mean level of 7.7796 and thus reduces the volatility. Using 3-month at-the-money options under the Black-Scholes model (i.e. the strike price K is set equal to F), the results of the differences between the option values of the MRL and Black-Scholes models presented in Figure 9 are also similar to those based on the 12-month options illustrated in Figure 4.

In summary, the observations in Figures 4 to 9 show that option values and the corresponding hedge parameters under the MRL model are very different from the market option values based on the Black-Scholes model. The mean-reverting process has material impact on the valuation of foreign exchange rate options and their hedge parameters.

V. DYNAMIC HEDGING PERFORMANCE

In this section we adopt a simple framework to evaluate the ability of the MRL model to capture the dynamic properties of the options and the forward exchange rates. If the MRL process proves to be a good model of the forward exchange rate dynamics, one should be able to create the payoff of an associated call or put option through trading the underlying asset continuously according to the model-implied hedge parameters, with the cost of such replication being close to the option prices given by the equations (12) and (17) respectively.

The HKD/USD exchange rate data from January 1996 to December 2005 is divided into ten non-overlapping one-year periods. For each one-year period we compile a forward exchange rate series expiring at the end of each period by interpolating forwards with different maturities. The 12-month HIBOR and LIBOR are used as the domestic and foreign risk-free interest rates respectively. The conditional mean of the forward exchange rate and the speed of mean reversion are set to be their estimated values 7.8225 and 1.94 respectively.

At the beginning of each period, we use equation (12) to price a 12-month at-the-money option to buy 1 US dollar with Hong Kong dollar (i.e. the strike price is the HKD/USD spot exchange rate at the beginning of the period). The realised volatilities of the forward exchange rate during each period are used, assuming investors have

perfect foresight. We also create a synthetic long position of such call options through delta-hedging, taking into account the interest cost of borrowing the domestic currency (HKD) and the carry earned by investing in the foreign currency (USD). The deltas are computed using equation (19) and the hedging portfolio is rebalanced on each trading day as the forward exchange rate moves. In this way our position in the underlying will always be equivalent to the delta.

At the end of each period, if the option matures out-of-the-money, we will have a position of zero US dollar. However, the hedging scheme will incur a cost. On the other hand, if the option matures in-the-money, we will end up with a position of one US dollar. The actual cost of having the position will be greater than the strike price, leading to a post-delivery cost. Either way this cost should come close to the theoretical option price given by the MRL model. As a benchmark we have also priced and replicate the options using the Black-Scholes model which assumes log-normality of the forward exchange rate (via its assumption on the spot rate).

To an option trader, the main risk of selling an option lies in the possibility of the cost required to hedge his position deviates significantly from the price he received from selling the option. Thus we use the following as a measure of the option pricing models' performances:

$$e = \frac{HC(F, \sigma, t) - P_{call}(F, \sigma, t)}{P_{call}(F, \sigma, t)}$$

where HC and P_{call} are the hedging cost and the call price respectively. The closer the error e is to zero, the better the model. The results are presented in Table 3. In general the costs required to hedge the options are lower than the option price. It is worth noting that the Black-Scholes model leads to serious underpricing in a couple of cases, while the underpricing problem of the MRL model is relatively moderate. On average the MRL model produces an absolute pricing error of 75%, while the Black-Scholes model (affected by the incidents of serious misprices) produces a pricing error of over 7,000%. However, with only 10 samples it is difficult to assess the performance of the two pricing models. The serious pricing error of the option as of January 2005 based on the Black-Scholes model is partly due to the introduction of a strong-side Convertibility Undertaking by the HKMA on 18 May 2005, that moved the forward rate from 7.7463 to 7.7635 in a day. The other serious pricing error of the option as of January 2001 based on the Black-Scholes model is partly due to the large movement of the forward rate from 7.8070 to 7.8330 on 19 October 2001. The mispricing resulting from the MRL model may partly be attributed to the measurement of the conditional mean and the speed of mean reversion of the forward exchange rate. We use the estimations based on the full sample, while F_0 and κ should change over time. The pricing/hedging evaluation is also conducted on 3-month options. With the 10-year data sample period producing

forty non-overlapping 3-month periods, the results are presented in Table 4. While the overall performance of the two models are much closer, the Black-Scholes model still produces occasional serious misprices, indicating that the forward rate does not follow the log-normal assumption at a shorter time horizon.

It should be noted the objective here is to evaluate the performance of the MRL model relative to the control Black-Scholes model and thus we keep the hedging method simple. In practice one would want to take into account among other factors such as time-varying volatility and hedging of the other hedge parameters. These are likely to reduce the above pricing errors to a more reasonable level.

VI. SUMMARY

This paper has presented a model for valuing European foreign exchange options when the forward foreign exchange rate follows a MRL process. The corresponding closed-form solutions for the option valuation are derived. The mean-reverting process has material impact on the foreign exchange rate option values and their hedge parameters. It limits the range of the exchange rate at the option's maturity and hence the uncertainty of its payoff. The effective volatility under the MRL model is less than the corresponding volatility under the Black-Scholes model. This tends to decrease the value of a simple put or call. On the other hand, the dynamics also keep the exchange rate in a small range around the mean level. As this is the region in which an option's intrinsic value is high, and the expected value $\tilde{F}$ of the forward exchange rate is higher (lower) than the level of its strike price, there is also a tendency for call (put) option values to be enhanced compared with the values under the Black-Scholes model. The numerical results show that both of these effects are important for realistic choices of model parameter values, in particular the speed of mean reversion. When the forward exchange rates follow the MRL process, the hedge parameters calculated from the Black-Scholes model may produce erroneous results even though traders consider that the market option values are correct in monetary-value terms.

Forward exchange rates and foreign exchange option prices are increasingly used to extract market expectations and views about monetary policy (for example, Bahra (1996), Söderlind and Svensson (1997), and Söderlind (1997)). As the dynamics of target exchange rates may not follow the standard lognormal process, monetary authorities who adopt target exchange rates may consider to analyse market information extracted from such derivative instruments in the light of the MRL process. For instance, information on the market's subjective probability distribution of the exchange rate can be extracted from option prices. However, the technique for extracting such information is based on the assumption that the exchange rate follows the

lognormal process. Such market information may overestimate or underestimate the risk reflected from the market data, if the exchange rate actually follows the MRL process.

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Figure 1. Spot and 3-month and 12-month forward exchange rates of HKD/USD from 2 January 1996 to 4 March 2005

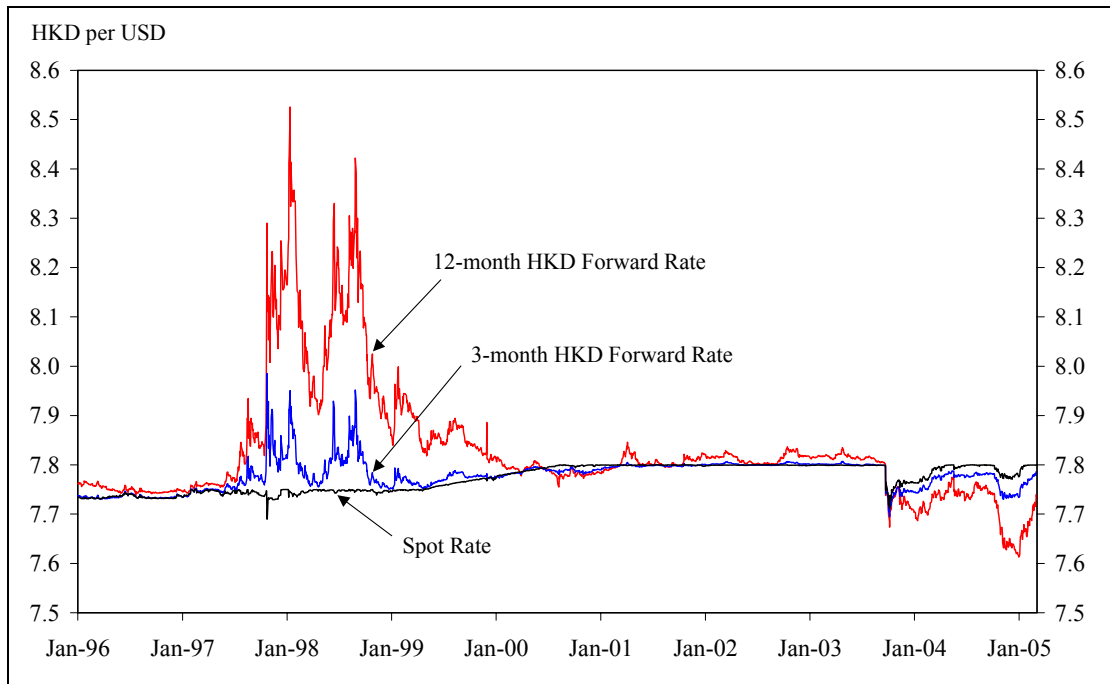


Figure 2. 3-month and 12-month $\tilde{F}$ of HKD/USD exchange rate with different value of κ with $F = 7.62$, $F_0 = 7.8$ and $\sigma = 3\%$

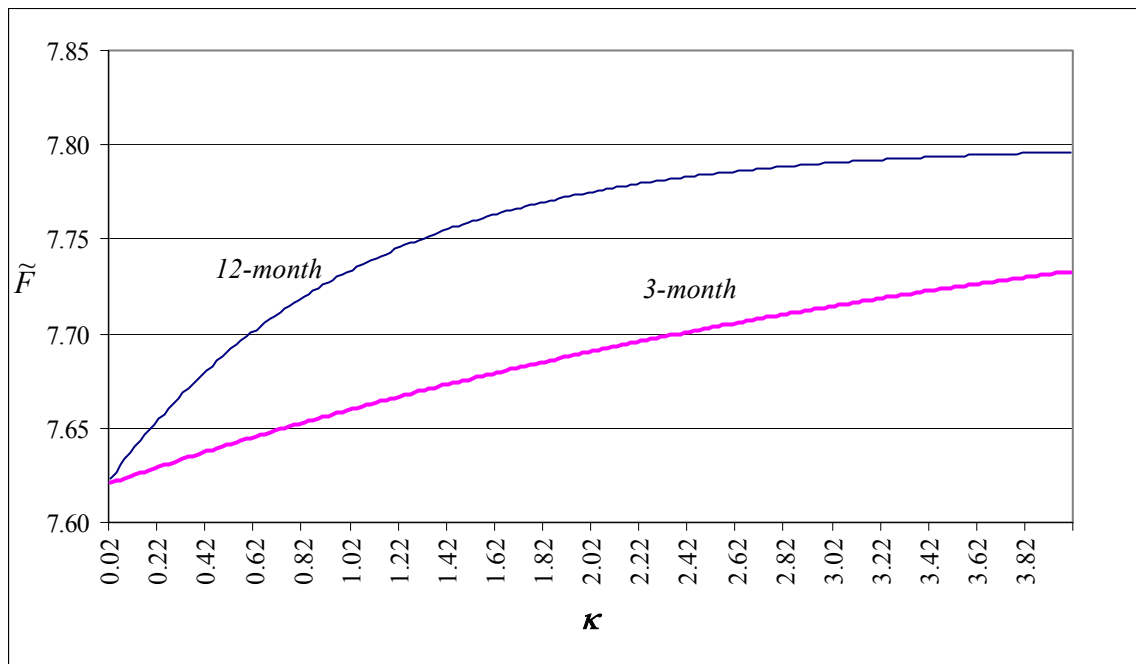


Figure 3. 12-month HKD/USD forward exchange rates F and the corresponding $\tilde{F}$ based on the MRL model with $\kappa = 1.94$ and 0.5 , and $F_0 = 7.8225$

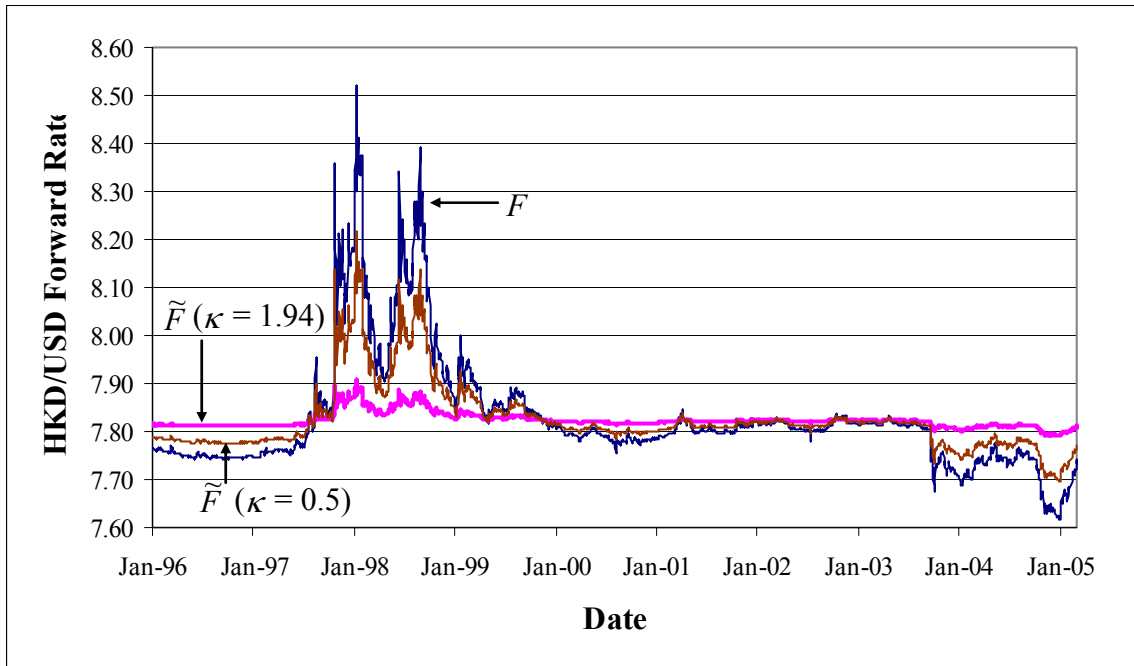


Figure 4. 12-month option values based on the MRL model with $\kappa = 1.94$ and $F_0 = 7.8225$, and the Black-Scholes (BS) model

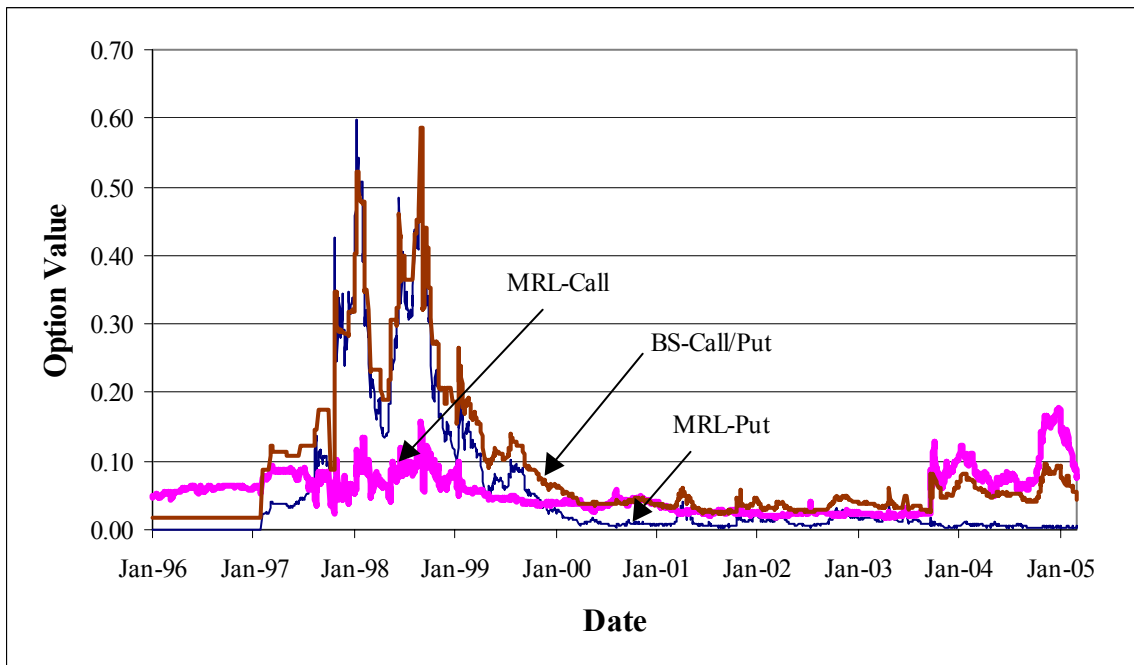


Figure 5. Deltas of the 12-month options under the MRL and Black-Scholes (BS) models

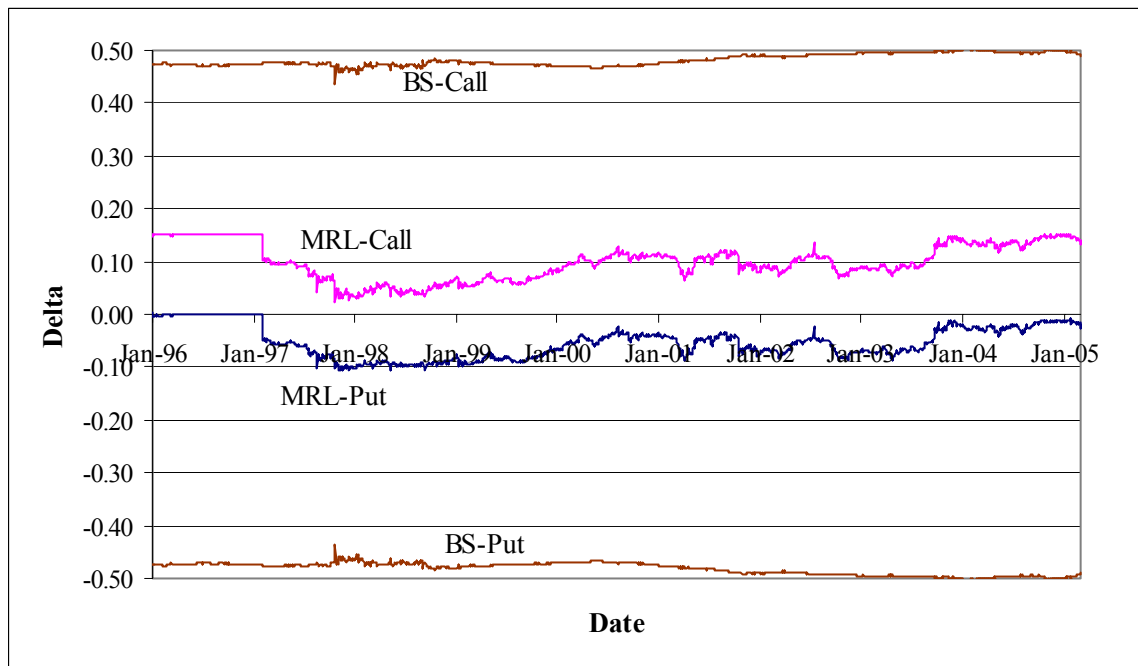


Figure 6. Gammas of the 12-month options under the MRL and Black-Scholes (BS) models

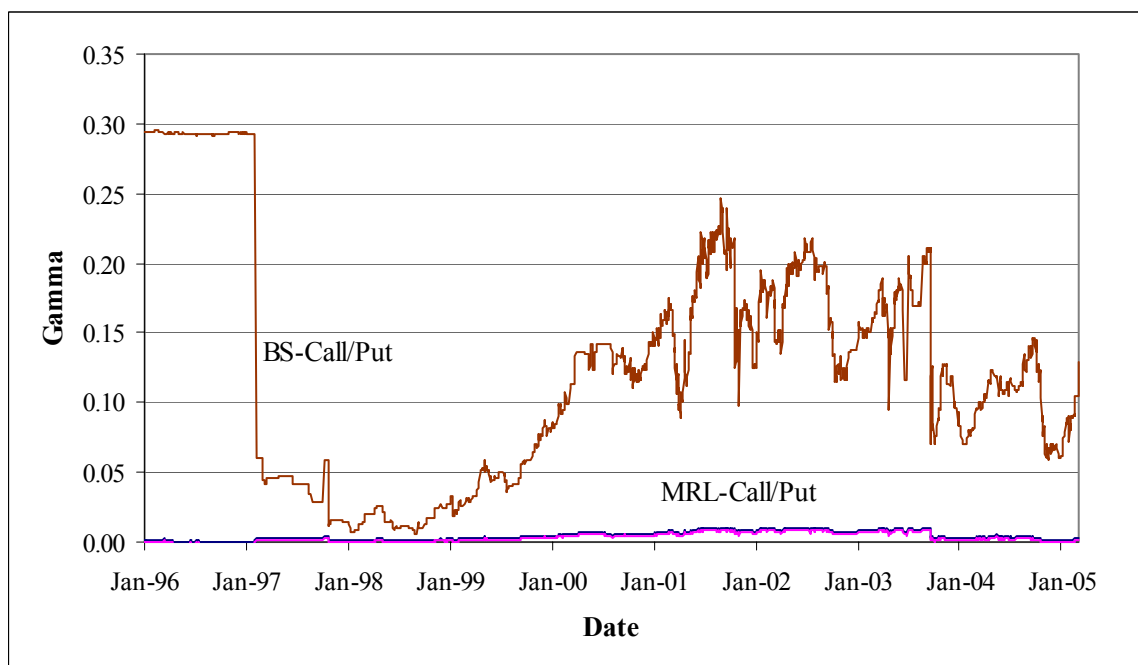


Figure 7. Vegas of the 12-month options under the MRL and Black-Scholes (BS) models

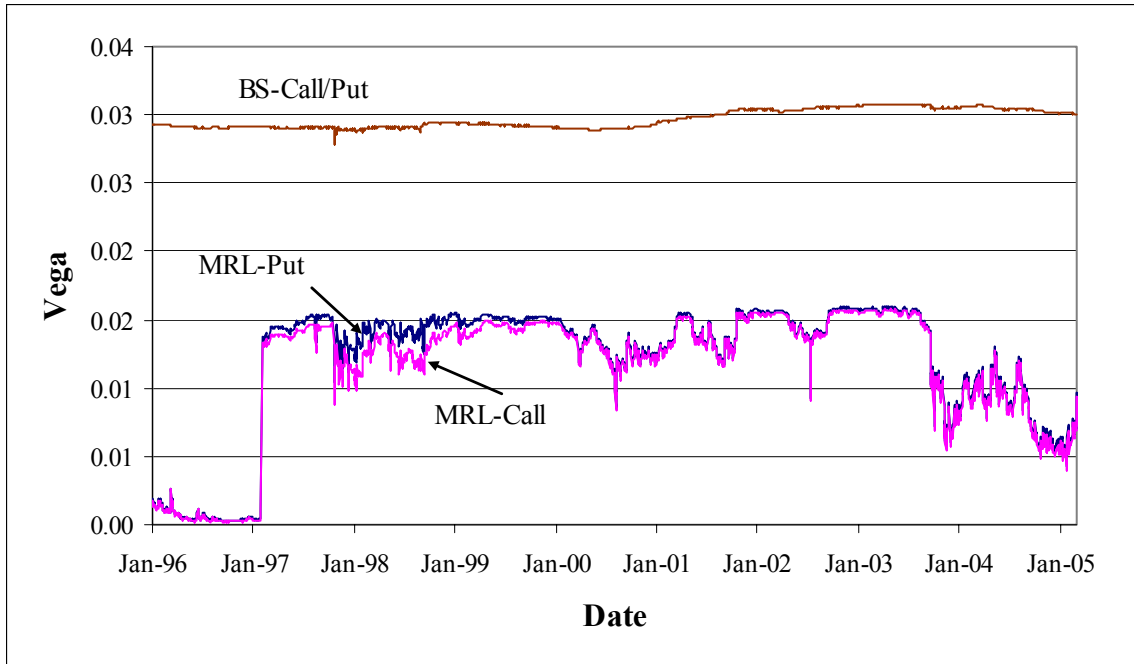


Figure 8. 3-month HKD/USD forward exchange rates F and the corresponding $\tilde{F}$ based on the MRL model with $\kappa = 7.03$ and $F_0 = 7.7796$

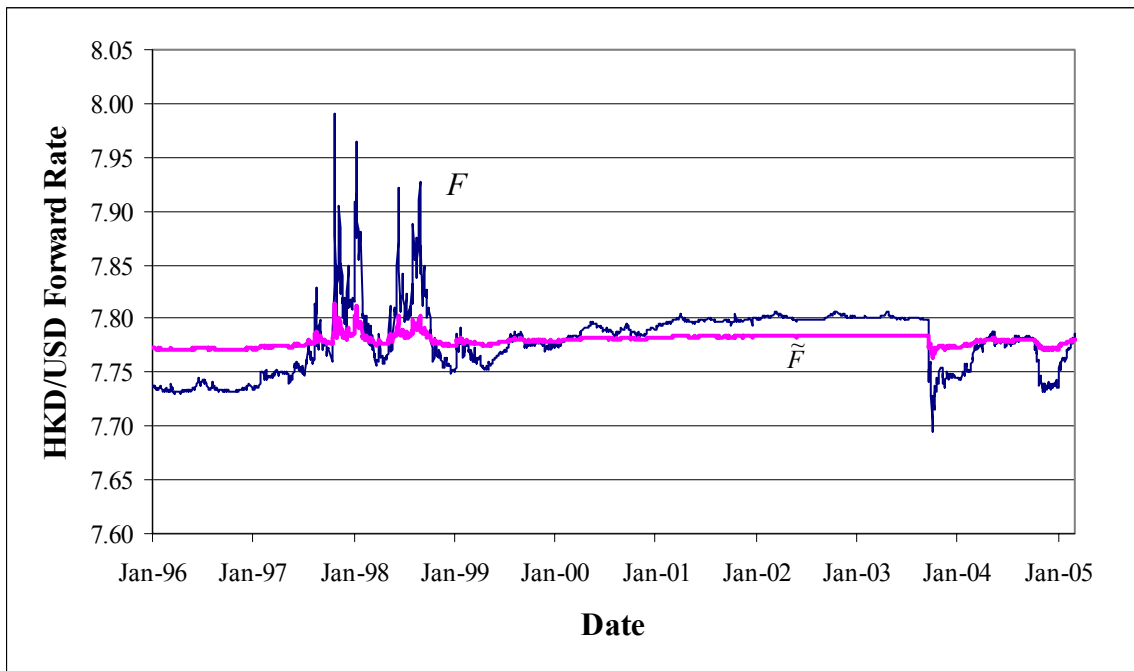


Figure 9. 3-month option values based on the MRL model with $\kappa = 7.03$ and $F_0 = 7.7796$, and the Black-Scholes (BS) model

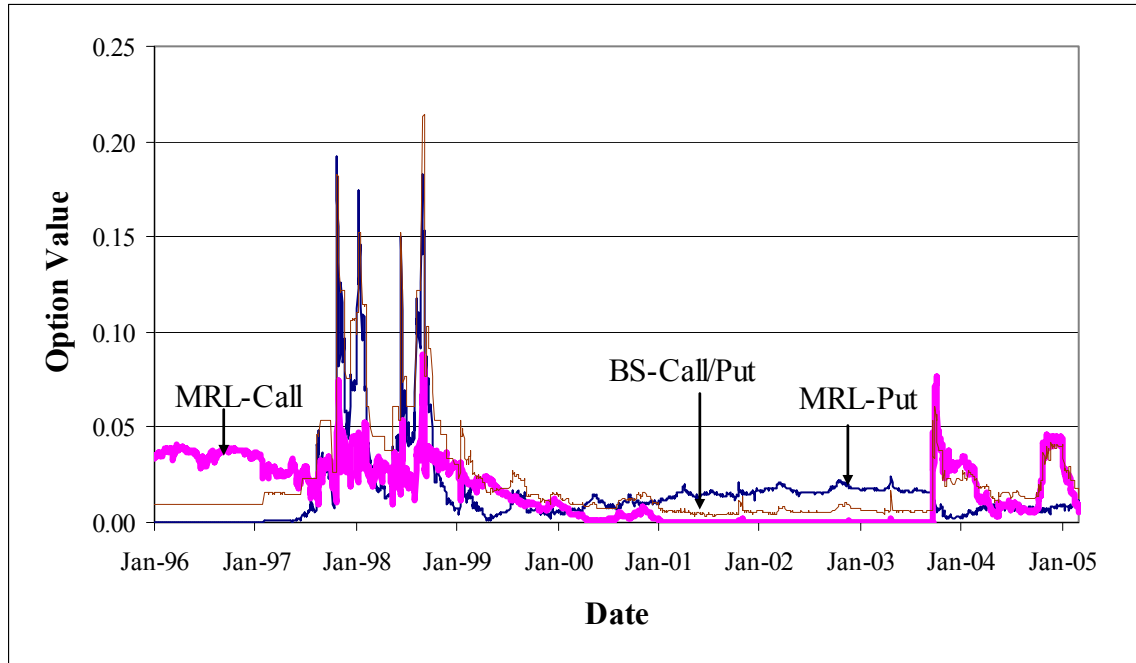


Table 1. Summary statistics of the time series of HKD/USD 3-month and 12-month forward exchange rates and the ADF test

<u>HKD/USD Forward Exchange Rates</u>								
	2 Jan 1996 – 4 Mar 2005				2 Nov 1998 – 4 Mar 2005			
	<u>3-month Rate</u>		<u>12-month Rate</u>		<u>3-month Rate</u>		<u>12-month Rate</u>	
	Level	Daily change	Level	Daily change	Level	Daily change	Level	Daily change
Mean	7.7794	1.95×10^{-5}	7.8264	-1.06×10^{-5}	7.7835	6.71×10^{-6}	7.7964	-1.5×10^{-4}
Maximum	7.9850	0.1025	8.5255	0.1675	7.8072	0.0322	7.9989	0.0782
Minimum	7.6947	-0.1100	7.6133	-0.1705	7.6947	-0.0563	7.6133	-0.0736
Standard Deviation	0.0299	0.0073	0.1218	0.0153	0.0192	0.0028	0.0578	0.0063
ADF test statistics	-3.91*	-25.31*	-3.06*	-22.85*	-2.96*	-12.61*	-3.14*	-46.56*
Observations	2,394	2,393	2,394	2,393	1,655	1,655	1,655	1,655

Notes: ADF statistics are from the Augmented Dickey-Fuller unit root test. The critical ADF value at the 5% significance level is -2.86 . * indicates significant at the 5% level.

Table 2. Estimation results of the discrete-time econometric model for the HKD/USD 3-month and 12-month forward exchange rates

Dependent variable: Log Difference of Forward Rates				
	3 Jan 1996 – 4 Mar 2005		2 Nov 1998 – 4 Mar 2005	
	3-month Rate	12-month Rate	3-month Rate	12-month Rate
α (annualised)	7.03* (0.0000) [0.0025]	1.94* (0.0000) [0.0017]	2.11* (0.0058) [0.0031]	2.11* (0.0000) [0.0016]
β	2.0515* (0.0000) [0.0011]	2.0570* (0.0000) [0.0100]	2.0518* (0.0000) [0.0019]	2.0514* (0.0000) [0.0027]
σ^2 (annualised in %)	1.44* (0.0000) [0.0000]	2.97* (0.0000) [0.0000]	0.56* (0.0000) [0.0000]	1.26* (0.0000) [0.0000]
Log likelihood	13,345.00	11,628.90	10,801.50	9,461.25
Schwarz criterion	-11.14	-9.71	-13.04	-11.42
Observations	2,393		1,655	
Wald test ($\beta = 2.0541$)	5.28*	0.08	1.49	1.05

Notes: * denotes coefficient significant at the 5% level. Figures in round parentheses are p-values and those in square brackets are standard errors. The Wald test statistics has a χ^2 distribution with one degree of freedom. The critical value of $\chi^2(1)$ at the 5% level is 3.842.

Table 3. Hedging performance of 12-month options under the Black-Scholes and MRL models

1-year option written in	BS	MRL
Jan 1996	-68%	-87%
Jan 1997	-38%	13%
Jan 1998	-96%	22%
Jan 1999	-93%	-25%
Jan 2000	-50%	-94%
Jan 2001	18347%	-121%
Jan 2002	-80%	-66%
Jan 2003	-13%	-14%
Jan 2004	714%	-172%
Jan 2005	54068%	-140%
Average of the absolute values	7357%	75%
Average of absolute values excluding samples with extreme pricing errors (> 100% or < -100%)	62%	46%

Note: negative means the option is overpriced

Table 4. Hedging performance of 3-month options under the Black-Scholes (BS) and MRL models

3-month option written in	BS	MRL		3-month option written in	BS	MRL
Jan 1996	-65%	-88%		Jan 2001	645%	-191%
Apr 1996	12%	-103%		Apr 2001	-31%	-108%
Jul 1996	-17%	-78%		Jul 2001	24%	-123%
Oct 1996	-32%	-97%		Oct 2001	74%	-106%
Jan 1997	26%	-80%		Jan 2002	-59%	-66%
Apr 1997	15%	-178%		Apr 2002	-33%	-93%
Jul 1997	-28%	-16%		Jul 2002	11%	-103%
Oct 1997	-34%	34%		Oct 2002	-23%	-56%
Jan 1998	-40%	-235%		Jan 2003	-22%	-76%
Apr 1998	-19%	22%		Apr 2003	-11%	-71%
Jul 1998	-87%	80%		Jul 2003	14%	11%
Oct 1998	-66%	-38%		Oct 2003	28%	-95%
Jan 1999	-21%	-53%		Jan 2004	110%	-165%
Apr 1999	-45%	-70%		Apr 2004	131%	-183%
Jul 1999	-90%	-34%		Jul 2004	242%	-307%
Oct 1999	-8%	-85%		Oct 204	7%	-66%
Jan 2000	727%	-121%		Jan 2005	1389%	-278%
Apr 2000	-40%	-79%		Apr 2005	25%	-55%
Jul 2000	677%	-219%		Jul 2005	-17%	-78%
Oct 2000	118%	-205%		Oct 2005	6%	-90%

	BS	MRL
Average of the absolute values	127%	106%
Average of the absolute values excluding samples with extreme pricing error (> 100% or < -100%))	33%	64%

Note: negative means the option is overpriced