



**A VAR FRAMEWORK FOR FORECASTING HONG KONG'S
OUTPUT AND INFLATION**

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Abstract

This paper develops a multivariate time series model to forecast output growth and inflation in the Hong Kong economy. We illustrate the steps involved in designing and building a vector autoregression (VAR) forecasting model, and consider three types of VAR models, including unrestricted, Bayesian and conditional VARs. Our findings suggest that the Bayesian VAR framework incorporating external influences provide a useful tool to produce more accurate forecasts relative to the unrestricted VARs and univariate time series models, and conditional forecasts have the potential to further improve upon the Bayesian models. In particular, a six-variable Bayesian VAR including domestic output, domestic inflation, domestic investment, world GDP, the best lending rate, and import prices appears to generate good out-of-sample forecasts results.

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Executive Summary:

- *Central banks around the world are increasingly in favour of the ‘suite of models’ approach in forecasting future economic conditions. The forecasting work at the HKMA currently involves two types of models: a simultaneous-equations structural model producing forecasts for output as well as inflation on a quarterly basis, and an indicator model for forecasting inflation at monthly frequency.*
- *This paper develops a multivariate time series model to forecast the two main economic aggregates of concern – output growth and inflation. We illustrate the steps involved in designing and building a real-time vector autoregression (VAR) forecasting model for the Hong Kong economy. A large set of forecast experiments are carried out employing three types of VAR models, including unrestricted, Bayesian and conditional VARs.*
- *We find that a six-variable VAR including domestic output, domestic inflation, domestic investment, world GDP, the best lending rate, and import prices appears to generate good out-of-sample forecasts results. Our results also show that Bayesian VARs can generate more accurate forecasts relative to the unrestricted VARs and univariate time series models while conditional forecasts have the potential to further improve the Bayesian models’ forecast performance.*
- *In summary, our findings suggest that the Bayesian VAR framework incorporating external influences provide a practical and promising tool to produce more accurate forecasts for output growth and inflation in Hong Kong. This could serve to complement the structural model and help to improve our forecast accuracy by providing an alternative view on the existing forecasts.*

I. INTRODUCTION

Central banks around the world are increasingly in favour of the ‘suite of models’ approach in forecasting future economic conditions. Under this approach, forecasts generated from different types of models are simultaneously taken into consideration for economic projections, therefore improving the quality of forecasts by reducing information loss and model uncertainty associated with a single model. Macro-econometric or general equilibrium models are usually employed as core models, which are often supplemented with various types of statistical time series models.

The forecasting work at the HKMA currently involves two types of models: a simultaneous-equations structural model (SFM) producing forecasts for output as well as inflation on a quarterly basis, and an indicator model for forecasting inflation at monthly frequency. The SFM consists of a system of behaviour equations based on the error correction mechanism, where the output forecast is the aggregation of the forecasts of the GDP expenditure components, and the inflation forecast is based on a generalised Phillips curve.¹ The indicator model, on the other hand, is a pure statistical model employing techniques such as factor analysis and combination forecasts to exploit information embedded in a large number of economic indicators, and inflation is forecasted based on its past statistical relationships with various indicators.²

The purpose of this study is to develop a time series model to simultaneously forecast the two main economic aggregates of concern – output growth and inflation.³ This could serve to complement the structural model and provide an alternative view on our existing forecasts. Our model is based on the vector autoregression (VAR) method, which since its introduction in Sims (1980), has gained widespread acceptance as a useful forecasting tool and a convenient framework for illustrating short-term aggregate relationships among key variables of the economy.

¹ See Ha, Jiming, Cynthia Leung, and Chang Shu (2002), and Kong, Janet and Cynthia Leung (2004).

² See Liu, Li-gang, Jian Chang and Andrew Tsang (2006).

³ As discussed in Litterman (1986), one distinct advantage of the time series approach relative to the macro-econometric models is that the former does not require judgemental adjustment. “Thus, it is a scientific method which can be evaluated on its own, without reference to the forecaster running the model.”

A standard VAR model consists of a dynamic system of n linear equations for n endogenous variables, in which each variable is explained by its own lags, as well as the current and past values of the remaining variables. In contrast with structural models, VARs do not attempt to model the underlying structure of the economy. Instead, they try to identify statistical relationships based on variables' past interactions and then use this information to project likely future values. Like other time series models, they are relatively easy to construct and maintain, and have the advantage of not imposing a prior restriction on the interactions among variables.

The structure of the standard VARs, however, leads to a significant drawback.⁴ A large number of parameters have to be estimated even with only moderately large number of variables and lags. This "over-parametrisation" problem often results in good in-sample fit but poor out-of-sample forecast performance. Bayesian technique is therefore employed in this study. Proposed in Litterman (1981) and Doan, Litterman and Sims (1984), the Bayesian method imposes inexact prior restrictions on the coefficient matrix and has proved to be a useful way to reduce the dimensionality problem associated with unrestricted VARs and thus produce more accurate forecasts.⁵

In view of the highly open nature of the Hong Kong economy and its currency board exchange rate arrangement, we include in our six-variable VAR three exogenously-determined variables, together with three domestic variables. The three exogenous variables represent influences from external demand, the US interest rate, and external price, and the three domestic variables represent output, inflation, and an additional domestic activity indicator. Variants of this specification have been used to study relative importance of domestic and foreign shocks in the Hong Kong economy (Genberg, 2003 and Genberg and Pauwels, 2005).

We carry out a large set of forecast experiments employing three types of VAR models, including unrestricted, Bayesian and conditional VARs, and compare the models' out-of-sample forecast performance. Our findings suggest

⁴ A standard VAR model uses the same lag length for all the variables in the system, so the number of parameters to be estimated gets large very quickly as the number of lags increases, even though some of which may be insignificant.

⁵ There is another drawback of VAR models in general. Unlike structural forecasting models, the a-theoretical nature of non-structural VARs makes it difficult to explain the driving forces underlying the computed forecasts.

that unrestricted VARs tend to produce poor out-of-sample forecasts, while BVARs could generate more accurate forecasts relative to the unrestricted VARs and univariate time series models. In cases where relatively accurate forecasts of external variables could be obtained in advance, conditional BVARs (CBVARs) appear to be able to further improve upon the unconditional BVARs, evidencing the importance of external influences on the Hong Kong economy, particularly on the output dynamics.

The paper is organised as follows. Section II introduces various VAR models and discusses some model specifics. Section III lays out the model for the Hong Kong economy, including variables selection process, and estimation and forecasting procedures. Section IV describes model evaluation criterion and compares out-of-sample forecast performances of alternative model specifications and alternative candidate variables. Section V presents forecast results from some preferred models, and Section VI concludes.

II. METHODOLOGY

Unrestricted VAR

A reduced-form VAR with lag length p can be expressed as

$$Y_t = C + \Phi_1 Y_{t-1} + \dots + \Phi_p Y_{t-p} + \Psi_t \quad (1)$$

where Y_t denotes $n \times 1$ vector of variables to be forecasted, C represents a $n \times 1$ vector of constant terms, $\Phi_1, \dots, \Phi_p$ are $n \times n$ matrices of coefficients, and Ψ_t measures $n \times 1$ vector of white noise error terms. Equation (1) states that the current value of each of the n variables can be expressed as a weighted average of the recent past of all the variables plus a disturbance term that contains all the other exogenous disturbances that are not captured by the n -equation model.

Forecasting using VARs is straightforward. Each equation in the unrestricted VAR system can be estimated efficiently using ordinary least squares (OLS).⁶ Assuming the relationship identified in the past continues to hold in the

⁶ When we restrict our system so that not all equations contain the same variables (see equation (4) below), we also consider the Seemingly Unrelated Regression (SUR) estimator. In the presence of correlation among error terms, the SUR in principle should enable us to achieve some efficiency gains in estimation and forecasting.

forecast period, the h period ahead forecasts can be obtained by simply substituting the estimated coefficients and the past values of Y as in equation (2).

$$\begin{aligned}
 \hat{Y}_{t+1} &= \hat{C} + \hat{\Phi}_1 Y_t + \dots + \hat{\Phi}_p Y_{t-p+1} \\
 \hat{Y}_{t+2} &= \hat{C} + \hat{\Phi}_1 \hat{Y}_{t+1} + \hat{\Phi}_2 Y_t \dots + \hat{\Phi}_p Y_{t-p+2} \\
 &\vdots \\
 \hat{Y}_{t+h} &= \hat{C} + \hat{\Phi}_1 \hat{Y}_{t+h-1} + \dots + \hat{\Phi}_{h-1} \hat{Y}_{t+1} + \hat{\Phi}_h Y_t \dots + \hat{\Phi}_p Y_{t-p+h}
 \end{aligned} \tag{2}$$

However, this simple forecasting framework tends to result in large out-of-sample forecasts errors, as the “over-parameterisation” problem commonly observed in unrestricted VARs often leads to large standard errors on the estimated coefficients.⁷ Various solutions have been proposed in the forecasting literature, mostly concerning imposition of prior constraints on some parameters so that less information is required from the data. For example, p could be selected by minimising certain lag selection criterion (e.g., Akaike or Schwartz information criteria), which penalises the increase in the measured in-sample fit resulting from the addition of more lags. Such constraints are equivalent to restricting the coefficients beyond the optimally selected lag length to zero.

Bayesian VAR

An alternative solution is the Bayesian approach.⁸ The idea behind the Bayesian procedure is to assign less weight to the more distant lags but without necessarily restricting it to zero, while also allowing this assumption to be overturned if there is strong evidence from the data indicating otherwise. In practice, this is implemented by specifying prior distributions for the coefficients, which are treated as random variables with pre-specified means, and with the tightness of the distributions being determined by a set of hyperparameters. This information is then incorporated in the estimation.

⁷ This could be because the coefficients are actually zero as indicated, or the data might not be rich enough to provide sufficiently precise estimates of nonzero coefficients.

⁸ See Litterman (1981, 1986), Doan, Litterman, and Sims (1984), and Sims (1992).

Many Bayesian VARs are set up based on the so-called Minnesota (or Litterman) prior, which follows the prior assumption that most macroeconomic series can be described as a random walk. As detailed in Doan, Litterman and Sims (1984), each element in the coefficient matrix is an independent, normally distributed random variable, with the prior mean of Φ_1 set to equal the identity matrix and the prior mean of all the other coefficients $\Phi_l, l > 1$ set to be zero.

We follow Doan (2004) to specify the standard deviation of the prior distribution for lag l of variable j in equation i (i.e., the ij th element of the l th lag coefficient matrix Φ_l), denoted by $S(i, j, l)$, according to a simple form as equation (3),

$$S(i, j, l) = \frac{\{\gamma g(l) f(i, j)\} \sigma_i}{\sigma_j} \quad (3)$$

where $f(i, j) = g(l) = 1$, if $i = j$; and $f(i, j) = \lambda_{ij}$ with $0 \leq \lambda_{ij} \leq 1$, otherwise. Note γ is the standard deviation on the first own lag, measuring the overall tightness; $g(l)$ measures the tightness on lag l relative to lag 1 and is assumed to have a harmonic shape with a decay factor of d ; $f(i, j)$ represents the tightness on variable j in equation i relative to variable i , and σ_i, σ_j are the estimated standard error of the univariate autoregression for variable i, j with their ratio to correct for the different magnitudes of the variables in the system. This procedure therefore allows individual prior variances to be determined by only a small number of hyperparameters. Depending on the value of these hyperparameters, a BVAR model could collapse into an unrestricted VAR, random walk models or univariate autoregressive models.

Conditional forecasts

If strong priors about the future paths of certain variables (e.g., the exogenous variables) exist, the forecast quality of the VARs could potentially be improved by fixing the value of these variables over the forecast period in advance. Forecasts generated by imposing such constraints are called conditional forecasts. In real-time application, conditional forecasts could also be used to handle the staggered release of data. We follow the conditional forecasting procedure described in Doan (2004) and apply this technique to our Bayesian VARs. The corresponding VAR models are referred to as CBVARs.

III. VAR FORECASTING FOR THE HONG KONG ECONOMY

a. Model structure

Some recent research has pointed out the importance of external factors on the output and price determination in a small open economy like Hong Kong (Genberg, 2003 and Genberg and Pauwels, 2005). We therefore consider a system of structural equations taking account of the external influences as expressed in equation (4)

$$\begin{aligned} Ay_t &= Bx_t + A(L)y_{t-1} + B(L)x_{t-1} + u_t \\ Cx_t &= C(L)x_{t-1} + v_t \end{aligned} \quad (4)$$

where y_t and x_t are vectors of domestic and external variables of interests, respectively, u_t and v_t are vectors of residuals, and L is the lag operator. Domestic variables are assumed to be determined by both their own lagged values, and the current and past values of external variables. External variables are, however, viewed as exogenous, and assumed to follow a multivariate autoregressive process.

We can then derive the reduced-form VAR representation consisting of a domestic block and an exogenous block as equation (5),

$$\begin{pmatrix} y_t \\ x_t \end{pmatrix} = \begin{pmatrix} D_{11}(L) & D_{12}(L) \\ 0 & D_{22}(L) \end{pmatrix} \begin{pmatrix} y_{t-1} \\ x_{t-1} \end{pmatrix} + \begin{pmatrix} \varepsilon_t \\ \eta_t \end{pmatrix} \quad (5)$$

where $D_{ij}(L)$ capture the reduced form lag structure, and ε_t, η_t are reduced form errors. Equation (5) implies that development of domestic variables is assumed to have no impact on external variables. In this study, we focus on a six-variable VAR with three domestic variables and three external variables.

b. Forecast objective

Year-on-year growth rate of real GDP and the composite consumer price index (CCPI) are chosen to be our targeted output growth and inflation measures. Since RGDP data are only available each quarter, our VAR models are

at a quarterly frequency. A forecast horizon of one year is the focus of this study, in part because experiences show that VAR models tend to perform well around this horizon.⁹ The models are therefore forecasted four-quarter ahead to generate dynamic forecasts at $t+4$. Regarding estimation sample, we consider both an entire sample starting from 1985:1, shortly after the adoption of the Linked Exchange Rate system, and a rolling window of 10 years extracted from the complete sample. Our forecast evaluation period is 20 quarters. Table 1 summarises these model specifics.

Table 1. Model specifics

Forecast Object	RGDP and CCPI
Measure of Object	Year on year growth rate
Forecast Horizon	Four quarter ahead
Model Frequency	Quarterly
Forecast Evaluation	2001:3 - 2006:3
Estimation Sample	Recursive estimation starting from 1985:1 Rolling estimation with 10-year fixed window

c. Variable pre-selection

While the final specification of a VAR is largely determined by the data, the choice of variables to include in the system is still underpinned by theoretical foundation. Our selection of both domestic and external variables is largely based on theory as well as previous studies in which certain variables are identified to be useful in explaining or forecasting Hong Kong's output or inflation.¹⁰ All candidate variables used in the VARs, BVARs, and CBVARs are presented in Table 2. Our final choice of baseline variables is based on BVAR models' out-of-sample forecast performance.

As a small open economy with strong reliance on external demand, HK's GDP is significantly affected by its major trading partners' growth conditions. For our first external indicator, we therefore consider the GDP measures of a few

⁹ Lupoletti and Webb (1986) show that the VAR forecasts were more accurate at four and six quarters ahead as compared to shorter horizons relative to forecasts made by forecasting services. On the other hand, forecast performance of VARs tends to deteriorate rapidly as forecast horizon lengthens further.

¹⁰ For example, Genberg and Pauwels (2005) conduct a block exogeneity test in a VAR system and found foreign factors (the US 3-month Treasury bill rate, the unit value of HK imports) have significant effects on domestic prices as measured by the GDP deflator and some other domestic variables including nominal wage rates, the nominal property prices, and the unemployment rate.

foreign economies, including the mainland China, the US, and a weighted average of Hong Kong's major trading partners.¹¹

Table 2. Description of variables used in the VAR

Variable	Description	ADF results
CCPI	Composite consumer price index	I(2)
RGDP	Real GDP	I(1)
PCER	Real personal consumption expenditure	I(1)
GDFCFR	Real domestic fixed capital formation	I(1)
PIV	Real private investment	I(1)
DD	Domestic demand	I(1)
NX	Net exports	I(1)
UMR	Unemployment rate	I(0)
PROP	Nominal property price index	I(1)
WAGEN	Nominal wage rate	I(2)
JVACANCY	Job Vacancy index	I(1)
NEERI	Nominal effective exchange rate index	I(1)
REERI	Real effective exchange rate index	I(1)
FFTR	US Federal Funds target rate	I(0)
USTB3M	US 3-month Treasury bill rate	I(0)
HIBOR3M	HK 3-month interbank lending rate	I(0)
BLRA	Best lending rate	I(0)
UVIM	Unit value of imports into HK	I(0)
UVIRM	Unit value of retained imports into HK	I(1)
WCPI	World CPI, trade-weighted	I(1)
CNCPI	The mainland China's CPI	I(2)
WGDP	World GDP, trade-weighted	I(2)
CNRGDP	The mainland China's real GDP	I(2)
USRGDP	US real GDP	I(1)

Source: C&SD and staff estimates.

Also, external interest rate movements are often found to play an important role in explaining the economic activities in HK (Genberg, 2003 and Genberg and Pauwels, 2005).¹² Under the Linked Exchange Rate system, HK's interest rates have largely tracked the development of the US interest rates. However, especially in recent years, the domestic liquidity conditions sometimes contribute to a persistent wedge between the two interest rates. For the second external indicator, we therefore consider both US Federal Funds target rate,

¹¹ World RGDP and China's GDP have been included in the export equations in the SFM.

¹² For countries with independent monetary policy, an alternative channel is the direct response of interest (policy) rates to the economic conditions measured by inflation and/or output.

US 3-month Treasury bill rate, and some measures of domestic interest rates. The US interest rates certainly satisfy the exogeneity condition, while the local interest rates measure more closely the domestic financing conditions. Although variables in the latter group are not strictly exogenous, they are still expected to be largely dependent on the movement of the US interest rates under the current exchange rate arrangement.

Lastly, external prices could influence consumer prices directly through imported consumption goods, and indirectly through its impact on domestic production to the extent that imports are used as intermediate inputs. For our final external indicator, we therefore consider a variety of external price variables including import price index, retained import price index, a weighted average of consumer prices in the major trading partners, and consumer price for Mainland China.¹³ Effective exchange rates, which in theory could affect both output and inflation through the external trade channel, are also included as candidate variables.¹⁴

In addition to our subjects of forecast – output and inflation, we contemplate various candidates for the third domestic activity indicator. Motivated by the VAR forecasting literature, in which different components of GDP (particularly investment) are used, we consider measures of domestic demand, domestic consumption, investment, and net exports. On the other hand, since property price accounts for around 30% of HK's CCPI, property price index is chosen as another candidate. Some domestic demand variables such as the unemployment rate (according to the Phillips curve), job vacancy indicator and nominal wage rate (which were identified as important in forecasting the non-rental component of the CCPI inflation) are also considered.^{15,16}

d. Variable transformation

All variables are adjusted for seasonality before feeding into the model. We use the x-12 method as specified in the E-views 5.0, except for the unemployment rate, which is already a seasonally adjusted series from C&SD, and various interest rates. Seasonally adjusted data appears to be particularly

¹³ China's CPI, world CPI and property prices have been included in the inflation equation in the SFM.

¹⁴ Given the Linked Exchange Rate arrangement, nominal effective exchange rate is determined exogenously, depending on exchange rates between HK's trading partners and the US.

¹⁵ See Liu, Li-gang, Jian Chang and Andrew Tsang (2006).

¹⁶ Genberg and Pauwels (2003) argue that both the nominal wage rate and property prices are important for the evolution of inflation in Hong Kong.

compatible with the BVAR models since, as Hamilton (1994, p.362) pointed out, the Minnesota prior is not well suited for seasonal data. It is therefore suggested seasonally adjusted data be used or seasonal dummies be included in the regression before employing this prior distribution.

All variables are measured in logarithms, except for interest rates and the unemployment rate. While the Augmented Dickey Fuller or the Phillips-Perron tests sometimes suggest different degree of stationarity for different variables, we consider both level and first-difference specifications in the estimation where appropriate. Note that Sims et al. (1990, p.136) state that “Because the Bayesian approach is entirely based on the likelihood function, which has the same Gaussian shape regardless of the presence of nonstationarity, Bayesian inference need take no special account of nonstationarity”.

e. Estimation and forecasting

Our baseline model consists of RGDP, CCPI, investment (gross domestic capital formation), world GDP, the best lending rate, and unit value of imports into HK.¹⁷ We also incorporate a SARS dummy to capture the sharp decline of GDP in 2003 Q2 as a result of the SARS epidemic. To reduce computational burden, we pre-fix the same number of lags for all independent variables in unrestricted VARs.¹⁸ Note that VAR forecasting results appear to be sensitive to the number of lags used, we therefore select the optimal lag lengths based on their out-of-sample forecast performance.

In estimating the BVARs, the block triangular structure between domestic and external variables is imposed as in equation (5). We follow Doan (2004) in choosing 0.1 and 0.2 for the overall tightness γ and 1 and 2 for the harmonic lag decay parameter d , which have proved to work well in practice. The entire $f(i, j)$ function is first specified in which higher (lower) weights are assigned to variables that are believed to be more (less) important. It is suggested in the literature that although specifying an exact restriction is possible, the best forecasting performance usually occurs when the prior restrictions are not imposed exactly. The final choices of hyperparameters for the BVARs are based on the models' out-of-sample forecast performance.

¹⁷ Evaluation of alternative candidate variables under some best-performing Bayesian models is reported and discussed in Appendix I. Our results suggest that the baseline choice of variables exhibit higher predictive power in general relative to alternative candidate variables.

¹⁸ This is also motivated by previous finding that formal tests (Akaike or Schwartz information criteria) for the optimal lag lengths often produce different results (see Genberg, 2005).

In the case of conditional forecasts, we focus on one experiment where the future paths of all external variables in the BVARs are assumed to be known at the time of the out-of-sample forecast, and investigate the potential improvement that could arise from such assumptions. These CBVARs share the merits of BVARs, and also address the issue of less accurate forecasts for exogenous variables owing to the exclusion of their determinants (except for their own lags) from our VAR system. Note that our assumption can be justified on the ground that in real-time application, it may be possible to obtain relatively accurate forecasts on the US interest rates and world GDP growth from external information providers who specialise in providing such forecasts.

Estimations based on two types of estimating sample are considered. In addition to recursive estimation, which holds the starting period of the regression sample unchanged, we also consider rolling estimation with a fixed rolling window of 10 years. In both cases, parameters are estimated recursively by constantly updating the information set when the forecast moves forward in time and dynamic forecasts are generated as would be needed in actual forecasting practice. Specifically, in recursive estimation, the model is first estimated using data from 1985:1 to 2000:3, and a four-quarter-ahead forecast for 2001:3 is generated. Moving ahead one quarter, the model is re-estimated with data ending 2000:4 and the coefficients are updated before producing the forecast for 2001:4. This iterative process is carried out until the end of the estimation period. The rolling estimation, on the other hand, selects a fixed length of data window on which the estimation is based by excluding observations from the distant past as the estimation goes forward. Compared with the recursive estimation, the rolling estimation uses the most updated information and in principle could better accommodate the possibility of structural changes over the complete sample, but at the cost of incorporating less information in the sample.

IV. FORECAST PERFORMANCE COMPARISON

a. Forecast accuracy criterion

In order to evaluate the forecasts of output growth and inflation generated from various models, we focus on the accuracy (and reliability) of forecasts over some historical period. Naturally, a forecasting scheme would be given more weight if it can generate relatively accurate forecasts comparing with other schemes.

Depending on the purpose (final use) of the model, e.g., whether to produce more accurate forecasts or to better capture the turning points, different accuracy criteria, which might yield different rankings of specifications, should be used. The most commonly used accuracy criteria are some measures of the averaged forecast error, typically the root mean squared error (RMSE) or mean absolute error, and they are usually computed out of sample to mimic the real-time forecasting experience.

In this study, forecast performance is assessed using the RMSE criterion. The RMSE is defined as $\sqrt{\frac{1}{T} \sum_{t=1}^T (\Delta^4 y_{t+4} - \Delta^4 \hat{y}_{t+4|t})^2}$, measuring the standard deviation of the forecasted year-on-year output growth or inflation ($\Delta^4 \hat{y}_{t+4|t}$) from the actual data ($\Delta^4 y_{t+4}$) over the specified forecast period.

In addition to the three types of VAR models, we consider two univariate time series models: a “naïve” random-walk model (RM) where the year-on-year growth forecast is equal to the growth rate four quarters ago, as well as a univariate autoregressive model (AR) which only uses lagged dependent variable for forecasting.¹⁹ Various VAR models are evaluated against the “naïve” model. The relative RMSE between a VAR model and the “naïve” model is considered as our model evaluation criterion.

b. Evaluation of models

This section describes the forecast performance of various specifications of our six-variable VAR. In all the experiments considered, models estimated in levels produce smaller forecast errors for the year-on-year growth measures than those estimated in first-differenced data. Models including the SARS dummy generally perform better than those that do not. The Seemingly Unrelated Regression models could improve upon the unrestricted VAR models in some cases, but only marginally at best. In the following, we focus our discussion on various VAR models with level specifications.²⁰

¹⁹ RW: $\Delta^4 y_{t+4} = \Delta^4 y_t + \varepsilon_{t+4}$, and AR: $\Delta y_{t+1} = \mu + \sum_{i=1}^p \phi_i \Delta y_{t+1-i} + \varepsilon_{t+1}$.

²⁰ Forecast results of the above findings are not reported here due to space limits, and are available on requests.

Table 3 compares forecast accuracy of the best-performing VAR models against benchmark univariate time series models. Recall that γ and d denote overall tightness and decay factor in a Bayesian model. All VAR models use the same baseline choice of variables. The standard deviations of the averaged forecast errors (RMSEs) over the period from 2001:3 to 2006:3 are presented. Columns 1,3 (2,4) report the RMSEs produced by recursive (rolling) regression, while the numbers in parentheses are the RMSEs of the associated forecasts relative to the “naïve” forecasts. A value less than one indicates that the given model produces more accurate forecasts than the “naïve” model.

Table 3. RMSE of VAR forecasts 2001 – 2006^a

	RGDP growth		Inflation	
	Recursive	Rolling	Recursive	Rolling
RW	4.68 (1.00)	4.68 (1.00)	1.69 (1.00)	1.69 (1.00)
AR ^b	3.73 (0.80)	4.80 (1.03)	2.18 (1.29)	2.16 (1.28)
VAR ^b	3.82 (0.82)	5.47 (1.17)	2.03 (1.20)	2.82 (1.67)
BVAR ($\gamma=0.1, d=1$)	2.66 (0.57)	2.98 (0.64)	1.83 (1.08)	1.08 (0.64)
BVAR ($\gamma=0.2, d=1$)	2.01 (0.43)	2.39 (0.51)	1.68 (0.99)	1.36 (0.80)
CBVAR ($\gamma=0.1, d=1$)	1.54 (0.33)	2.43 (0.52)	1.32 (0.78)	1.06 (0.63)
CBVAR ($\gamma=0.2, d=1$)	1.48 (0.32)	2.13 (0.46)	1.46 (0.86)	0.99 (0.59)
CBVAR ($\gamma=0.2, d=2$)	1.49 (0.32)	2.59 (0.55)	0.95 (0.56)	1.13 (0.67)

Note:

a. RMSEs are measured in %, and the numbers in parentheses are the ratio of the corresponding RMSE to the RMSE of the RW forecasts.

b. Lag lengths are fixed at $p=4$.

Source: staff estimates.

Several findings arise from Table 3. In terms of model specifications, all except for one VAR model produce more accurate forecasts for output than the “naïve” model, although it seems more difficult for the VAR models to outperform the “naïve” model in inflation forecasting. Within the class of VAR models, unrestricted VARs tend to perform poorly, recording larger out-of-sample forecast errors, possibly due to problems of over-parameterisation and over-fitting explained in section II. Significant improvement is observed once we move to the Bayesian method. Table 3 shows that the best-performing BVAR models improve considerably over the RW model and the unrestricted VARs, with $BVAR(\gamma = 0.1, d = 1)$ and $BVAR(\gamma = 0.2, d = 1)$ outperforming the “naïve” model in terms of RMSEs by 57% and 36% respectively for the RGDP growth and inflation forecasting.

In addition, our results found that the best-performing Bayesian models for the output and inflation may not be the same, depending on choice of estimation sample and specification of prior distributions for the coefficient matrix. For unrestricted VARs, recursive estimation appears to perform better for both output and inflation forecasts, possibly because more data help to ease the associated degree of freedom problem.²¹ For the same BVARs, rolling estimation with more updated information tends to generate more accurate forecasts for inflation and recursive estimation with more information tends to produce better forecasts for output.

Within the BVAR models, relatively loose prior ($\gamma = 0.2$) combined with recursive estimation appears to perform the best for RGDP growth forecasting (Figure 1), while relatively tight prior ($\gamma = 0.1$) together with rolling estimation appears to perform the best for inflation forecasting (Figure 2), both recording impressive reductions of forecast errors relative to the “naïve” model, especially over recent years.

²¹ This is consistent with the results reported in Liu, Li-gang, Jian Chang and Andrew Tsang (2006): models estimated by recursive regression generally produce smaller RMSEs than those estimated by rolling regression in forecasting HK’s inflation when using unrestricted time series models.

Figure 1. BVAR and RW forecast of real GDP growth

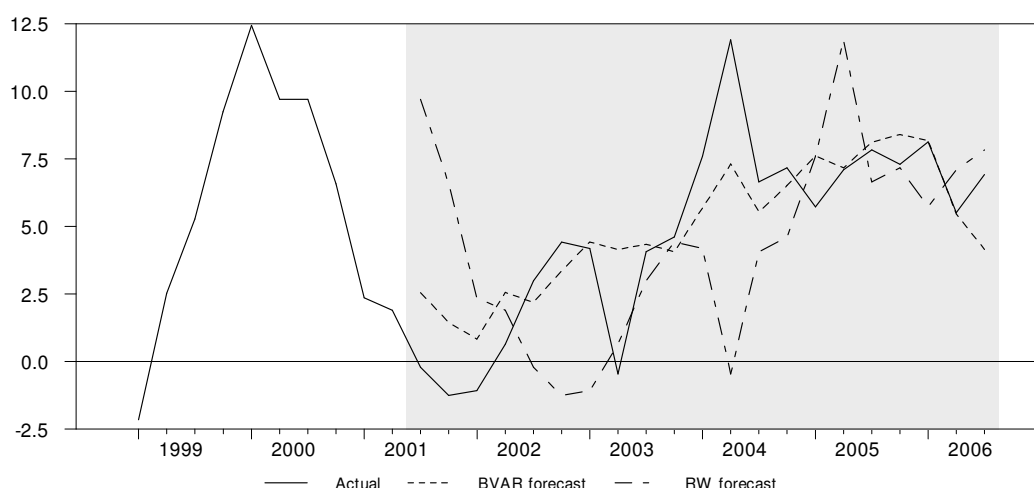
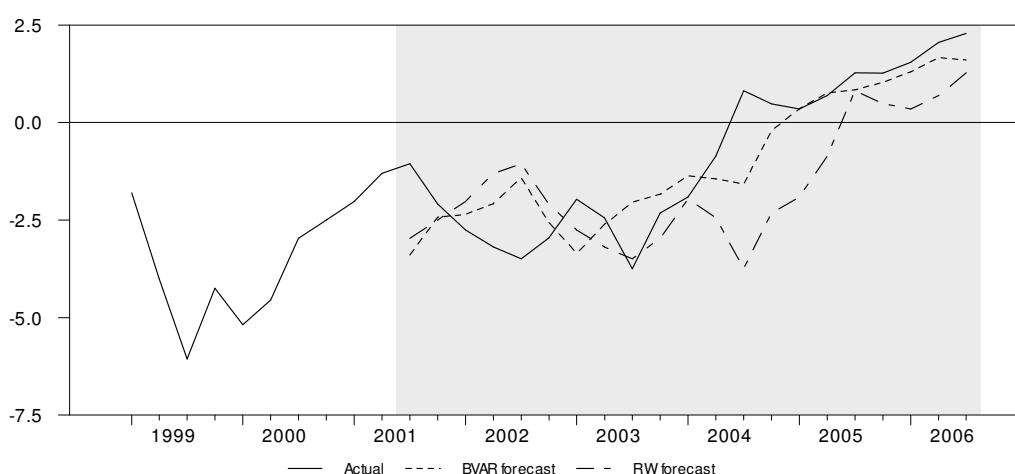


Figure 2. BVAR and RW forecast of CCPI Inflation



Source: C&SD, and staff estimates.

Moreover, the last three rows in Table 3 show that the Bayesian VAR framework exhibits further improvement in forecast accuracy, once the actual values of exogenous variables are fed into the estimation.²² Relatively looser prior appears to produce the best forecasts for both output growth and inflation within the class of CBVAR models, possibly reflecting that such a prior permits a larger role played by exogenous variables and such variables are indeed useful in explaining the movement of out target variables.

²² Conditioning on the actual observed values of the exogenous variables will of course not be possible in a real-time forecasting exercise. It is nevertheless reassuring to observe that more accurate forecasts of the exogenous variables do indeed improve the forecasts of the endogenous variables. Furthermore, as noted above, organisations specialising in the production of forecasts for external variables such as US GDP growth should be able to provide forecasts that are better than those of the simple model of these variables implied by our (B)VAR.

In combination with recursive estimation, such CBVARs could generate a reduction of 68% in RMSE relative to the “naïve” model in RGDP growth forecasting (Figure 3), an additional 11% compared with the best BVAR model, evidencing the importance of external variables in affecting Hong Kong’s RGDP growth. Forecast accuracy for inflation improves as well, albeit to a lesser extent, with the best CBVAR model reducing the RMSE by 44% relative to the “naïve” model (Figure 4). This may be less of a surprise as the exogenous foreign variables are probably more important in explaining domestic output growth than domestic inflation. Possibly reflecting this, our results also find that a BVAR model with tighter prior could produce almost as accurate inflation forecast as a CBVAR model (Table 3).

Figure 3. CBVAR and RW forecast of real GDP growth

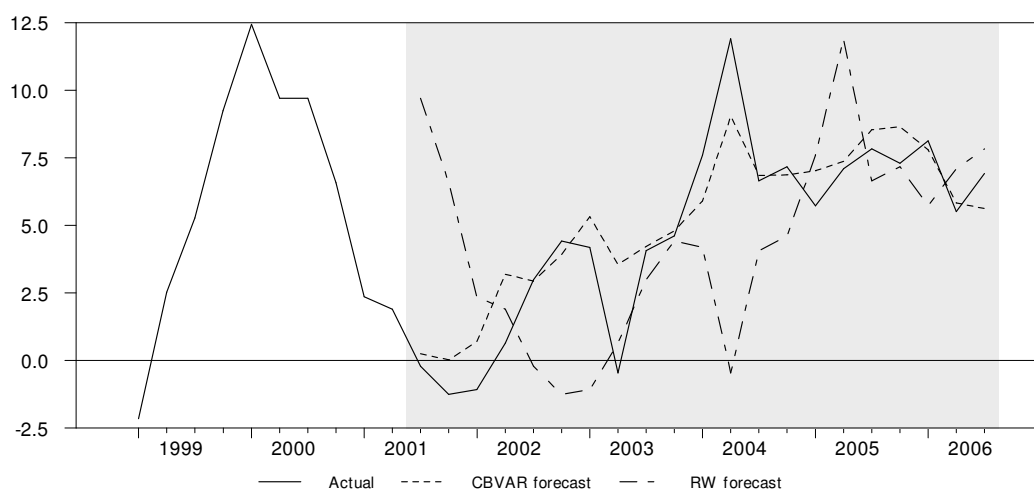
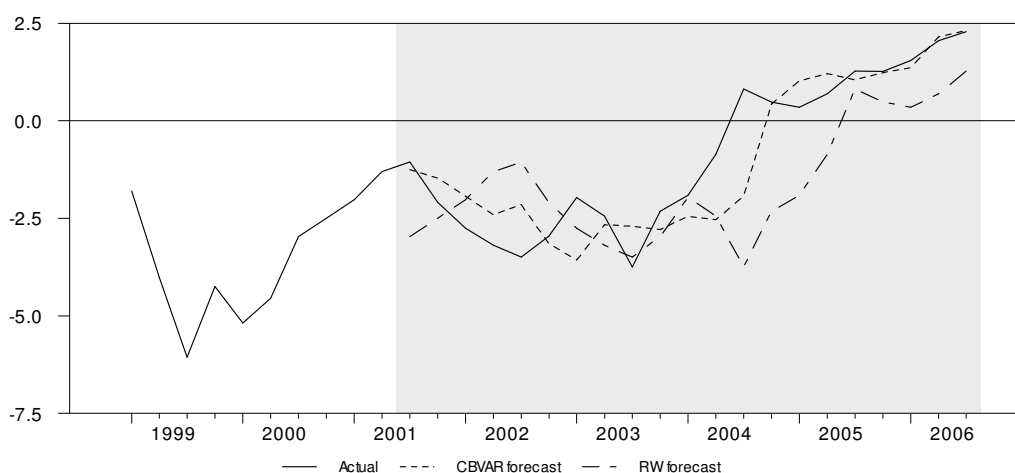


Figure 4. CBVAR and RW forecast of CCPI inflation



Source: C&SD, and staff estimates.

Moreover, while in general we find rolling estimation tends to produce better forecasts for inflation, the last row in Tables 3 shows that a CBVAR model with a relatively loose prior in combination with recursive estimation appears to produce the most accurate inflation forecast on average. This possibly reflects the benefit of richer data in generating more efficient estimates, when a large number of parameters would need to be estimated as more lags of exogenous variables potentially become useful in a Bayesian model with looser prior.

V. CONCLUSION

VAR models have become increasingly popular among central banks for forecasting purpose. This paper illustrates the steps involved in designing and building a real-time VAR forecasting model for output growth and inflation in Hong Kong. We consider three types of VAR models, namely unrestricted VARs, Bayesian VARs and conditional VARs. These models are estimated using quarterly data on domestic output, domestic inflation, and alternative candidate variables representing a domestic activity indicator, interest rate, external price and external demand. We compare the out-of-sample forecast performance of various models over the period 2001:3 – 2006:3, and investigate the forecast ability of various candidate variables under the Bayesian VAR models.

In comparison to unrestricted VAR models and alternative univariate time series models, BVAR models in general provide more accurate forecasts for both output and inflation. This method greatly reduces the dimensionality problem of unrestricted time series models by imposing inexact prior restrictions on the model parameters, therefore resulting in efficiency gains in the parameter estimation and, consequently, smaller out-of-sample forecast errors. Moreover, feeding in more accurate foreign variables forecasts appears to improve further the BVAR models in a conditional forecasting framework, confirming the importance of external variables in determining Hong Kong's economic activity.

Within the class of BVAR models, forecast accuracy appears to depend on the specification of the prior distribution function, in particular the standard deviation, of the coefficient matrix. The best-performing BVAR models for output and inflation tends to differ in their degree of tightness and estimation sample. Relatively tighter prior in combination with rolling estimation appears to produce lower RMSEs for inflation forecasts, while relatively loose prior in

combination with recursive estimation appears to produce lower RMSEs for output growth forecasts. Once conditional forecast is considered, however, a CBVAR model with a relatively loose prior in combination with recursive estimation appears to produce the most accurate inflation forecast.

Among the various candidate variables, our baseline choice, including domestic investment, the best lending rate, import prices and world GDP, appears to generate better out-of-sample forecasts than many other sets of alternative variables. On the other hand, mainland China's GDP, US interest rates, effective exchange rates and other domestic GDP components are also found useful for our evaluation period.

In summary, our results suggest that the Bayesian VAR framework incorporating external influences provide a practical and promising tool to produce reasonably accurate forecasts for key macroeconomic variables in the Hong Kong economy. This framework could be a useful supplement to our suite of models, allowing us to cross-check forecasts made from our core model, thus potentially improving our forecast accuracy of future economic conditions.

REFERENCES

- Doan, Thomas A. (2000). RATS User's Guide, Version, 5.0. Illinois: Estima.
- Doan, Thomas A. (2004). RATS User's Guide, Version, 6.0. Illinois: Estima.
- Doan, Thomas, Robert B. Litterman and Christopher A. Sims (1984), "Forecasting and Conditional Projections Using Realistic Prior Distributions," *Econometric Reviews*, Vol. 3, pp. 1-100.
- Felix, Ricardo M. and Luis C. Nunes (2003), "Forecasting Euro Area Aggregates with Bayesian VAR and VECM Models," *Working Paper 4-03*, Banco De Portugal.
- Genberg, Hans (2003), "Foreign versus Domestic Factors as Sources of macroeconomic Fluctuations in Hong Kong," *Working Paper*, September 2003, Hong Kong Institute for Monetary Research.
- Genberg, Hans and Laurent Pauwels (2005), "Wage-Price Dynamics and Deflation in Hong Kong," *Pacific Economic Review*, Vol. 10, pp. 191-216.
- Ha, Jiming, Cynthia Leung, and Chang Shu (2002), "A Small Macroeconomic Model of Hong Kong", *Research Memorandum 07/2002*, Hong Kong Monetary Authority.
- Hamilton, James D. (1994), *Time Series Analysis*. Second Edition, Princeton: Princeton University Press.
- Kong, Janet and Cynthia Leung (2004), "Revised Small Forecasting Model for Hong Kong", *Research Memorandum 13/2004*, Hong Kong Monetary Authority.
- Litterman, Robert B. (1981), "A Bayesian Procedure for Forecasting with Vector Autoregressions," *Unpublished Manuscript*, Department of Economics, Massachusetts Institute of Technology.
- Litterman, Robert B. (1985), "Forecasting with Bayesian Vector Autoregressions – Five Years of Experience," *Working Paper 274*, Federal Reserve Bank of Minneapolis.
- Litterman, Robert B. (1986), "Forecasting with Bayesian Vector Autoregressions – Five Years of Experience," *Journal of Business and Economic Statistics*, Vol. 4, No. 1, pp. 25-38.

- Liu, Li-gang, Jian Chang, and Andrew Tsang (2006), "Forecasting the Non-rental Component of Hong Kong's CCPI Inflation – an Indicator Approach", *Research Memorandum* 03/2006, Hong Kong Monetary Authority.
- Lupoletti, William M. and Roy H. Webb (1986), "Defining and Improving the Accuracy of Macroeconomic Forecasts: Contributions from a VAR Model," *Journal of Business*, Vol. 59. No. 2, pp. 263-285.
- Robertson, John C. and Ellis W. Tallman (1999), "Vector Autoregressions: Forecasting and Reality," *Economic Review*, First Quarter 1999, pp. 4-18, Federal Reserve Bank of Atlanta.
- Sims, Christopher A., James H. Stock, and Mark W. Watson (1990), "Inference in Linear Time Series Models with Some Unit Roots," *Econometrica*, Vol. 58, pp. 113-144.
- Sims, Christopher A. and Tao Zha (1998), "Bayesian Methods for Dynamic Multivariate Models," *International Economic Review*, Vol. 39, No. 4, pp. 949-968.
- Smets, Frank and Raf Wouters (2004), "Forecasting with a Bayesian DSGE model – an application to the Euro Area," *Working paper*, the European Central Bank, No. 389.
- Stock, James H. and Mark W. Watson (2001), "Vector Autoregressions", *Journal of Economic Perspectives*, Vol. 15, No. 4, pp. 101-115.
- Waggoner, Daniel F. and Tao Zha (1999), "Conditional Forecasts in Dynamic Multivariate Models," *The Review of Economic and Statistics*, Vol. 81, No. 4, pp. 639-651.

Appendix I. Forecast performance evaluation for alternative variables

As part of the robustness check, we compare forecast performance evaluation for various candidate variables, holding other model specifications constant. In tables below, we present RMSEs for alternative variables under the best-performing BVARs and CBVARs identified in the previous section, and focus on comparisons of output growth forecast performance in Table A.1, and inflation forecast performance in Table A.2. Detailed model specifications and estimation methods are listed at the top of each column. Our results indicate that within our preferred Bayesian VAR models, our baseline choice of variables consistently exhibit higher predictive power in general relative to alternative candidate variables, whose performance appears to vary under different model specifications.

Table A.1 RMSE for output growth forecasts under preferred models

	BVAR($\gamma=0.2$, $d=1$)	CBVAR($\gamma=0.1$, $d=1$)	CBVAR($\gamma=0.2$, $d=1$)	CBVAR($\gamma=0.2$, $d=2$)
	Recursive	Recursive	Recursive	Recursive
Baseline variables	2.01 (0.43)	1.54 (0.33)	1.48 (0.32)	1.49 (0.32)
To replace GDFCFR				
PCER	2.17 (0.46)	1.73 (0.37)	1.77 (0.38)	1.49 (0.32)
PIV	2.07 (0.44)	1.58 (0.34)	1.54 (0.33)	1.52 (0.32)
DD	1.99 (0.43)	1.53 (0.33)	1.68 (0.36)	1.56 (0.33)
NX	2.13 (0.46)	1.72 (0.37)	1.55 (0.33)	1.46 (0.31)
UMR	2.96 (0.63)	2.41 (0.51)	3.13 (0.67)	3.16 (0.68)
PROP	2.60 (0.56)	1.79 (0.38)	1.75 (0.37)	1.97 (0.42)
WAGEN	2.29 (0.49)	2.03 (0.43)	1.92 (0.41)	1.80 (0.38)
JVACANCY	2.29 (0.49)	1.89 (0.40)	1.98 (0.42)	1.51 (0.32)
To replace WGDGP				
USRGDP	3.05 (0.65)	2.66 (0.57)	2.34 (0.50)	2.32 (0.50)
CNRGDP	1.94 (0.41)	2.04 (0.44)	1.74 (0.37)	1.77 (0.38)
WGDPexCN	3.12 (0.67)	1.61 (0.34)	2.00 (0.43)	1.90 (0.41)
To replace USTB3M				
USFFTR	2.15 (0.46)	1.89 (0.40)	1.97 (0.42)	1.67 (0.36)
USTB3M	2.17 (0.46)	1.62 (0.35)	1.90 (0.41)	1.51 (0.32)
HIBOR3M	2.39 (0.51)	1.67 (0.36)	1.97 (0.42)	1.59 (0.34)
To replace UVIM				
UVIRM	3.11 (0.66)	2.26 (0.48)	2.28 (0.49)	2.28 (0.49)
WCPI	3.21 (0.69)	2.49 (0.53)	2.60 (0.56)	2.78 (0.59)
CNCPI	3.34 (0.71)	2.53 (0.54)	2.67 (0.57)	2.89 (0.62)
REERI	4.31 (0.92)	3.04 (0.65)	3.57 (0.76)	4.11 (0.88)
NEERI	3.94 (0.84)	2.69 (0.57)	3.36 (0.72)	3.39 (0.72)

Note: RMSEs are measured in %, and the numbers in parentheses are the ratio of the corresponding RMSE to the RMSE of the RW forecasts.

Source: staff estimates.

Table A.1 shows that regarding output growth forecasting, measures of GDP components appear to perform better on average than all other domestic activity indicators. While GDFCFR generally produce smaller forecast errors than other GDP components, domestic demand, private investment and net exports appear to perform almost as well in the three preferred models. World GDP produces more accurate forecasts than other external demand variables such as USGDP and WGDPEXCEN. However, CNGDP outperforms WGDGP by 7% in terms of the RMSE in the BVAR model and performs almost as well as the WGDGP in the other two CBVAR models, reflecting high predictive power of growth in mainland China on growth in Hong Kong. As for various interest rate measures, HK's best lending rate generates the most accurate forecasts, outperforming the US interest rates by considerable margins in some cases. The baseline choice of import price appears to perform considerably better for output forecast than all the other candidate external price variables.

Table A.2 RMSE for inflation forecasts under preferred models

	BVAR ($\gamma=0.1$, $d=1$)	CBVAR($\gamma=0.1$, $d=1$)	CBVAR($\gamma=0.2$, $d=1$)	CBVAR($\gamma=0.2$, $d=2$)
	Rolling	Rolling	Rolling	Recursive
Baseline variables	1.08 (0.64)	1.06 (0.63)	0.99 (0.59)	0.95 (0.56)
To replace GDFCFR				
PCER	1.33 (0.79)	1.40 (0.83)	1.74 (1.03)	2.32 (1.37)
PIV	1.09 (0.64)	1.06 (0.63)	1.04 (0.62)	1.00 (0.59)
DD	1.19 (0.70)	1.23 (0.73)	1.45 (0.86)	1.94 (1.15)
NX	1.47 (0.87)	1.46 (0.86)	1.93 (1.14)	1.84 (1.09)
UMR	1.90 (1.12)	1.59 (0.94)	2.12 (1.25)	3.13 (1.85)
PROP	1.35 (0.80)	1.34 (0.79)	1.50 (0.89)	2.17 (1.28)
WAGEN	1.61 (0.95)	1.66 (0.98)	1.70 (1.01)	1.55 (0.92)
JVACANCY	1.67 (0.99)	1.44 (0.85)	1.92 (1.14)	1.71 (1.01)
To replace WGDGP				
USRGDP	1.07 (0.63)	1.23 (0.73)	1.01 (0.60)	1.01 (0.60)
CNRGDP	1.08 (0.64)	1.09 (0.64)	1.06 (0.63)	0.92 (0.54)
WGDPeXCEN	1.07 (0.63)	1.17 (0.69)	1.00 (0.59)	0.99 (0.59)
To replace USTB3M				
USFFTR	1.08 (0.64)	1.10 (0.65)	1.31 (0.78)	0.96 (0.57)
USTB3M	1.08 (0.64)	1.21 (0.72)	1.49 (0.88)	0.94 (0.56)
HIBOR3M	1.08 (0.64)	1.04 (0.62)	1.36 (0.80)	1.03 (0.61)
To replace UVIM				
UVIRM	1.15 (0.68)	1.16 (0.69)	1.19 (0.70)	1.89 (1.12)
WCPI	1.23 (0.73)	1.39 (0.82)	1.95 (1.15)	1.79 (1.06)
CNCPI	1.17 (0.69)	1.57 (0.93)	1.95 (1.15)	1.82 (1.08)
REERI	1.10 (0.65)	1.13 (0.67)	1.40 (0.83)	1.51 (0.89)
NEERI	1.10 (0.65)	1.32 (0.78)	1.60 (0.95)	1.62 (0.96)

Note: RMSEs are measured in %, and the numbers in parentheses are the ratio of the corresponding RMSE to the RMSE of the RW forecasts.

Source: staff estimates.

Table A.2 suggests that, among all candidate domestic activity indicators, property prices appear to be able to improve inflation forecast more than the UMR, WAGEN, and JVACANCY in most cases, although measures of investment again generate the most accurate forecast for inflation. Other GDP components also perform well in models with relatively tighter priors.

External demand variables and interest rates variables appear to have little impact on the out-of-sample forecast performance for inflation in the unconditional Bayesian model with relatively tight prior, while their influences increase once forecasts are made conditional on the actual value of these external variables. In a CBVAR model with a relatively loose prior, CNGDP appears to generate slightly better forecast compared with most other variables, and recording a reduction of around 2% in RMSE relative to the baseline choice in the model with the loosest prior.

The unit value of imports appears to produce the lowest forecast errors compared with all other external price variables, although both nominal and real effective exchange rates generate almost equally accurate forecasts for inflation in the unconditional Bayesian model with relatively tight prior, with the RMSEs produced by the three models differing by less than 2%. The performance of most external price candidates, however, deteriorates significantly once we move to conditional forecasts.