

### Research Memorandum 03/2019

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## **REVISITING US RECESSION PROBABILITY MODELS**

### Key points

- In recent months, the flattening of the US Treasury yield curve has sparked intense market discussions about recession risks in the US. This paper seeks to contribute to these discussions by re-examining the predictive power of Treasury term spreads and selected macroeconomic indicators in forecasting US recessions.
- Using a variety of forecast accuracy metrics that take into account both false positives and false negatives, this paper finds that decomposing term spreads into a near-term spread (either 2-year minus 3-month, or the "near-term forward yield spread" by Engstrom et al. (2018)) and a far-term spread (10-year minus 2-year) helps improve prediction quality. But this is not the case for term premium. The in-sample forecasting power of these models may be further enhanced by adding a measure of corporate bond market sentiment (by Gilchrist et al. (2012)) and the Conference Board's Composite index of Leading Indicators (CLI).
- In February 2019, our preferred model implied only about 18% probability of the US economy transitioning into a recession in the next 12 months.

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The views and analysis expressed in this paper are those of the authors, and do not necessarily represent the views of the Hong Kong Monetary Authority.

### I. INTRODUCTION

Several participants (...) noted that the slope of the Treasury yield curve was unusually flat by historical standards, which in the past had often been associated with a deterioration in future macroeconomic performance.<sup>1</sup>

According to the National Bureau of Economic Research (NBER), the current economic expansion in the US is the second longest on record, having continued for 116 months by February 2019. At this juncture, it appears the current recovery may still have further room to go as the expansion has been quite modest in pace and the economy does not appear "over-heated". In addition, research generally finds that economic expansions do not necessarily "die of old age" (e.g. Diebold et al. (1990)). Nonetheless, a number of financial market indicators seem to signal otherwise, particularly the sharp flattening of the US Treasury yield curve since early 2017, which sparked intense market discussion as an inverted yield curve usually foreshadows a recession.<sup>2</sup> The signal conveyed by the flattening of the yield curve is all the more perplexing when one takes into account the solid US economic performance until late 2018.

In contributing to the recent discussions on US recession risks, our paper systematically re-examines the statistical power of the yield curve and other economic and financial indicators in predicting recessions. Indeed, while the notion of an inverted yield curve foreshadowing a recession is widely accepted, there is considerable disagreement over the appropriate choice of term spreads — measures of the steepness of the yield curve — that should be considered. We find that, instead of being fixated on one of the commonly cited term spreads (e.g. the 10-year / 2-year spread), it is better to consider term spreads involving both the long end and the short end of the Treasury yield curve, as well as making use of information conveyed by corporate credit spreads and leading economic indicators.

This memorandum proceeds as follows. The next section provides a brief review of the literature on statistical predictions of US recessions, and describes how our work contributes to this field of study. Section III

<sup>&</sup>lt;sup>1</sup> From the minutes of the Federal Open Market Committee (FOMC) meeting in January 2019.

<sup>&</sup>lt;sup>2</sup> The spread between 10-year and 2-year nominal Treasury constant-maturity yields, a widely cited measure of the slope of the yield curve, narrowed from an average of 122 basis points in January 2017 to just 19 basis points in February 2019, the smallest difference since July 2007.

discusses data and model specifications. Section IV presents the empirical results, and the last section concludes.

#### **II. LITERATURE REVIEW AND CONTRIBUTIONS**

There is a vast amount of literature on the statistical modelling of recession probability in the US (see Filardo (1999) for a partial survey). Among a wide variety of models, probit models that make use of Treasury term spreads gained prominence following the work by Estrella et al. (1996), which established the predictive power of the 10-year minus 3-month Treasury yield spread in forecasting recessions two to six quarters ahead. Since then, the robustness of Treasury term spreads as a predictor of recessions has been repeatedly confirmed, e.g. Rudebusch et al. (2009). As explained by Estrella et al. (2006), the observed predictive power of term spreads in forecasting recessions is not merely a statistical coincidence; there are strong theoretical foundations that underpin their relationship via channels such as investor expectations and the impact of monetary policy on the yield curve.

Nonetheless, in spite of the extensive work on yield-curve-based recession prediction models, there is still considerable disagreement over the appropriate choice of term spreads to be considered. For instance, when calculating term spreads involving the 10-year and 2-year Treasury yields, consideration should be given to whether the term premium component, which is still likely to be compressed by the Federal Reserve's large asset holdings (Bonis et al. (2017)), should be deducted from the spot yield.<sup>3</sup> In another strand of the literature, Engstrom et al. (2018) advocated the use of the "near-term forward yield spread", which is defined as the difference between the current implied forward rate on 3-month Treasury bills six quarters from now and the current yield on 3-month Treasury bills, as a superior leading indicator of recessions compared with conventional term spread measures.

At the same time, a number of macroeconomic variables other than Treasury term spreads were also found to be useful in predicting recessions. For instance, Neftci (1982) developed a recursive statistical model that translates monthly readings of the Conference Board's Composite index of Leading Indicators (CLI) into a probability of imminent recession. More recently, Gilchrist et al. (2012) constructed a measure of corporate bond market

 $<sup>^3</sup>$  Rosenberg et al. (2008) argued that the term premium component of Treasury yields is not a leading indicator of recessions, and its removal from the term spreads may give sharper predictions. On the other hand, Bauer et al. (2018) finds that recession prediction models using the 10-year / 3-month Treasury term spread adjusted for term premium performed worse than those without adjustments.

sentiment known as "excess bond premium" (EBP), and asserted its usefulness in predicting the likelihood of US recessions within a 12-month horizon.

In view of the vast amount of literature on recession predictions, this paper's contribution mainly lies in synthesising the findings from prior studies about the optimal combination of term spread(s) and macroeconomic indicators. In particular, we wish to address the long-standing debate on whether term premiums matter in recession forecasting, and whether Engstrom et al. (2018)'s "near-term forward yield spread" retains its advantages over traditional term spreads in the presence of other variables, namely the EBP and CLI. To this end, instead of relying on readily available, but potentially misleading summary statistics (e.g. pseudo- $R^2$ ), we assess the desirability of our models by using more formal statistical benchmarks, such as the Brier score, the Area under the receiver operating characteristics curve (AUC) and the Bayesian Information Criterion (BIC). In particular, the Brier score and the AUC capture the intrinsic trade-offs between false positives (i.e. predicting a recession when there isn't one) and false negatives (i.e. failing to give advance signals of recessions) inherent in any early warning systems, while the BIC provides the ability to compare the plausibility of individual models. These metrics suggest the inclusion of the EBP and the CLI in addition to the term spreads generates desirable in-sample forecasting results.

## **III. EMPIRICAL MODELS**

# 3.1 <u>Data</u>

Monthly data on US Treasury spreads and control variables from January 1973 to December 2017 are used to predict the probability of entering into an NBER recession in the US any time in the next 12 months. Observations in which the effective lower bound is binding are dropped for the estimation (January 2009 to December 2015). And, as the model aims to examine the probability of transitioning into recession, the estimation also drops any observations in which the economy was already in recession in the previous month<sup>4</sup>.

Detailed explanations on US Treasury spread variables and control variables used in the empirical analysis are described in Table 1. Detailed calculations on estimating US Treasury spot yields and forward spreads can be found in Appendix A.1.

<sup>&</sup>lt;sup>4</sup> Following practices in Engstrom et al (2018).

Catagorias	Variables				
Categories	Variables	Descriptions A binary variable that equals			
Dependent Variable	Recession probability $Pr(Recession_{t+1,t+12} = 1   Recession_t = 0)$	to 1 if an NBER recession happened any time between the next 1 to 12 months, conditional on the economy not currently in recession.			
	Long-term yield spread ( $spread_t^{10y2y}$ ) or ( $spread_t^{10y3m}$ )	Monthly average of (daily zero-coupon yield on 10-year Treasury bond minus daily zero-coupon yield on 2-year Treasury note (for $spread_t^{10y2y}$ ) or the daily secondary market 3-month Treasury bill rate <sup>5</sup> (for $spread_t^{10y3m}$ ))			
Independent Variable: US Treasury spreads	Short-term yield spread $(spread_t^{2y3m})$	Monthly average of (daily zero-coupon yield on 2-year Treasury note minus daily secondary market rate on 3-month Treasury bill <sup>5</sup> )			
	Near-term forward spread (spread <sup>6q1q</sup> )	Monthly average of (implied forward rate on 3-month Treasury bill six quarters from now minus the current secondary market rate on 3-month Treasury bill <sup>5</sup> )			
Independent Variable: US Treasury term premium	Treasury bond term premium $(TP_t^{10y})$ or $(TP_t^{2y})$	Monthly average of daily 10-year (or 2-year) US Treasury term premium Estimated using the ACM methodology by Adrian et al.			

 <sup>&</sup>lt;sup>5</sup> The discount-basis secondary-market rate on the 3-month Treasury bill is converted into the bond-equivalent basis following the practice in Estrella and Trubin (2006).
<sup>6</sup> Source: https://www.newyorkfed.org/research/data\_indicators/term\_premia.html

Categories	Variables	Descriptions		
Independent Variable: Controls	Excess bond premium (EBP <sub>t</sub> )	Corporate bond spreads net of estimated default risk, as a proxy of corporate bond market sentiment. A spike in the EBP suggests that investors' attitude towards corporate credit risk deteriorates and will likely indicate a higher probability of recession. <sup>7</sup> Estimated by Gilchrist and Zakrajsek (2012)		
	Leading indicators (Δcli <sub>t</sub> )	Group of real-side variables used as a proxy for macroeconomic conditions (12-month difference of the Conference Board's Composite index of Leading Indicators at month t) <sup>8</sup>		

# 3.2 <u>Model specification</u>

Eight models are examined using the probit regressions<sup>9</sup>. Models 1-6 employ only US Treasury spread variables, while Models 7-8 add additional real variables and excess bond premium as controls.

(1) 10-year minus 3-month term spread

 $Pr(Recession_{t+1,t+12} = 1 | Recession_t = 0) = \Phi(\alpha_0 + \alpha_1 spread_t^{10y3m})$ 

<sup>&</sup>lt;sup>7</sup> Following the practice in Favara et al (2016) of using excess bond premium as a predictor for the US entering into recession for the next 12 months.

<sup>&</sup>lt;sup>8</sup> Obtained from The Conference Board Leading Economic Index website. Detailed components of the index can be found in https://www.conference-board.org/data/bcicountry.cfm?cid=1

<sup>&</sup>lt;sup>9</sup> Where  $\Phi$  is the standard cumulative Gaussian distribution function.

(2) 10-year minus 2-year term spread

$$Pr(Recession_{t+1,t+12} = 1 | Recession_t = 0) = \Phi(\gamma_0 + \gamma_1 spread_t^{10y2y})$$

(3) 10-year minus 2-year term spread and 2-year minus 3-month term spread  $Pr(Recession_{t+1,t+12} = 1 | Recession_t = 0)$   $= \Phi(\theta_0 + \theta_1 spread_t^{10y2y} + \theta_2 spread_t^{2y3m})$ 

(4) 10-year minus 2-year term spread and near-term forward spread  $Pr(Recession_{t+1,t+12} = 1 | Recession_t = 0)$   $= \Phi(\lambda_0 + \lambda_1 spread_t^{10y2y} + \lambda_2 spread_t^{6q1q})$ 

(5) 10-year minus 3-month term spread, adjusted for term premium

$$Pr(Recession_{t+1,t+12} = 1 | Recession_t = 0)$$
$$= \Phi\left(\beta_0 + \beta_1(spread_t^{10y3m} - TP_t^{10y})\right)$$

(6) 10-year minus 2-year term spread, adjusted for term premium

$$Pr(Recession_{t+1,t+12} = 1 | Recession_t = 0)$$
$$= \Phi\left(\delta_0 + \delta_1\left(spread_t^{10y2y} - \left(TP_t^{10y} - TP_t^{2y}\right)\right)\right)$$

(7) 10-year minus 2-year term spread, 2-year minus 3-month term spread, excess bond premium and Conference Board's CLI

$$Pr(Recession_{t+1,t+12} = 1 | Recession_t = 0) \\ = \Phi(\mu_0 + \mu_1 spread_t^{10y2y} + \mu_2 spread_t^{2y3m} + \mu_3 EBP_t \\ + \mu_4 \Delta cli_t)$$

(8) 10-year minus 2-year term spread, near-term forward spread, excess bond premium and Conference Board's CLI

$$Pr(Recession_{t+1,t+12} = 1 | Recession_t = 0) \\ = \Phi(\omega_0 + \omega_1 spread_t^{10y2y} + \omega_2 spread_t^{6q1q} + \omega_3 EBP_t \\ + \omega_4 \Delta cli_t)$$

### **IV. ESTIMATION RESULTS**

Consistent with the literature, the models are estimated in-sample to find which one performs the best in predicting the probability of the US economy entering into recession in the next 12 months. The models are grouped by those utilising only Treasury spreads (Models 1 - 6) and those utilising both Treasury spreads and other variables (Models 7 and 8). Table 2 below also provides statistics on multiple model selection criteria, with a full explanation in Appendix A.2.

Results from Table 2 show that while the conventionally used 10-year minus 3-month and 10-year minus 2-year spreads appear to be reasonably good predictors of recessions (Models 1 and 2 respectively), they are statistically dominated by Models 3 and 4 (in terms of BIC, AUC and Brier score) that derive information from different segments of the yield curve. Interestingly, results from Models 3 and 4 suggest that short-term spreads (*spread*<sub>t</sub><sup>6q1q</sup>) are more important than the long-term spread (*spread*<sub>t</sub><sup>10y2y</sup>), judging by the statistical significance of their associated coefficients. But the latter retains its importance in the presence of the 2-year / 3-month spread.

For the impact of term premium on recession forecasting, Models 5 and 6 imply that subtracting the term premiums from the term spreads actually results in a deterioration in the forecasting performance (in terms of lower AUC, higher BIC and higher Brier score), compared with their unadjusted counterparts (Models 1 and 2).<sup>10</sup> A possible explanation is that the term premiums do not only capture the impact of the Fed's quantitative easing on Treasury yields, but may also contain useful information embedded in investor expectations that help predict future recessions. Our finding is in line with Engstrom et al. (2018), who also find that term premium should not be subtracted from the spot yields. Therefore, from this point onwards, we will not consider term-premium-adjusted spreads.

In selecting the best performing model, it appears that models with both term spreads and control variables (Models 7 - 8) outperform those with only term spreads (Models 1 - 6), as indicated by higher AUC and lower BIC (and Brier score) of the former. However, between Models 7 and 8, it is not entirely evident which is superior, as model selection criteria give mixed

<sup>&</sup>lt;sup>10</sup> An astute reader may note that Model 5 has a lower Brier score than Model 1. However, the Diebold-Mariano test fails to reject the null hypothesis that Models 1 and 5 have the same forecast accuracy (p-value = 0.35).

results<sup>11</sup>. We decide that Model 8, the one that employs the long-term yield spread( $spread_t^{10y2y}$ ), the near-term forward spread ( $spread_t^{6q1q}$ ), the excess bond premium ( $EBP_t$ ) and the 12-month change in the Conference Board's CLI ( $\Delta cli_t$ ), is our preferred choice, as the near-term forward spread ( $spread_t^{6q1q}$ ) appears to be more intuitive in measuring how investors see the future path of policy rates<sup>12</sup>. Thus, Model 8 will be a better candidate in forecasting the likelihood of the US economy entering into a recession over a shorter horizon, such as the next 12 months.

<sup>&</sup>lt;sup>11</sup> The lower Brier score of Model 8 indicates the model's outperformance vis-à-vis Model 7, while its slightly lower AUC and higher BIC suggest otherwise. Diebold-Mariano test results suggest that the two models have statistically identical forecasting accuracy (p-value = 0.47), while a test of equality between the AUCs of the two models is also inconclusive (p-value = 0.13).

<sup>&</sup>lt;sup>12</sup> Intuitively, a negative value for the near-term forward spread suggests a future policy easing, as future rates are projected to be lower than current ones.

#### **Table 2: Recession Model Results**

							Models with term		
	Models with term spreads only							spreads and controls	
spread <sup>10y3m</sup>	(1) -1.255*** (-8.07)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	
$spread_t^{10y3m} - (TP_t^{10y} - TP_t^{2y})$					-1.231*** (-8.35)				
$spread_t^{10y2y}$		-1.584*** (-7.31)	-0.596*** (-2.66)	-0.189 (-0.69)			-1.301*** (-4.62)	-1.080*** (-3.07)	
$spread_t^{10y2y} - (TP_t^{10y} - TP_t^{2y})$						-0.914*** (-9.17)			
$spread_t^{2y3m}$			-2.108*** (-6.60)				-1.319*** (-3.51)		
$spread_t^{6q1q}$				-1.498*** (-7.09)				-0.858*** (-4.38)	
EBP <sub>t</sub>							1.395*** (2.62)	1.342*** (4.15)	
$\Delta cli_t$							-0.249** (-2.43)	-0.256*** (-3.62)	
Ν	391	391	391	391	391	391	391	391	
BIC	185.2	247.0	178.6	182.2	197.1	301.1	146.4	149.8	
AUC	0.947	0.901	0.954	0.954	0.940	0.806	0.977	0.975	
Brier score	0.125	0.132	0.125	0.123	0.115	0.144	0.052	0.050	

Dependent variable:  $Pr(Recession_{t+1,t+12} = 1 | Recession_t = 0)$ 

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Notes: Bootstrapped t-statistics in parentheses. \*, \*\*, \*\*\* denote statistical significance at 10%, 5% and 1% respectively. The dependent variable is a binary variable that equals 1 if the US economy enters into a recession (defined by NBER) in any period within the next 12 months. BIC stands for Bayesian Information Criterion, and a smaller value indicates a greater likelihood that the model is a correct fit for data. AUC stands for area under the receiver operating characteristics curve, and a value closer to one indicates better predictions. Brier score is a proper score function measuring probabilistic prediction accuracy, with a value closer to zero indicating better accuracy. The constant terms are not reported for brevity.

Chart 1 below shows the predicted probability of the US economy entering into a recession in the next 12 months estimated by Model 8, our preferred model. In February 2019, the model suggested there was only a 18% probability of the US economy falling into recession, thus indicating there was no imminent risk of transitioning into a recession.



Chart 1: Predicted probability of recession

Chart 2 shows the historical time series of the 10-year / 2-year term spread and the "near-term forward spread" used in our preferred model. As shown in this chart, the near-term forward yield spread turned negative prior to each and every NBER recession (shaded in grey) since the 1970s, while the 10-year / 2-year spread was a less decisive predictor (for example, it did not turn negative prior to the 2008 global financial crisis). Yet, their less-than-perfect correlation (+0.61) suggests that each of them may contain useful and non-overlapping information about the state of the US economy. Meanwhile, Charts 3 and 4 show the EBP and the CLI tend to rise (fall) prior to recessions, illustrating their predictive power.

Note: Shaded periods represent NBER recessions.



**Chart 2: Near- and far-term spreads** 

Note: Shaded periods represent NBER recessions.



Chart 4: 12-month change in CLI



Note: Shaded periods represent NBER recessions.

#### V. CONCLUSION

In this study, we investigate the power of various Treasury term spreads in forecasting recessions. We find that a combination of long-term spreads and short-term spreads statistically dominate other commonly cited term spread measures in recession prediction models. We also find that term premiums should not be subtracted when predicting recessions using the long-term spread. Finally, the combination of 10-year minus 2-year term spreads and 6-quarter minus 1-quarter forward spreads, when augmented by Gilchrist et al (2012)'s measure of corporate bond spread and the Conference Board's CLI, produced the probit model that performs best in terms of prediction quality (in terms of Brier score and AUC). This preferred model currently predicts only about 18% probability of the US economy transitioning into a recession in the next 12 months, which appears to resonate with the present still-solid performance of the US economy.

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#### Appendix

#### A.1 Calculation on US Treasury zero-coupon yield spread and forward spread

All measures of US Treasury zero-coupon spreads start with daily estimates of the continuously-compounded zero-coupon nominal US Treasury curve, by Gürkaynak et al (2007). The daily yield curve consists of yields at maturities from one to ten years, at a one-year interval. To obtain the zero-coupon yields at incremental maturities, a Nelson-Siegel-Svensson model is used to fit the daily zero-coupon yield curve. The parameters of the Nelson-Siegel-Svensson model are provided by Kim and Wright daily<sup>13</sup>. We take monthly averages of the fitted daily yields and then calculated long-term and near-term spreads by taking differences respectively.

Forward rates are calculated from the zero coupon yield curve using the standard formula:

$$f_t^{n,1} = (n+1)y_t^{n+1} - ny_t^n$$

where  $f_t^{n,1}$  is the forward rate in date (t) from quarter (n) to (n+1), and  $y_t^n$  is the zero-coupon yield for maturity n (in annual rate) <sup>14</sup>. We take monthly averages of the fitted daily yield data and converted the rates from the continuously compounded basis to a semi-annually compounded basis (bond-equivalent yield (BEA) basis,  $f(BEY)_t^{n,1}$ ) using the formula below:

$$f(BEY)_t^{n,1} = 200 * \ln(1 + \frac{f_t^{n,1}}{200})$$

<sup>&</sup>lt;sup>13</sup> https://www.federalreserve.gov/pubs/feds/2006/200628/200628abs.html

<sup>&</sup>lt;sup>14</sup> For the forward rate six quarters ahead, it is inferred from the yield to maturity on Treasury notes maturing six quarters from now and seven quarters from now.

### A.2 Details of the model selection criteria

- BIC refers to Bayesian Information Criterion and is an estimator for the relative quality of statistical models for a given set of data. BIC measures the information loss for a model. A lower BIC suggests less information loss, and thus a better model. BIC is a relative measure and its actual value does not tell the absolute quality of a statistical model.
- AUC is defined as the area under the Receiver Operating Characteristics (ROC) curve. The ROC curve is generated by plotting sensitivity (on the vertical axis) against one minus sensitivity (on the horizontal axis) as the cut-off c is varied. Sensitivity is the fraction of observed positive-outcome cases that are correctly classified (True positive rate, TPR); specificity is the fraction of observed negative-outcome cases that are correctly classified (False positive rate, FPR). The ROC curve plots parametrically TPR(c) against FPR(c), with c set to 1 at start of the curve (0,0) and c set to 0 at end of the curve (1,1).

Thus, an AUC close to 1 suggests that the model is close to having 100% true positive rate, which is desirable in model selection, while an AUC close to 50% suggests that the model has a predicting power close to zero.

• Brier score is a scoring rule that evaluates the predictive accuracy of probabilistic models on binary outcomes. Brier score can be viewed as a cost function that measures the mean squared difference between the predicted probability of one possible outcome and the actual outcome.

The Brier score is a relative measure that takes on the value between 0 and 1, with a score closer to zero suggesting the more superior model. As a relative measure, Brier score's actual value provides limited information on prediction accuracy. When assessing the predictive accuracy of dichotomous events, Brier score can be used to compare model performances (Rufibach, 2010).