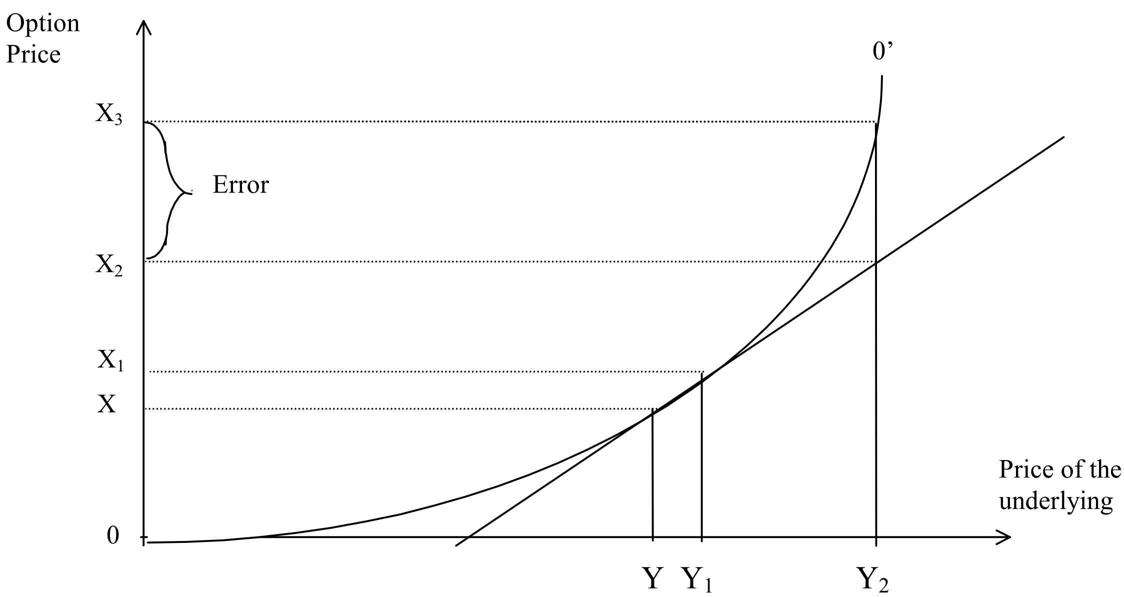
CHAPTER ELEVEN

The Gamma of an Option

In the previous two chapters, we discussed option's delta and volatility (vega) and their application in options trading. In this chapter, we will talk about another feature of option – gamma, also a Greek alphabet.

The gamma of a portfolio of options on an underlying asset is the rate of change of the portfolio's delta with respect to the price of the underlying asset. In other words, gamma is the second derivative with respect to the underlying.



For a small change in the underlying, say from Y to Y_1, the option price changes from X to X_1. In this case, delta does a relatively good job because the curvature of OO' is small in this price range. However, for a larger change in the underlying, the curvature of the OO' line becomes larger. Using delta alone will create a hedging error as the graph indicates. The magnitude of the error depends on the curvature of OO'. Gamma measures this curvature. If gamma is large in absolute terms (more curvature), delta is highly sensitive to the price change of the underlying.

The relation of delta and gamma is similar to that of duration and convexity. If you go back to Chapter 7, you will find that the above graph and the graph in that chapter illustrate the same point: the curvature of the actual price line makes duration and delta an inaccurate measurement tool, and convexity and gamma can correct this inaccuracy.

In Chapter 9, we say that a call option with a delta of 0.5 means that when the price of the underlying increases by $1, the price of the option increases by approximately $0.5. Now, adding the gamma element into the picture, if the price of the underlying increases by $1, the price of the option with a delta of 0.5 and a gamma of 0.1 will increase by approximately $0.6 instead of $0.5.

Option traders usually express gamma in change of delta per one point change in the underlying. Also, traders express a delta of 1.00 as 100 deltas and a change of 0.1 delta a change of 10 deltas. For example, if a trader holds 10 option contracts with a delta of 0.1, he is said to be holding a position of 100 deltas.

You will quickly realise how important gamma is to a trader's hedge position. A trader has to increase his hedges in order to stay within his trading limit, if he has a position with a delta of 0.45 and a gamma of 0.45. Assume he has 10 such contracts and a risk limit equivalent to the amount of 500 deltas, he may appear within his risk limit (450 deltas) if considering only the delta. However, he is in a danger zone of exceeding his limit if the market moves. If the underlying moves by one point, he will exceed his risk limit by 400 deltas because he will have a 900 delta position (10 contracts x (45 + 45) = 900). Thus, it is very important to set a gamma limit.

The profile of gamma changes against the underlying depend very much on the time remaining until expiration of the option. This means that when an *at-the-money* option approaches its expiration closer and closer, its gamma becomes bigger and bigger for every unit change of the underlying.

An article written by John Braddock and Benjamin Krause has an excellent description about the relation between gamma and the option's remaining time to expiration:

> Suppose there is a basketball game. Team A and Team B both are of equal strength. At the time when the game starts, the score is 0 and 0 and both team's chance of winning the game is 50 percent. This is analogous to purchasing an at-the-money call option with a delta of 0.5.
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> During the game, there are times when Team A is in front by a few points and it has a higher probability of winning the game. This is equivalent to the price of the underlying stock goes up and so does the delta of the call option.
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> How much should the delta go up? This is the same question as the one we will ask regarding how big a chance it is for Team A win the game at this time when it is leading by a few points.
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> It all depends on how much time remains in the game if both teams are still of (approximately) equal strength. If Team A is leading by 3 points at half-time, its winning chance surely is higher than 50 percent, but probably not by much, say, 55 percent. However, if Team A is leading by 3 points with only 30 seconds left in the game, its chance of winning is probably very high, say, 95

percent. Although the margin of the lead for Team A is the same, the chance of winning is different.

It is same for options. Assume that for an at-the-money option with three months to the expiration date, the delta is 0.5 and the gamma is 0.02. After one day, the price of the underlying changes by one point. At this time, the delta may change from 0.50 to 0.52 - a relatively small change because there is plenty of time for the price of the underlying to go either way. However, if the time is now one day before the expiration date, and the price of the underlying changes from at-the-money to one point in-the-money, the delta of the option may change from 0.50 to 0.95 because it surely looks good for the option to be in-the-money before it expires. Gamma estimates the rate of change.

For options which are far in-the-money (or far out-of-the-money), gamma probably does not matter much because the delta is already close to 1 (or close to zero for far out-of-the-money options). Just like a basketball game, if one team is leading by 30 points at the half, its chance of winning is very high. Whether it gains a couple of points or loses a couple of points in the second half shall not affect the result very much.