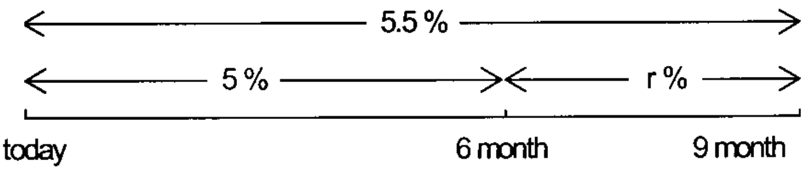
CHAPTER 4

Forwards and Futures [1]

Pricing Interest Rate Futures and Forward Rate Agreements

In the last chapter we have looked at some theories about the yield curve. In this chapter, we will look at some applications. The simplest kinds of interest rate derivatives are futures and forward rate agreements (FRAs). These two types of contracts are essentially identical; one major difference is that a futures contract is an exchange-traded contract and has fixed terms for the notional amount, length of contract, expiry date etc. whereas an FRA is an over-the-counter (OTC) contract which is a binding agreement between two parties. Another difference is that, as with other exchange-traded products, a minimum margin payment is required for the futures contract, whereas the actual payment for the FRA would only be settled at the expiry date. Other than these differences, the two types of instruments are priced in the same way.

For instance, today is 24/1/97 and assume that we have the following term structure of interest rate: 6-month rate of 5%, 9-month rate of 5.5%. What is the 3-month forward rate in 6-months' time, using today's market rate?



Again we use our method of having two strategies which should arrive at the same result (i.e. a no-arbitrage method). If we assume the forward rate to be r, starting with $1 today, at the end of 9 months we would either get

$$(1 + 5.5\% \times \tfrac{9}{12})$$ [A straightforward 9-month fixed rate deposit]

or

$$(1 + 5\% \times \tfrac{6}{12})(1 + r\% \times \tfrac{3}{12})$$

[6-month fixed rate deposit, rollover for another 3 months]

giving $r = 6.34\%$. This is the interest rate for the period between 24/7/97 and 24/10/97 as of today, and is equivalent to a quoted futures price of $(100 - 6.34) = 93.66$.

[1] *This chapter was written by Mr Chau Ka-lok.*

If today's date becomes 24/4/97. At this day, the 3-month rate has become 6%, whereas the 6-month rate is 6.5%. The forward rate for the period between 24/7/97 and 24/10/97 is then calculated by:

$$(1 + r\% \times \tfrac{3}{12}) = \frac{(1 + 6.5\% \times \tfrac{6}{12})}{(1 + 6\% \times \tfrac{3}{12})}$$

which gives $r = 6.90\%$ (or a futures price of 93.10).

What is the implication on the profit-and-loss of the trade? If this is marked to market, it implies that there is a 56 basis points loss (93.66–93.10) if the position in the futures contract is to be closed out immediately. If this is a US dollars futures contract and the notional amount is US$1,000,000, the loss converts to $\$1{,}000{,}000 \times 0.0056 \times \tfrac{3}{12} = \$1{,}400$.

The above examples give an illustration of a method of calculating interest rate forwards. Alternately, we can express the calculation in a more general way. Recalling the definition of the discounting factor (DF, the amount today which represents a future value of $1 using today's interest rate), we see that in the first example the discounting factors for 6-month and 9-month are

$$DF_{6-mth} = \frac{1}{(1 + 5\% \times \tfrac{6}{12})} = 0.9756, \quad DF_{9-mth} = \frac{1}{(1 + 5.5\% \times \tfrac{9}{12})} = 0.9604$$

The equation to calculate the forward rate between 24/7/97 and 24/10/97 is then

$$(1 + r\% \times \tfrac{3}{12}) = \frac{(1 + 5.5\% \times \tfrac{9}{12})}{(1 + 5\% \times \tfrac{6}{12})} = \frac{DF_{6-mth}}{DF_{9-mth}}$$

In other words, if we have to calculate the forward rate between time a and b (where a is before b and a, b are expressed in years), the formula to use is

$$\boxed{[1 + r\% \times (b - a)] = \frac{DF_a}{DF_b}}$$

Real market situations are seldom as simple as that. The first complication is when the discounting factor of more than one year is required, a compounding formula has to be used instead of calculating it as a simple interest. Other than that the same principle could be used and the above general formula could still apply. Secondly, when marking-to-market, the rates for the start and end dates are seldom given directly. In the above example, we assume that 3-month periods are exactly 0.25 year. To be more accurate, the time period should be calculated based on the day convention of the market. If the day convention is actual/365, the time period should be calculated by the difference in the number of days between the start and the end divided by 365. Assume that today is 10/3/97 and the forward rate between 24/7/97 and 24/10/97 is required. In the market, only rates for 1, 3, 6, 9, 12 months are quoted. To calculate the discounting factors at 24/7/97 (which is 136 days from today) and 24/10/97 (which is 228 days from today), some kind of interpolation is required. Using the figures in the above example (3-month rate is 6%, 6-month rate 6.5%), and assuming the 9-month rate is 6.8%, we work out the 136-day rate (r_1) and 228-day rate (r_2) using a simple linear interpolation,

$$\frac{r_1 - 6\%}{6.5\% - 6\%} = \frac{(24/7/97 - 10/6/97)}{(10/9/97 - 10/6/97)}, \quad \frac{r_2 - 6.5\%}{6.8\% - 6.5\%} = \frac{(24/10/97 - 10/9/97)}{(10/12/97 - 10/9/97)}$$

giving $r_1 = 6.239\%$, $r_2 = 6.645\%$. Different banks use different methods in interpolating for the rates between fixed dates and the answer could be slightly different (e.g. interpolate on the product of rate times maturity instead of on the rate, or fitting a curve to all the fixed points on the curve). There is no absolutely "correct" way to get to the right answer. Using these rates, the forward rate for the period 24/7/97 to 24/10/97 can be calculated as follows:

The discounting factors are:

$$DF_1 = \frac{1}{(1 + 6.239\% \times \frac{136}{365})} = 0.9773, \quad DF_2 = \frac{1}{(1 + 6.645\% \times \frac{228}{365})} = 0.9601$$

The forward rate is calculated using

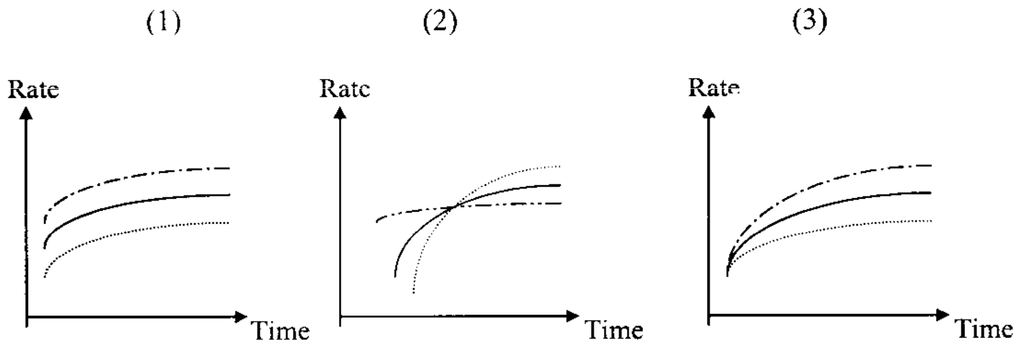
$$[1 + r\% \times \frac{(228 - 136)}{365}] = \frac{DF_1}{DF_2}$$

which gives $r = 7.11\%$.

Movements of the Yield Curve

Before we move on to other types of derivatives, there is one final concept to be introduced here. We often hear from the news that "central banks are cutting the interest rates". Have you ever wondered what exactly this means? It does not imply that the central bank has cut the interest rates across all maturities by the same margin. If so, the curve would always move in a parallel fashion. In reality only one reference rate (usually an overnight rate) is changed. The effect on the rates with long maturities is usually different from that of the short maturities. There are three *important* ways that the yield curve can move:

1) Parallel shift - the whole curve moves up or down by the same margin.
2) Change of slope - the curve moves in opposite direction around a pivot point (The curve "steepens" or "flattens".)
3) Change of convexity - rates of different maturities move by different magnitudes (The curve "twists".)



In the first part of this chapter, it is shown that the price of an interest rate forward can be obtained using today's market data. That is what the rate is expected to be given today's information. However, for some more complicated derivatives, we need to know what the yield curve would possibly look like in the future. For example, we are asked to price an option today of the 3-month forward given in the above example. In that case, although we know that the expected forward rate given by today's data is 6.34%, it is also very likely that the yield curve will move in the next 6 months. In order to price the option we need to know how this curve will move. This is considerably more difficult than the pricing of derivatives on other assets, because here we have to model the movement of the whole curve, whereas for other asset classes only the spot variable (e.g. the exchange rate or the stock price) needs to be modelled.

People have carried out research as to how these yield curves move over time in practice. Empirical results using US data show that a combination of parallel shift and change of slope can explain over 90% of the movements of the curves. This is an important piece of information because by knowing that two kinds of movements can account for most of the changes in the yield curves, we can simplify the modelling process and approximate the movements by summarising these using one or two "factors", which makes the life of the "rocket scientist" [2] much easier.

[2] *A nickname for those people who used to study Astrophysics and similar subjects and then apply this knowledge to design exotic financial derivative products.*