HONG KONG MONETARY AUTHORITY

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A STRUCTURAL APPROACH TO ASSESSING THE CREDIT RISK OF HONG KONG'S CORPORATE SECTOR

Key Points:

- *Given the close relationship between corporate vulnerabilities and the occurrence of banking and financial crises, regulators need to adopt a financial stability monitoring and surveillance framework which includes the assessment of the credit risk of the corporate sector.*

- *This paper illustrates how to assess the default risk of the non-financial corporate sector in Hong Kong by constructing an aggregate market indicator of default probabilities (PDs) using a structural approach (i.e. the Merton model).*

- *Rather than relying solely on accounting data, the Merton model quantifies the default risk of the corporate sector as well as credit conditions of different industry sectors using up-to-date market-based information such as equity prices.*

- *The aggregate PD of the non-financial constituent companies of the Hang Seng Index rose sharply a few months before the burst of the internet bubble in late February 2000. This appears to suggest that the PD may serve as an early warning signal for monitoring the potential vulnerability in the corporate sector.*

- *The study shows that the aggregate PDs derived from the Merton approach reflect the corporate default risk arising from economic shocks. The industry-specific PDs reveal areas of potential weaknesses in different industry sectors. Overall, the PD has proven to be an effective monitoring tool to gauge vulnerability in the corporate sector.*

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I. INTRODUCTION

The assessment of the credit risk of the non-financial corporate sector has become part of a broad financial stability monitoring and surveillance framework adopted by central banks in recent years. While the stability of the banking sector continues to be the key to financial stability as a whole, policymakers also acknowledge the fact that corporate failures would exert severe distress on the banking sector. This is especially the case when the corporate sector relies heavily on bank financing. Because of such close financial linkage between the corporate and the banking sectors, widespread default by companies may threaten financial stability. Aware of this, policymakers have been increasingly paying more attention to the monitoring of the credit quality of the corporate sector in their ongoing surveillance work (IMF, 2002).

The development of an effective monitoring tool that can help detecting corporate vulnerability in terms of default risk is important for central banks to forewarn the potential risk to financial stability arising from corporate failures. Different methods for measuring default risks are discussed in the literature of credit risk modelling.[1] Among them, the structural approach (also known as the contingent claim approach) proposed by Merton (1974) is frequently used by market participants as a framework to help assess the default risk of corporation. In fact, the Merton-type model has gained popularity with central banks and international organisations. Since late 2004, both Bank of England and European Central Bank have published their estimated indicators based on the Merton-type model as part of the measures of banking sector vulnerability in their regular *Financial Stability Review*.[2] Similar indicators are also found in *Global Financial Stability Report* by the International Monetary Fund (IMF).[3] The IMF finds that "the distance-to-distress has proven to be a powerful measure to gauge corporate balance sheet vulnerability. The analysis has also shown to be rather useful in predicting bank rating downgrades" (IMF 2004).

There are three main elements in the Merton model for quantifying the default risk of a company: (a) the market value of the company's assets, (b) the volatility of its asset value and (c) the liabilities of the company. The default event is determined by the market value of the company's assets in conjunction with its liability structure.

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1 A comparative analysis of different credit risk models that are commercially available can be found in Crouhy et al. (2000).

2 Examples of these indicators include the "distance-to-default" and the implied default probability (Bank of England (2004) and European Central bank (2004)). Other central banks like the Swedish Riksbank and the Norges Bank also use these indicators as a measure of corporate vulnerability.

3 See IMF (2004) and IMF (2005). In addition to the "distance-to-default" indicator of banking sector, the *Global Financial Stability Report* also presents indicators of government and non-financial corporate sectors in selected countries. The uses of "distance-to-default" to assess default risk of sovereign, corporate and banking sectors can be found in Gapen et al. (2005), Gapen et al. (2004) and Chan-Lau et al. (2004) respectively.

When the value of the assets falls below a critical threshold (usually defined by its liabilities), the company is considered to be default. The difference between the values of assets and liabilities combined with the volatility of the assets then determines the model-dependent estimate of the company's default probability.

The Merton approach uses both accounting information from corporate balance sheets and equity prices, which contain forward-looking information about the credit quality of the company, to estimate the default probability of a company. This approach has several advantages compared with other traditional default risk indicators based on bond market data and accounting information. For instance, a default spread (or a yield spread), which is calculated as the yield differential between a corporate bond and its corresponding government paper, is not a direct and clean measure of expected default as other factors such as tax and liquidity concerns may contribute a significant share of an observed spread (Tudela and Young (2005); Morttinen (2005); Elton et al. (2001)). On the other hand, default risk indicators based on accounting models (such as the Altman (1968) *Z*-score and Ohlson (1980) *O*-score) may not be suitable for monitoring purposes because a) these indicators cannot be frequently updated as most accounting data are only available annually and b) they lack the ability to pick up signals of default in a timely manner as accounting data are inherently backward-looking.

This study presents the Merton approach and applies it to estimate the default risk of publicly-listed non-financial companies in Hong Kong.[4] Like other central banks, an indicator based on aggregating the default probabilities of individual companies is constructed to assess the overall credit quality of Hong Kong's corporate sector, including major sectors such as the property sector, the commerce & industry sector, and the utility sector. The purpose of this analysis is to illustrate how the Merton approach can be used to estimate the default probabilities of individual companies and how an aggregate indicator based on these probabilities can be used to assess corporate credit risks for surveillance purposes.

The paper is organised as follows. Section II discusses the methodology of the structural approach and the data. The option pricing model is applied to the stock prices and balance sheet information of two groups of listed companies in Hong Kong. Technical details of the model are given in the Appendix. Section III presents the results of the estimation and examines the behavior of these indicators. Section IV concludes.

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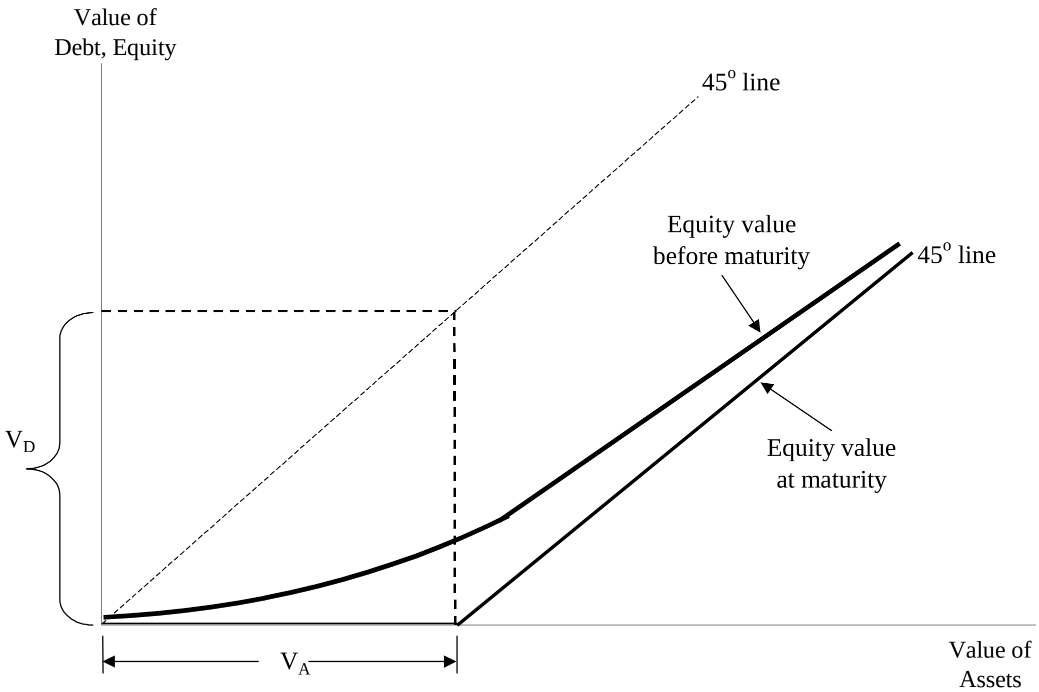
[4] Financial companies are excluded in this study because of their distinct characteristics. For example, due to the presence of derivatives and many off-balance sheet items, it is very difficult to determine the true size of their assets and liabilities (Crosbie and Bohn, 2002).

II. METHODOLOGY AND DATA

(a) The Model

The structural approach proposed by Merton (1974) assumes that the default process is related to the capital structure of a company. In this study, we consider the case of a company whose capital structure only comprises debt and equity. For this company, its equity component can be regarded as a call option written on its underlying assets with the strike price being the book value of its debt. Chart 1 provides an illustration. At the time when the debt matures, the value of the equity, like an option, depends on the final value of the asset.[5] When the value of the asset (V_A) is less than the value of debt (V_D), the option becomes worthless and the company is considered to be in default. Before the debt matures, the equity still has value (shown as the curve in Chart 1) even though the value of the asset is below that of the debt.[6]

Chart 1. Value of Assets, Equity and Debt



The company's equity price thus contains information about the probability of the value of its assets falling below the book value of its debt at the end of a specific period of time, which is equal to the default risk of the company.[7] Using market data of

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[5] One insight of the Merton model is that the payoffs to the shareholders of a company are similar to those they would have received from a purchased call option on the value of the company with a strike price given by the amount of debt outstanding.

[6] This is similar to the time value of an option before its expiry date.

[7] This study follows the original Merton model that assumes the company will pay the debt at maturity. Effectively, under this assumption, the equity can be treated as a European call option.

equity values and accounting data, the default probability at the end of a period of time t (PD_t) is, according to the Merton model, given by:[8]

$$PD_t = N\left[-\frac{\ln\frac{V_A}{X_t}+\left(\mu-\frac{\sigma_A^2}{2}\right)t}{\sigma_A\sqrt{t}}\right] \tag{1}$$

where V_A is the current market value of the company's assets,

μ and σ_A are the drift rate and volatility of V_A respectively,

X_t is the book value of its debt due at time t, and

N is the standard cumulative normal distribution ($N(0,1)$).

A difficulty in computing the probability in equation (1) is that neither the market value nor the volatility of a company's underlying assets is observable. However, these two unknowns can be estimated by solving the following two equations simultaneously:

$$V_E = V_A N(d1) - e^{-rt} X_t N(d2) \tag{2}$$

where X_t is the book value of the debt due at time t, V_E is the equity value and

$$d1 = \frac{\ln(V_A / X_t) + (r + \sigma_A^2 / 2)t}{\sigma_A \sqrt{t}},$$

$$d2 = d1 - \sigma_A\sqrt{t}, \text{ and}$$

$$\sigma_E = \eta_{E,A}\sigma_A = (V_A / V_E)(\Delta)\sigma_A \tag{3}$$

where $\eta_{E,A} = (V_A / V_E)(\partial V_E / \partial V_A)$ is the elasticity of equity value to asset value and $\Delta = \partial V_E / \partial V_A$ is the hedge ratio, $N(d1)$, from (2).

Equation (2) gives the value of this option based on the Black-Scholes option pricing formula. Assuming that the market value of the company's assets follows a geometric lognormal process, equation (3) describes the relationship between the equity and asset volatility (Bensoussan et al. (1994)). Technical details on how the default probability is computed are discussed in the Appendix.

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[8] The "distance-to-default" measure (DD_t) used in the surveillance work of some central banks is very similar to the PD_t estimated from the Merton model. In fact, both measures are related to each other by the following equation $PD_t = N[-DD_t]$ where $N[.]$ is the standard cumulative normal distribution.

(b) Data

Next we illustrate the use of the model, using data on the publicly-listed non-financial companies of the Hang Seng Index (HSI) from January 1991 to October 2005. For each company, its PD in the coming 12 months (t = 1) is estimated on a monthly basis. As discussed above, the data needed for the estimation include the market value of equity (V_E), the volatility of equity (σ_E), the debt level (X) and the 1-year risk-free interest rate (r). The market value of equity is equal to the product of the outstanding number of shares and stock price.[9] The volatility of equity prices is estimated by the exponentially weighted moving average (EWMA) method.[10]

To calculate the debt level of the company, "short-term loans", "due to creditors", "long-term loans" and "other long-term liabilities" in the balance sheet are considered.[11] The face value of debt is equal to (short-term loans + due to creditors) plus half of (long-term loans + other long-term liabilities).[12] As audited balance sheet data are available on an annual basis, monthly figures are estimated by using a cubic interpolation routine.[13] With the time period under consideration being one year, 12-month Hong Kong Interbank Offered Rate (HIBOR) is used to represent the risk-free interest rate in the estimation. For simplicity, the expected growth rate of asset is assumed to be zero, given the short estimation horizon. Once the PDs of individual companies are estimated, the aggregate PD of the non-financial constituent companies of the HSI and its three constituent sectors are constructed by summing up the PDs, weighted by their market capitalisations.[14]

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9 Outstanding number of shares and stock prices are obtained from Bloomberg and Datastream.

10 Equity volatility is given by $\sigma_t^2 = (1-\lambda)R_t^2 + \lambda\sigma_{t-1}^2$, where R_t is the monthly return of equity price and λ is the decay factor which is set to be 0.94. The initial σ_t^2 is estimated from the average of the first 12 observations of the data series. We recognise that there are other ways to estimate the volatility. A comparison of different volatility estimation methods can be found in J.P. Morgan & Co. (1995).

11 The data are from Thomson Financial.

12 The reason to use short-term loans and due to creditors is that debt due within one year is more likely to cause default. The reasons for including long-term liabilities in the calculation are two-fold, (i) companies need to service their long-term debt, and these interest payments are part of their short-term liabilities; and (ii) the size of the long-term debt may affect the ability of a company to roll over its short-term debt. A factor of 0.5 is used for the long-term debt because the default point is found to lie generally somewhere between total liabilities and short-term liabilities (Crosbie and Bohn, 2002).

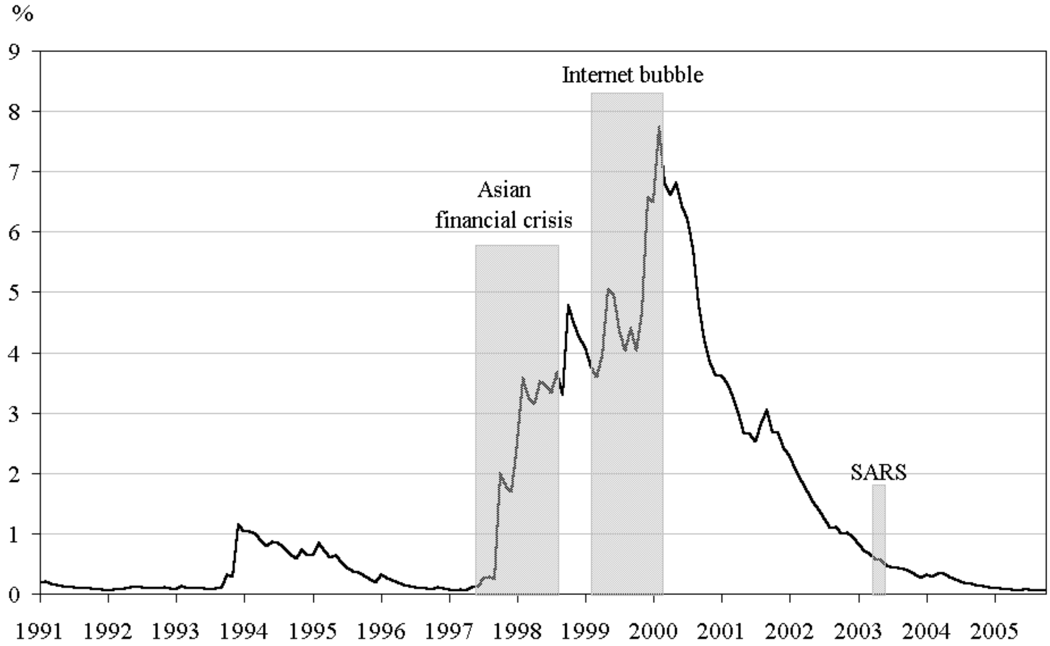
13 The same cubic smoothing method is also applied to extrapolate the debt levels of a company for recent months when the current balance sheet data are not available.

14 There are alternative ways to obtain an aggregate PD. For example, the Bank of England (2005) presents both unweighted and liability-weighted one-year implied probabilities of default for UK-quoted companies. The European Central Bank advocates the asset-weighted distance-to-default indicators (see Morttinen et al. (2005)). Despite some differences in the final numbers obtained from different measures, all these aggregate indicators are aimed at reflecting the systematic risk faced by the corporate sector as a whole.

III. EMPIRICAL RESULTS

The market capitalisation-weighted PDs of the HSI constituent companies covered in this study are computed on a monthly basis from January 1991 to August 2005 and illustrated in Chart 2.

Chart 2. Aggregate Default Probability of HSI Non-Financial Constituent Companies



Note: Shaded areas represent major events or crises.
Source: HKMA staff estimates.

As shown in the chart, the aggregate PD did not move much before the Asian financial crisis. The aggregate PD started to climb in mid-1997, jumping from 0.25% to about 5% during the Asian financial crisis. The PD came down briefly after the crisis and then returned to the up-trend when the dot.com mania stormed through the stock market in late 1999. In the few months before the internet bubble burst in late February 2000, the PD rose sharply from 4% in October 1999 to 7.7% in February 2000.[15] The sharp rise prior to the bursting of the bubble suggests that the PD may serve as an early warning signal for monitoring potential vulnerability in the corporate sector.[16] Starting from early 2000, the PD has been on a downward trend. The outbreak of the

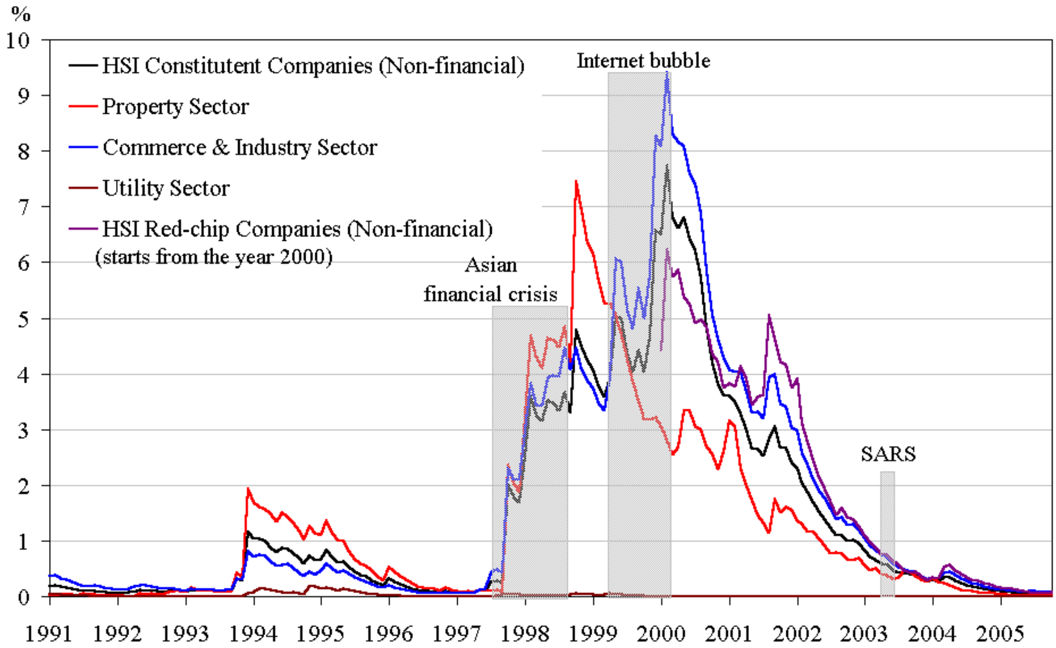
[15] The market capitalisation of the Internet Stock Index, an authoritative and widely cited index for publicly traded Internet stocks in the US, fell in value by approximately 45% from February to May 2000, signalling the bursting of the internet bubble.

[16] Various studies acknowledge that credit risk models signal default risks more timely than credit rating changes. For instance, the study by Oderda et al. (2003) shows that with respect to rating agencies, credit risk models predict default on average ten months in advance. Gropp et al. (2004) indicate that the distance-to-default indicator is able to predict individual bank fragility with considerable lead-time.

SARS epidemic in 2003 had little impact on the corporate vulnerability as a whole.[17] Since 2004, the PD has stayed at a level very close to that prior to the Asian financial crisis, showing that the likelihood of corporate default has eased substantially from the peak in early 2000.

To reveal areas of vulnerability within the HSI constituent sectors and examine their relative resilience to the macroeconomic shocks since the Asian financial crisis, similar PDs are derived for the HSI red-chip constituent stocks as well as the HSI's three constituent sectors, namely the property sector, the commercial & industry sector, and the utility sector, and presented in Chart 3.[18]

Chart 3. Aggregate Default Probabilities of HSI and its Constituent Sectors



Note: Shaded areas represent major events or crises.
Source: HKMA staff estimates.

[17] During the SARS episode, industry-specific companies such as those in the aviation and retail businesses might have been severely affected. However, for the corporate sector as a whole, the aggregate PD did not increase substantially as the aggregation process may reduce the effect of the idiosyncratic component of risk of individual companies on the aggregate PD. In fact, the HSI was mainly at a range trading between 8,400 and 9,000 during the peak of the SARS epidemic (from late March to early May 2003).

[18] Note that the finance sector of the HSI is excluded in this study, as the estimation of the default probability for financial institutions may be different due to the possible contagion effect among these institutions. Given the importance of the banking sector to financial stability, however, the credit risk of financial institutions will be examined in the future. Red-chip stocks are companies that incorporated in Hong Kong and listed on the Hong Kong Stock Exchange which have at least 35% of their shareholdings held, directly or indirectly, by state-owned organisations or provincial or municipal authorities in the Mainland. The data series used here starts from 2000 as there was only a small number of red-chip companies included in the HSI before 2000.

The first observation from Chart 3 is the resilience of the utility sector in terms of its financial health and soundness. In fact, the maximum PD of the utility sector over the period 1991 – 2005 was a mere 0.2%, which was recorded in November 1994. From late 1995 onwards, the PD of the utility sector has consistently stayed at less than 0.1%, despite the occurrence of Asian financial crisis, the burst of the internet bubble and the outbreak of the SARS epidemic. PDs of the property sector and the commercial & industry sector rose during the Asian financial crisis. The further increase in default risk of the property sector after the Asian financial crisis might reflect the continuous slump in the property market and the relatively high leverage of property developers' balance sheets. Nonetheless, the PD of the property sector declined gradually from 1999 onwards, partly attributed by the effort of the property developers to improve their leverages and debt levels. Yet the improvement in the credit quality of the property sector was not a smooth one, as we witnessed a lot of hiccups over the course of time. Moreover, it should be noted that the burst of the internet bubble seems to have little impact on the sector's default likelihood.

In contrast, after a brief drop in the aftermath of the Asian financial crisis, the default risk of the commercial & industry sector rose substantially to as high as 9.4% during the internet-bubble period between 1999 and late February 2000. In fact, the rise in the default risk of the commercial & industry sector outweighed the credit improvement in the property sector, and thus led to the sharp increase in the default risk of the aggregate HSI companies to its highest level of 7.7% over the study period. The PDs of the red-chip companies were consistently higher than those of the other HSI constituent sectors. However, the differences have narrowed since 2003.

In line with the movement in the default likelihood of the corporate sector as a whole, the PDs of both the property sector as well as the commercial & industry sector were mostly on a downward trend since 2001, despite the occurrence of the SARS epidemic in 2003. Since 2004, the financial health of all industrial sectors has returned to their pre-Asian crisis level at a mere 0.01% to 0.08%.

The sectorial analysis reveals areas of vulnerability within the HSI constituent sectors. The aggregate PDs derived from the Merton model reflect the impacts of economic shocks on the default risk of different corporate sectors in Hong Kong.

Mainly due to the scarcity of data caused by the lack of actual default events in Hong Kong, the accuracy of the PDs estimated in this study has not been tested empirically. Nonetheless, as discussed in the surveillance work of other central banks, these PDs are more indicative of possible systemic vulnerabilities of the corporate sector as a whole, which are related to macroeconomic shocks to the whole economy, than companies' individual idiosyncratic risks.[19] In our assessment of the Merton approach, we note that the PDs derived from the approach reflect the impacts of economic shocks and events, suggesting that the PDs are indeed indicative of systemic risk.

IV. CONCLUSION

The close relationship between the vulnerability of the corporate sector and the occurrence of banking and financial crises has made the assessment of corporate credit risk an important challenge to regulators responsible for monitoring banking and financial stability. One of the credit risk models that has been widely adopted by central banks is the Merton model. This paper presents the Merton model for credit risk analysis and applies the model to the non-financial listed companies of the Hang Seng Index in Hong Kong during the period of January 1991 to October 2005.

Compared with other market indicators, the Merton approach has the advantage of providing high frequency information that can be used to assess the credit condition of the corporate sector. This study has shown that the aggregate PDs derived from the Merton approach reflect the corporate default risk arising from economic shocks, and in some cases, provide early warning signals to the potential vulnerability in the corporate sector. Sectorial analyses based on the industry-specific PDs reveal areas of potential weaknesses in different industry sectors. Thus, the Merton approach can be used to monitor the overall credit quality of the corporate sector and also credit conditions of different industry sectors. The PD derived from this approach is considered as an effective monitoring tool to aid the ongoing surveillance work of regulators.

[19] The aggregation process should reduce the idiosyncratic component of risk of individual companies.

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Appendix. Technical Details for Deriving the Default Probability Based on the Structural Approach

To derive the default probability using the Merton approach, the market value of the company's underlying assets is assumed to follow the stochastic process:

$$dV_A = \mu V_A dt + \sigma_A V_A dz \qquad \text{(A1)}$$

where V_A , dV_A are the company's asset value and change in asset value,

μ , σ_A are the company's asset value drift rate and volatility, and

dz is a Wiener process.

The probability of a company default is given by the likelihood that the market value of the company's assets will be less than the book value of the company's liabilities by the time when the debt matures, that is:

$$PD_t = \text{Prob}[V_A^t \leq X_t \,|\, V_A^0 = V_A] = \text{Prob}[\ln V_A^t \leq \ln X_t \,|\, V_A^0 = V_A] \qquad \text{(A2)}$$

where PD_t is the probability of default by time t, V_A^t is the market value of assets at time t, and X_t is the book value of debt due at time t.

Given (A1), the value of the company's assets at time t, V_A^t, is:

$$\ln V_A^t = \ln V_A + \left(\mu - \frac{\sigma_A^2}{2}\right)t + \sigma_A \sqrt{t}\varepsilon \qquad \text{(A3)}$$

where V_A the company's current asset value,

μ is the expected return on the company's assets, and

ε is the random component of the company's return.

Combining (A2) and (A3) gives the PD as

$$PD_t = \text{Prob}\left[\ln V_A + \left(\mu - \frac{\sigma_A^2}{2}\right)t + \sigma_A \sqrt{t}\,\varepsilon \leq \ln X_t\right]$$

or, alternatively

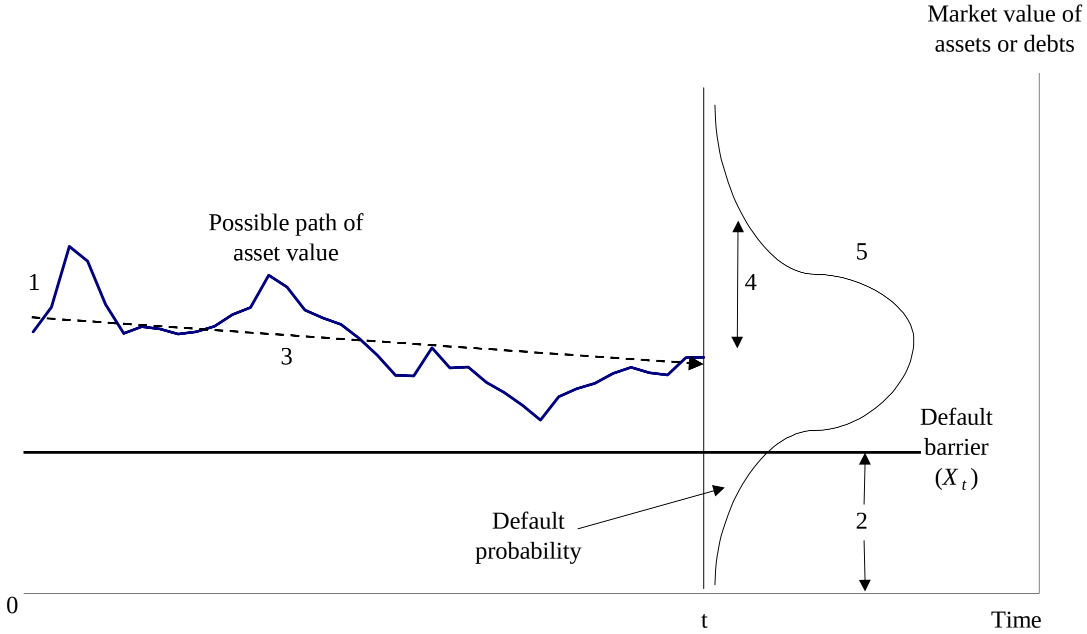
$$\text{PD}_t = \text{Prob}\left[-\frac{\ln\frac{V_A}{X_t}+\left(\mu-\frac{\sigma_A^2}{2}\right)t}{\sigma_A\sqrt{t}} \geq \varepsilon\right] \qquad \text{(A4)}$$

The Black-Scholes model assumes that the random component of the asset return is normally distributed, $\varepsilon \sim N(0,1)$ and as a result the probability of default can be defined in terms of the cumulative normal distribution as:

$$\text{PD}_t = N\left[-\frac{\ln\frac{V_A}{X_t}+\left(\mu-\frac{\sigma_A^2}{2}\right)t}{\sigma_A\sqrt{t}}\right] \qquad \text{(A5)}$$

See Chart A1 for an illustration.

Chart A1. Asset Value, Default Barrier and Default Probability



Notes:
1. The current market value of assets ($V_{A,}$).
2. The level of default barrier, the book value of debts due at time t (X_t)
3. The expected rate of growth in the asset value (μ).
4. The volatility of asset (σ_A).
5. The distribution of asset value (N(0.1)).

There are two unknowns, V_A and σ_A, in (A5) for the estimation of the default probability. These can be obtained by simultaneously solving equations (A6) and (A7).

$$V_E = V_A N(d1) - e^{-rt} X_t N(d2) \qquad \text{(A6)}$$

where r is the risk free interest rate,

$$d1 = \frac{\ln(V_A / X_t) + (r + \sigma_A^2 / 2)t}{\sigma_A \sqrt{t}},$$

$$d2 = d1 - \sigma_A \sqrt{t},$$

$N(.)$ is the standard cumulative normal distribution.

and

$$\sigma_E = \eta_{E,A} \sigma_A = (V_A / V_E)\, \Delta\, \sigma_A \qquad \text{(A7)}$$

where $\eta_{E,A} = (V_A / V_E)(\partial V_E / \partial V_A)$ is the elasticity of equity value to asset value,[20]

σ_E is the volatility of the company's equity value, and

Δ is the hedge ratio, $N(d1)$, from equation (A6).

Equation (A6) is the Black-Scholes pricing formula, which relates the market value of equity (V_E) to the market value and volatility of the company's underlying assets (V_A and σ_A), given that the company's capital structure is only composed of equity and debt, and given X_t the book value of the debt which is due at time t.

Equation (A7) links the volatility of equity value with that of the company's assets value which is assumed to follow the stochastic process shown in equation (A1).

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[20] See Bensoussan et al. (1994).