Hong Kong Monetary Authority

12 December 2003

VOLATILITY LINKAGES BETWEEN FINANCIAL MARKETS IN HONG KONG

Key Points:

- *This paper examines the volatility linkages between the stock and quasi-government bond markets in Hong Kong and the Hong Kong dollar forward exchange market. The understanding of volatility linkages between asset markets is important to policy makers who are concerned about the maintenance of financial stability.*

- *Estimation results from a regime switching ARCH (SWARCH) model confirm the presence of structural shifts in the volatility processes of Hong Kong financial markets, as well as volatility co-movement between financial markets.*

- *The average of a stay in a high-volatility state for a pair of financial markets is found to be between five to seven weeks. For a major shock such as the Asian financial crisis, the duration is at least six months.*

- *The results are helpful for gauging the possible duration of disruption in the financial system under a severe shock and in developing effective policies to deal with financial crises.*

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* The authors are grateful to participants at the Hong Kong Institute for Monetary Research (HKIMR) seminar on August 21, 2003 and the 8th Annual Australasian Macroeconomics Workshop, hosted by the HKIMR, on September 22-23, 2003, for helpful suggestions and comments.

I. Introduction

The analysis of financial market volatility and the links between asset markets have gained growing interest among researchers and policy makers in recent years, especially after the stock market crash in October 1987 and the Asian financial crisis in 1997-1998. Most studies in the empirical and theoretical investigations of the relationship between asset markets are concentrated on the linkages across markets of the same types.[1] In many cases, the research focuses on how a shock in one market will affect returns and volatility in other geographically distinct markets. Within an economy, the interest is mainly on the volatility linkage between cash and futures markets, especially that of the stock market. Few attempts have been made to understand the volatility phenomenon and linkages between different financial markets, such as that between the bond and foreign exchange markets, within an economy.[2] This study examines the volatility linkages between pairs of financial markets in Hong Kong amongst the stock market, the quasi-government bond market and the Hong Kong dollar forward exchange market.

There are several reasons for studying the volatility linkages between pairs of Hong Kong financial markets. First, the information on the volatility association between different assets is useful for risk management by investors. In particular, the results can be used to develop effective hedging or portfolio management strategies against shocks spreading across markets. Secondly, for policy makers, the results have implications on financial stability and risk monitoring. For instance, if volatility movements are highly synchronised across markets within the same economy, a shock developed in one asset market might have destabilising impacts on the entire financial system. The lack of a thorough understanding of these linkages among financial markets may reduce the effectiveness of policy actions against any undesirable volatility.

This paper contributes to the analysis of financial market volatility in two ways. First, it examines the volatility transmission across different domestic asset classes rather than between markets of the same nature but in different economies. Secondly, it allows for the incorporation of structural shifts in the investigation of volatility linkage between markets with the specification of a bivariate regime switching ARCH (SWARCH) model.[3] The inclusion of regime switching is important for modeling volatility in the Hong Kong financial markets as structural shifts in these markets have been common in the last decade. In addition, the results of the SWARCH model can be

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[1] For example, Lin et al. (1994) focus on equity markets, while Engle et al. (1990) and Fleming and Lopez (1999) concentrate on the foreign exchange market and the US Treasury market respectively.

[2] Examples include Fleming et al. (1998), Darbar and Deb (1999) for the US, and Ebrahim (2000) for Canada.

[3] As it turns out, the estimation of SWARCH model is extremely intensive in computation time and the condition of a positive-definite conditional variance-covariance matrix during the process of optimisation is not always guaranteed. In order to minimise the dimensionality problem and to keep the number of parameters tractable, a 2-regime bivariate SWARCH model is considered in this study.

used to derive the transition probability between different volatility states. Such information provides useful reference to policy makers for gauging the expected duration of high market volatility arising from extreme shocks to the financial system.

The remainder of this paper is organised as follows. Section II discusses the model specification and introduces the regime switching ARCH models. In section III, the data and some preliminary analyses on the volatility pattern of each financial market are examined. Empirical results on the volatility linkages are presented and discussed in section IV. A conclusion is provided in the final section.

II. The Regime-Switching ARCH Model

While the family of ARCH and GARCH models has been widely applied to modeling variance of financial variables, Lamoureux and Lastrapes (1990) show that these models may not be appropriate if structural breaks are present. As pointed out by Hamilton and Susmel (1994), ARCH models are inadequate when the data are characterised more by structural shifts leading to switches in variance regimes than persistent shocks. Cai (1994) and Hamilton and Susmel (1994) propose a regime switching ARCH or SWARCH model that is time-variant and allows for the conditional volatility process to switch stochastically among a finite number of regimes.[4] They demonstrate that this formulation leads to a significant reduction in the degree of volatility persistence compared to standard GARCH models.

To illustrate the features of the SWARCH model, a univariate case is first considered. For any financial market, the return of an asset at time *t* is represented by y_t and the residual with respect to the information set Ω_{t-1} is denoted as ε_t. The process ε_t obtained from a first-order autoregression for y_t under a SWARCH(*K, q*) model is specified as:

$$
\begin{aligned}
& y_t = w_0 + w_1 y_{t-1} + \varepsilon_t && \varepsilon_t \mid \Omega_{t-1} \sim N(0,\ \sigma_t^2) \\
& \varepsilon_t = \sqrt{g_{s_t}}\, u_t && \\
& u_t = h_t v_t && \\
& h_t^2 = c_0 + \sum_{i=1}^{q} \alpha_i u_{t-i}^2 && i = 1, 2, ..., q,\ and\ s_t = 1, 2, ..., K
\end{aligned} \tag{1}
$$

[4] While Gray (1996) introduces the generalised regime-switching (GRS) model, in which the ARCH and GARCH parameters are regime-dependent, the incorporation of regime switches into the GARCH term introduces tremendous estimation problems, especially in a bivariate setting. In this study, the empirical analysis of regime switches is confined to the ARCH process only.

where q is the number of ARCH terms, K is the number of regime states, and the g_{s_t} are scale parameters that capture the size of volatility in different regimes. It follows that the underlying ARCH variable u_t is multiplied by the scale parameter $\sqrt{g_1}$ when the process is in the regime represented by s_t = 1. It is multiplied by $\sqrt{g_2}$ when s_t = 2 and so on. The scale parameter for the first state g_1 is normalised at unity with $g_{s_t} \geq 1$ for s_t = 2, 3, …, K. The K-state regime switching is assumed to follow a Markov process, where

$$\begin{aligned} &\text{Prob}(s_t = j \mid s_{t-1} = i,\ s_{t-2} = k,\ \ldots;\ y_t, y_{t-1}, y_{t-2}, \ldots) \\ &= \text{Prob}(s_t = j \mid s_{t-1} = i) = p_{ij} \end{aligned} \tag{2}$$

for i,j,k = 1, 2, …, K. Under this specification, the transition probabilities, the p_{ij}s, are constant. In a two-state setting, for example, if the financial time series was in a high-volatility state in the last period (s_t = 2), the probability of it changing to the low-volatility state (s_t = 1) is a constant p_{21}. In addition, the estimation of the model gives the "smoothed probability", Prob(s_t | y_1, y_2, …, y_T), which provides information about the likelihood that the asset is in a particular volatility state at time t based on the full sample of observations.

There are several advantages of using the SWARCH specification to model volatility. First, the SWARCH model incorporates the possibility of regime shifts or structural breaks in the conditional variance process in explaining volatility persistence, a phenomenon that is commonly observed in the literature. Secondly, the SWARCH model can date the period of high volatility based on the smoothed probabilities. This will help detect whether periods of "high volatility" coincide across different financial markets. Finally the identification of breakpoints can also be used to "time" the impact of policy changes on financial markets.[5]

A limitation of the SWARCH model is the technically non-trivial and very time-consuming estimation process. In this study, the application of the SWARCH model is restricted to pairs of financial markets only, each with one ARCH term in the conditional variance process and two volatility states. Under this bivariate AR(1) SWARCH(2,1) specification, the total number of states is four. For instance, considering the stock market and the quasi-government bond market pair, the four possible states, s_t^*, are as follows:

[5] A review of the SWARCH model and its application can be found in Ramchand and Susmel (1998), Susmel (2000), Edwards and Susmel (2001) and Edwards and Susmel (2003).

s_t^* = 1: Stock market – low volatility, Quasi-gov't bond market – low volatility.

s_t^* = 2: Stock market – low volatility, Quasi-gov't bond market – high volatility.

s_t^* = 3: Stock market – high volatility, Quasi-gov't bond market – low volatility.

s_t^* = 4: Stock market – high volatility, Quasi-gov't bond market – high volatility.

Similar to the univariate case, the system for the bivariate model can be written as:

$$y_t = A + B\, y_{t-1} + e_t,\quad e_t \,\big|\, \Omega_{t-1} \sim N(0, H_t) \tag{3}$$

where $y_t = \begin{bmatrix} y_{1,t} \\ y_{2,t} \end{bmatrix}$ is a 2x1 vector of returns, $e_t = \begin{bmatrix} e_{1,t} \\ e_{2,t} \end{bmatrix}$ is a 2x1 vector of disturbances, which are assumed to follow a bivariate normal distribution with zero mean and a time varying conditional covariance matrix H_t. The time varying conditional covariance matrix H_t is specified as a constant correlation matrix where the diagonal elements follow a SWARCH(2,1) process. $A = \begin{bmatrix} w_{10} \\ w_{20} \end{bmatrix}$ is a 2x1 vector and $B = \begin{bmatrix} w_{11} & 0 \\ 0 & w_{22} \end{bmatrix}$ is a 2x2 matrix. The parameters of the bivariate AR(1) SWARCH(2,1) model are estimated using GAUSS by numerically maximising the likelihood function in the algorithm developed by Broyden, Fletcher, Goldfarb and Shanno (BFGS), subject to the constraints that $g_1 = 1$, $g_2 \geq 1$, $\sum_{j=1}^{2} p_{ij} = 1$ for i = 1, 2 and $0 \leq p_{ij} \leq 1$ for i,j = 1, 2.[6,7]

III. The Data and Preliminary Analyses

The data consist of weekly changes for three types of assets in the Hong Kong financial markets, namely the stock market (represented by the log differences (in percent) of the Hang Seng Index), the quasi-government bond market (the holding returns for 10-year Exchange Fund Notes) and the Hong Kong dollar forward exchange market (the log differences (in percent) of the 12-month Hong Kong dollar forward exchange rate).[8] The data set spans from January 1990 to March 2003, while the quasi-government bond market starts from November 1996.

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[6] GAUSS programs and most of the routines are obtained from websites of James Hamilton and Rauli Susmel.

[7] The BFGS algorithm is described in Press et al. (1988).

[8] The approximation for the weekly holding period return for government bond is based on Shiller (1979). For bonds selling at or near par value, Shiller suggests an approximate expression for the *n*-period holding period return $H_t^{(n)}$, where $H_t^{(n)} = (R_t^{(n)} - \gamma_n R_{t+1}^{(n-1)})/(1-\gamma_n)$, $\gamma_n = \gamma(1-\gamma^{n-1})/(1-\gamma^n)$, $\gamma = 1/(1+\overline{R})$, $R_t^{(n)}$ is the yield to maturity and $\overline{R}$ is the mean value of the yield to maturity.

Table 1 presents some descriptive statistics of the financial markets. The distributions of the series are skewed and have fat tails, as implied by the high kurtosis coefficients. Furthermore, the significant Jarque-Bera statistics indicate that the financial time series are not normally distributed. The data are confirmed to be stationary by the test results of the augmented Dickey-Fuller (ADF) unit root test. The Ljung-Box Q statistics up to the 6^{th} order provide evidence of serial correlation in the level and in the squared level respectively. This also suggests the presence of autoregressive conditional heteroskedasticity (ARCH) in all the series and justifies the use of an AR(1) term in the specification of the conditional mean equation.[9]

Table 1. Summary of Statistics of the Financial Markets

	Stock Market (in % return)	**Quasi-government Bond Market** (in % return)	**Forward Market** (in % change)
Mean	0.16	0.20	0.00
Maximum	13.92	5.68	2.92
Minimum	-19.92	-8.91	-2.41
Std.Dev.	3.67	1.48	0.30
Skewness	-0.39	-0.81	1.60
Kurtosis	5.65	9.69	32.33
Jarque-Bera	222.14	662.62	25,391.34
ADF statistics	-25.89*	-17.72*	-28.00*
Q (6)	7.90	12.18^{+}	28.85*
Q^2 (6)	20.56*	28.34*	111.25*
Observations	700	336	700

Notes: * indicates significance at the 5% level. $^{+}$ indicates significance at the 10% level. The Jarque-Bera statistic has a χ^2 distribution with two degrees of freedom under the null hypothesis of normally distributed errors. The critical value of χ^2 (2) at the 5% level is 5.99. The critical ADF value at the 5% level is –2.87. Q (6) and Q^2 (6) are the Ljung-Box statistics based on the levels and the squared levels of the time series respectively up to the 6^{th} order. Both statistics are asymptotically distributed as χ^2 (6). The critical value of χ^2 (6) at the 5% and the 10% level is 12.59 and 10.64 respectively.

[9] A general discussion of Hong Kong financial market volatility can also be found in Yu and Fung (2003).

As a first step of the analysis, a simple AR(1) GARCH(1,1) model for each series is estimated and the results are presented in Table 2. The estimated coefficients of ARCH (α) and GARCH (β) effects are highly significant in each asset. The sum of ARCH and GARCH coefficients ($\alpha + \beta$) in each estimation is close to or larger than one, suggesting that shocks to the conditional variance are highly persistent. That is, shocks that occurred in the distant past have an effect on the current conditional variance.

Table 2. Parameter Estimates and Specification Tests of Univariate AR(1) GARCH(1,1) Model

	Stock Market	Quasi-government Bond Market	Forward Exchange Market
w_0	0.303*	0.177*	-0.006
	(0.124)	(0.068)	(0.006)
w_1	0.021	0.058	-0.165*
	(0.039)	(0.074)	(0.080)
c_0	0.273	0.096	0.001
	(0.158)	(0.072)	(0.001)
α_1	0.078*	0.068*	0.493
	(0.031)	(0.034)	(0.303)
β_1	0.905*	0.890*	0.638*
	(0.035)	(0.051)	(0.114)
$\alpha_1 + \beta_1$	0.983	0.958	1.131
Log Likelihood	-1,860	-579	367
Q (6)	7.64	4.32	3.21
Q^2 (6)	2.17	0.85	1.21

Notes: Numbers in parentheses are standard errors. * indicates significance at the 5% level. Q (6) and Q^2 (6) are the Ljung-Box statistics based on the standardised residuals and the squared standardised residuals respectively up to the 6^{th} order. Both statistics are asymptotically distributed as χ^2 (6). The critical value of χ^2 (6) at the 5% level is 12.59.

In the last decade, Hong Kong financial markets witnessed such major events as the changeover of sovereignty, the Asian financial crisis and the burst of the technology bubble. It is therefore important to check whether financial market volatility has been affected by structural shifts or extreme events leading to switches in variance regimes. To take this condition into account, an AR(1) SWARCH(2,1) model is estimated for each series to identify periods of unusually high volatility. The results are presented in Table 3.

Table 3. Parameter Estimates and Specification Tests of Univariate AR(1) SWARCH(2,1) Model

	Stock Market	**Quasi-government Bond Market**	**Forward Exchange Market**
w_0	0.281*	-0.020*	-0.004
	(0.120)	(0.008)	(0.003)
w_1	0.018	0.027	-0.173*
	(0.048)	(0.060)	(0.034)
c_0	5.844*	0.014*	0.004*
	(0.600)	(0.002)	(0.001)
α_1	0.000	0.018	0.475*
	(0.067)	(0.064)	(0.114)
g_2	3.78*	11.34*	81.16*
	(0.46)	(2.58)	(18.64)
Log Likelihood	-1,854	118	448
Q (6)	7.24	5.34	6.06
Q^2 (6)	1.97	0.59	10.31

Notes: Numbers in parentheses are standard errors. * indicates significance at the 5% level. Q (6) and Q^2 (6) are the Ljung-Box statistics based on the standardised residuals and the squared standardised residuals respectively up to the 6th order. Both statistics are asymptotically distributed as χ^2 (6). The critical value of χ^2 (6) at the 5% level is 12.59.

Table 3 shows that the estimated ARCH parameters (α_1) for stock and quasi-government bond markets under the SWARCH model are insignificant. The finding is similar to Edwards and Susmel (2001) where the use of the SWARCH model causes the ARCH effect to be reduced or suppressed. All the estimated scale parameters for variance in state 2 (g_2), the high-volatility state, are significantly different from unity. The ARCH parameters are smaller and sometimes insignificant while the scale parameters of the high-volatility state are significant, suggesting the presence of a structural break and the appropriate use of the SWARCH model in specifying Hong Kong financial market volatility. The insignificant Q(6) and Q^2(6) statistics indicate that the financial series are adequately modeled without any serial correlation or ARCH effect.

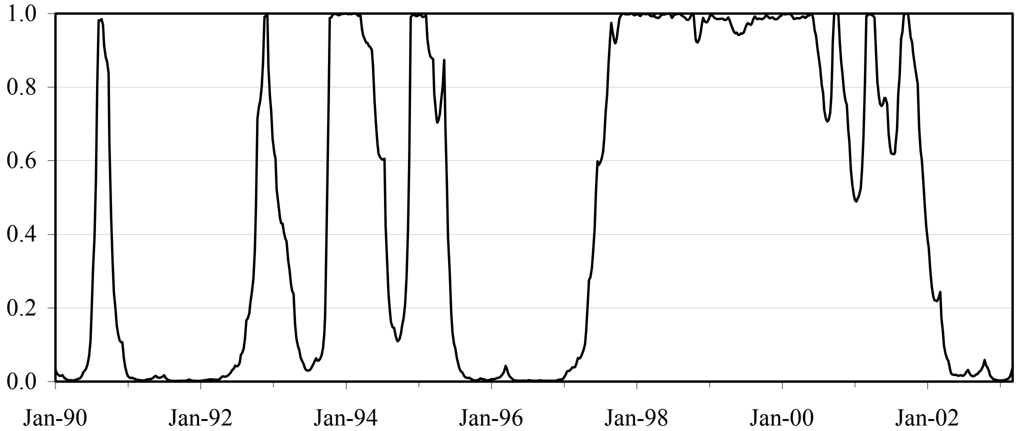
Chart 1 illustrates the smoothed probabilities of the high-volatility state in each financial market based on the univariate SWARCH model. During mid-1997 to end-2001, the stock market had been in the high-volatility state. In contrast, the quasi-government bond and forward exchange markets experienced mainly short-lived high-volatility episodes during the same period.

By comparing the pattern of the smoothed probability across different panels, one can examine whether the high-volatility state happened concurrently in different financial markets in the last decade. As shown in Chart 1, various financial markets in Hong Kong appear to experience high volatility simultaneously on a few occasions, suggesting that volatility linkage or co-movement may exist between financial markets. In order to examine the issue of volatility linkages between these markets and regime shift across different states of volatility in a more vigorous way, the univariate SWARCH model is extended to a bivariate one in the next section.[10]

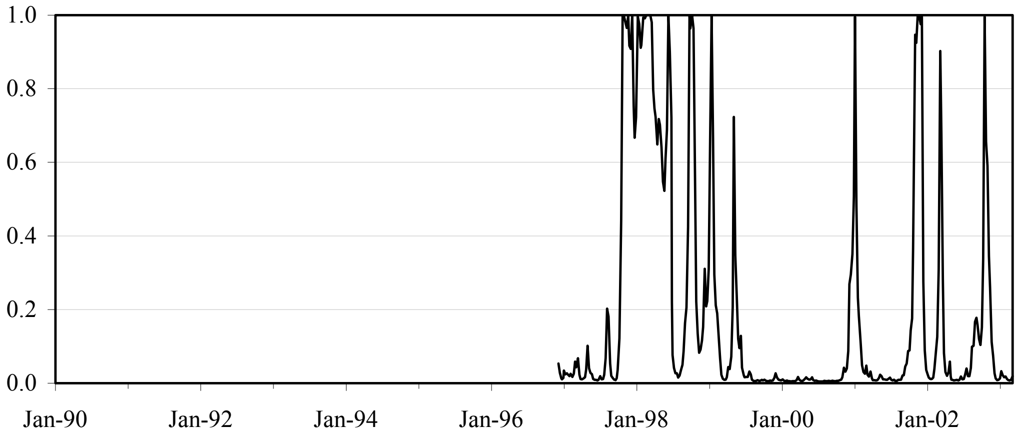
[10] By having the off-diagonal terms to be non-zero in the covariance-variance matrix, the bivariate SWARCH specification allows a more flexible structure in the modeling of the variance for the financial markets by capturing their cross-market dynamics.

Chart 1. Smoothed Probabilities of High-Volatility State

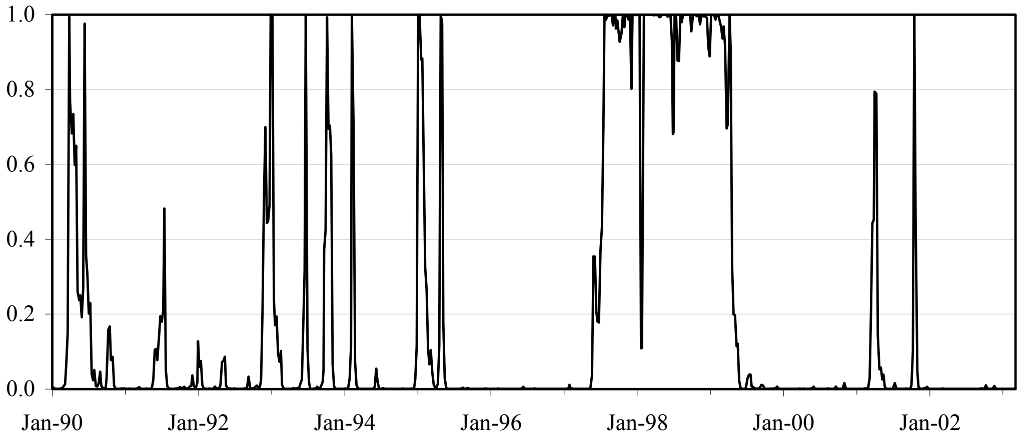
Stock Market



Quasi-government Bond Market



Forward Exchange Market



Note: Quasi-government bond series starts from November 1996.

IV. Volatility Linkages and Estimation Results

Estimation results of a bivariate AR(1) SWARCH(2,1) model are reported in Table 4. Diagnostic tests such as Q(6) and $Q^2(6)$ indicate that the data series are adequately modeled.

The scale parameters for volatility state two (g_2) are statistically significant in all markets for different pairs, suggesting that structural shifts need to be taken into account when modeling their volatility processes. As in the univariate case, the ARCH effect (α) in both the stock and quasi-government bond markets is suppressed and only the estimated ARCH term of the forward exchange market is significant. As shown by the g_2 parameters, the volatility shift in the forward exchange market is the largest among all markets. Its variance in the high-volatility state ($s_t = 2$) is over 80 times larger than that in the low-volatility state. On the other hand, this ratio is only about ten for the quasi-government bond market and three for the stock market respectively.

Table 4. Parameter Estimates of Bivariate AR(1) SWARCH(2,1) Model

	Stock – Quasi-government Bond	**Forward Exchange – Quasi-government Bond**	**Stock – Forward Exchange**
w_{10}	-0.112	-0.001	0.274*
	(0.718)	(0.004)	(0.119)
w_{11}	0.030*	0.082	0.017
	(0.076)	(0.071)	(0.040)
w_{20}	0.269*	0.267*	-0.004
	(0.065)	(0.062)	(0.003)
w_{22}	0.022	0.032	-0.173*
	(0.131)	(0.062)	(0.034)
c_{11}	7.310*	0.003*	5.863*
	(2.875)	(0.001)	(0.854)
c_{22}	0.741*	0.738*	0.004*
	(0.157)	(0.097)	(0.001)
α_{11}	0.000	0.535*	0.000
	(0.636)	(0.157)	(0.159)
α_{22}	0.029	0.026	0.471*
	(0.242)	(0.070)	(0.112)
$g_{2,1}$	2.937*	168.620*	3.771*
	(1.027)	(38.025)	(0.504)
$g_{2,2}$	9.633*	10.529*	81.385*
	(2.256)	(2.494)	(18.470)
Log likelihood	-1,472	-375	-1,408
$Q_1(6)$	4.43	3.38	7.27
$Q_2(6)$	5.87	5.36	6.07
$Q_1^2(6)$	1.86	0.66	1.97
$Q_2^2(6)$	0.63	0.66	10.28

Notes: Numbers in parentheses are standard errors. Standard errors are calculated from the inverse of the Hessian matrix. * indicates significance at the 5% level. Q (6) and Q^2 (6) are the Ljung-Box statistics based on the standardised residuals and the squared standardised residuals respectively up to the 6th order. Both statistics are asymptotically distributed as χ^2 (6). The critical value of χ^2 (6) at the 5% level is 12.59.

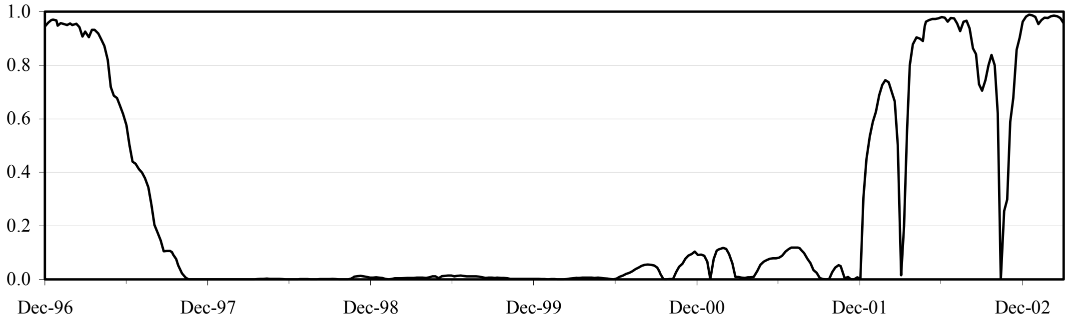
Charts 2 to 4 contain the smoothed probabilities of the four volatility states (s_t^*) described previously for three pairs of financial markets. The top panel is the smoothed probabilities when the selected pair of markets is in a low-volatility state, whereas the bottom panel is the smoothed probabilities when both of them are in a high-volatility state. The middle panels show the probabilities of them in opposite volatility states.

As shown in Charts 2 to 4, all market pairs started in a low-volatility state (top panel) before shifting to a high-volatility state from October 1997 onward (bottom panel). During the Asian financial crisis, the duration of all the three financial markets staying in a high-volatility state was over six months. Except for some brief periods, the high-volatility condition lasted till early 1999. This clearly demonstrates that periods of "high volatility" coincide across different financial markets, suggesting that there are strong volatility linkages during the crises. After the 1997-98 crisis, the stock market remained in a high-volatility state from mid-1999 to 2000 while the quasi-government bond and forward exchange markets moved back to a low-volatility state. All three markets shifted to a high-volatility state after the terrorist attack in the US in September 2001 but the disturbance was brief. Since late 2002, except for some temporary events, all market pairs remained in a low-volatility state (top panel). This shows that Hong Kong financial markets have regained their stability after such events as the Asian and Russian financial crises, the burst of the technology bubble and the terrorist attack in the US.

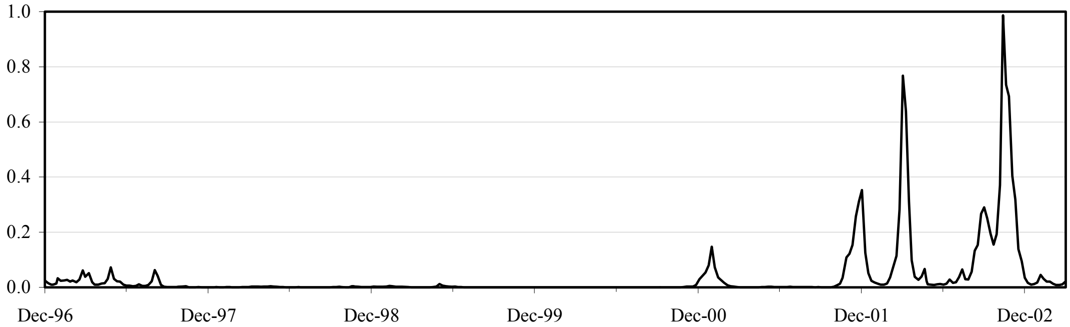
A strength of the SWARCH model is its ability to identify breakpoints and capture the reaction of different financial markets to news and events. Charts 2 to 4 show that the volatility of the stock and forward exchange markets shifted from a low state in early 1997 to a high one in mid-1997, while that of the quasi-government bond market remained in a low-volatility state. This may signal the higher sensitivity of the stock and forward exchange markets to the changeover of sovereignty in July 1997, while the quasi-government bond market appeared to be less reactive. By October 1997, all markets responded significantly to the Asian financial crisis and swiftly moved into a high-volatility state.

Chart 2. Stock Market – Quasi-government Bond Market Volatility States

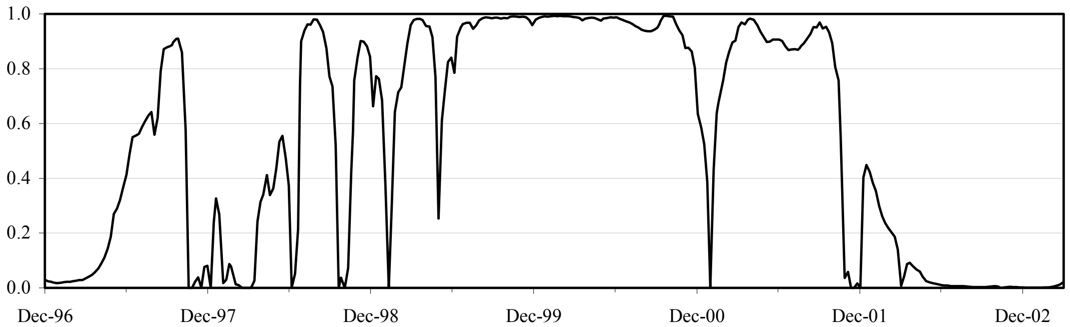
State 1: Stock Market - low volatility, Quasi-gov't Bond Market - low volatility



State 2: Stock Market - low volatility, Quasi-gov't Bond Market - high volatility



State 3: Stock Market - high volatility, Quasi-gov't Bond Market - low volatility



State 4: Stock Market - high volatility, Quasi-gov't Bond Market - high volatility

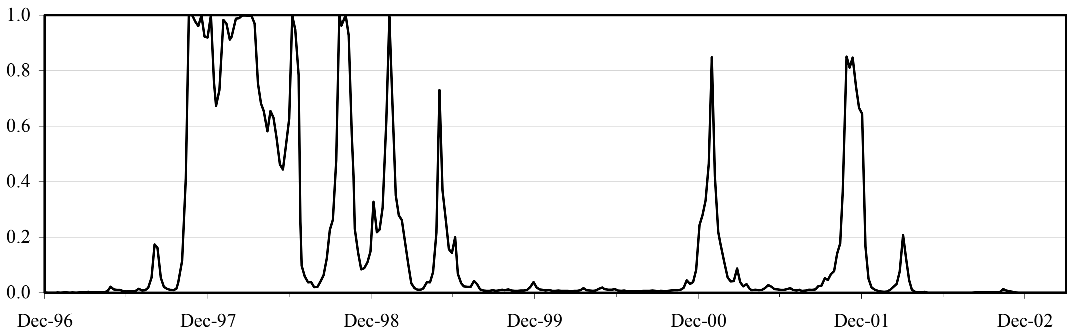
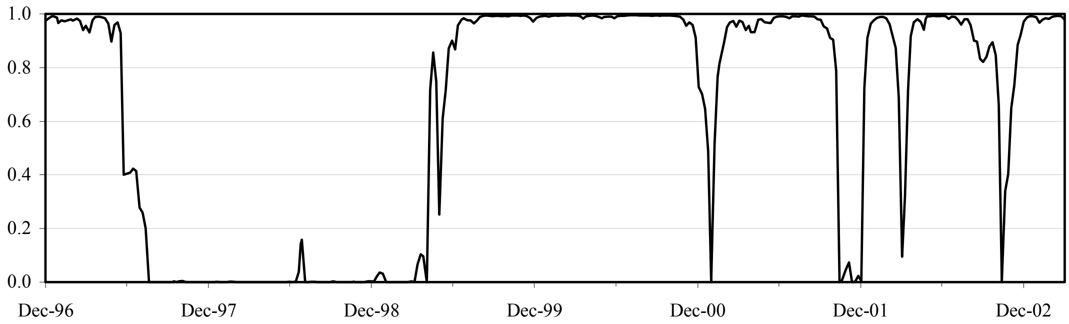
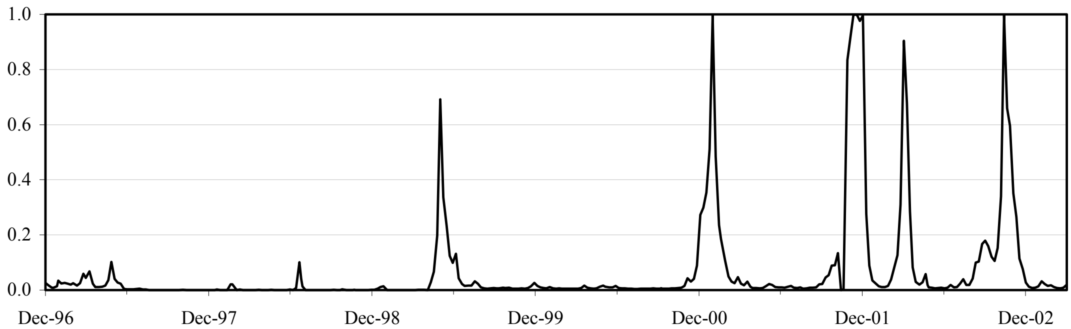


Chart 3. Quasi-government Bond Market – Forward Exchange Market Volatility States

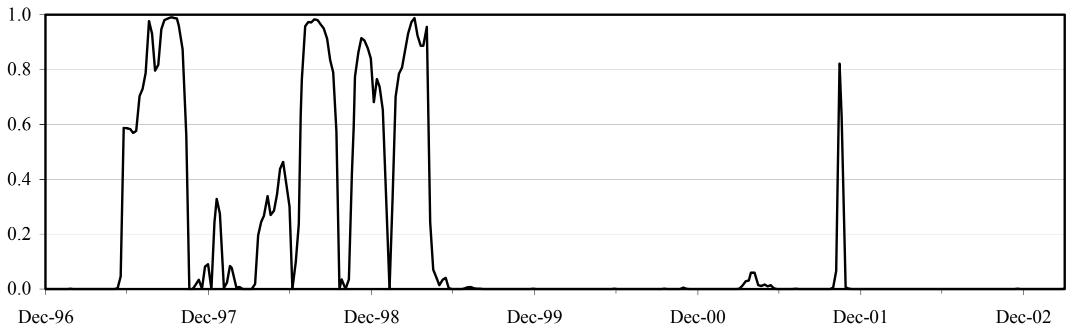
State 1: Forward Market - low volatility, Quasi-gov't Bond Market - low volatility



State 2: Forward Market - low volatility, Quasi-gov't Bond Market - high volatility



State 3: Forward Market - high volatility, Quasi-gov't Bond Market - low volatility



State 4: Forward Market - high volatility, Quasi-gov't Bond Market - high volatility

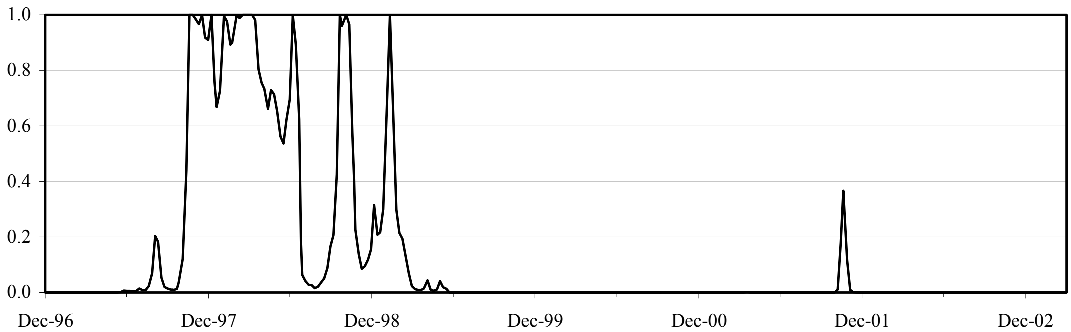
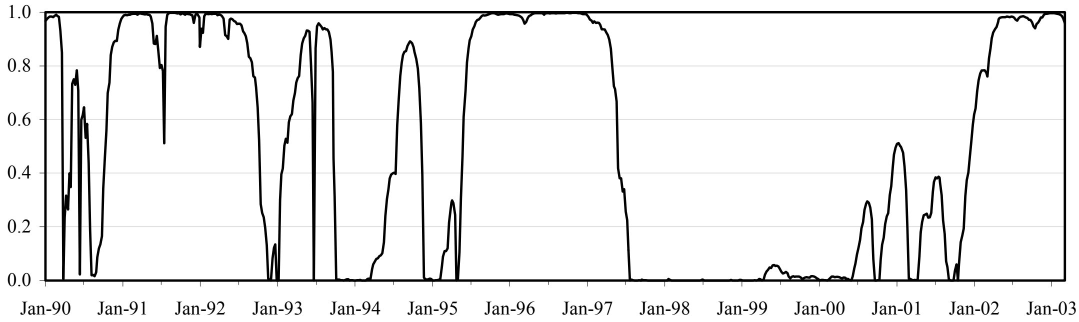
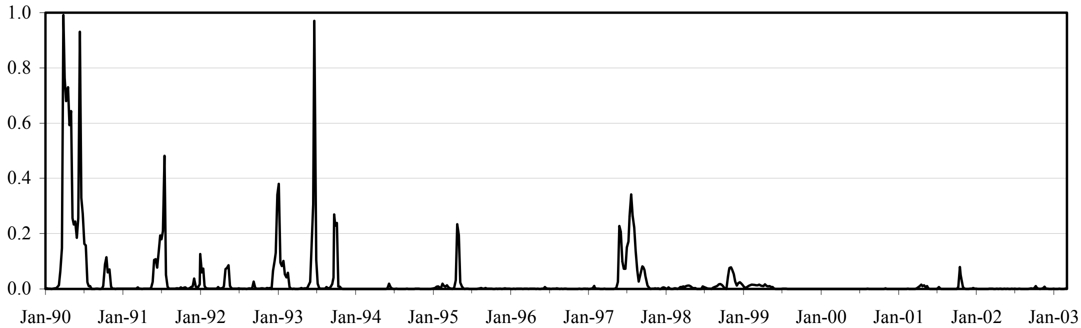


Chart 4. Stock Market – Forward Exchange Market Volatility States

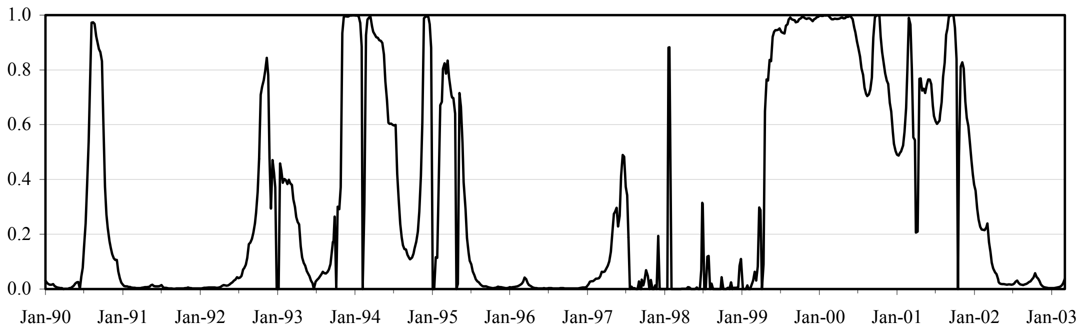
State 1: Stock Market - low volatility, Forward Exchange Market - low volatility



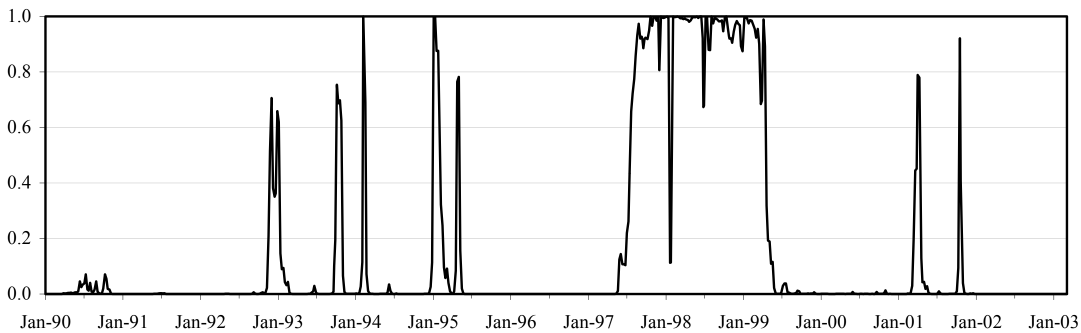
State 2: Stock Market - low volatility, Forward Exchange Market - high volatility



State 3: Stock Market - high volatility, Forward Exchange Market - low volatility



State 4: Stock Market - high volatility, Forward Exchange Market - high volatility



The expected duration of financial markets to stay in a high-volatility state is important to policymakers who are concerned about the maintenance of financial stability. This can be derived from the transition probability estimated by the SWARCH model.[11] Table 5 gives the transition probability and the respective expected duration of a pair of financial markets to being in a particular volatility state. As shown in Table 5, the transition probability is quite high for most market pairs and the expected duration for a market pair to stay in the same volatility state can be at least five weeks. For instance, the transition probability for both the stock and quasi-government bond markets (first column) to be jointly in the high-volatility state ($s_t^* = 4$) is 0.811. This can be translated into an expected duration (or volatility-state persistence) of 5 weeks (= $(1 - 0.811)^{-1}$). That means, on average, the stock and quasi-government bond markets are expected to stay in the high-volatility state for about 5 weeks before they shift into other states of volatility. On the other hand, for the stock and forward exchange market pair (third column), the transition probability for both markets to start and remain in the high-volatility state is 0.856, such that the expected duration is 7 weeks (= $(1 - 0.856)^{-1}$). In general, the transition probability for the stock and forward exchange market pair is the largest among all market pairs. Hence, the volatility for the stock and forward exchange market pair is, on average, the most state-persistent among all market pairs.

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[11] Given that a financial market pair is currently in a volatility state *j* ($s_t^* = j$), *D* is the duration of volatility state *j* and p_{jj} is the transition probability for the financial market pair to start and stay in the volatility state *j*, then we have

$D = 1$, if $s_t^* = j$ and $s_{t+1}^* \neq j$; Prob$[D = 1] = (1 - p_{jj})$

$D = 2$, if $s_t^* = s_{t+1}^* = j$ and $s_{t+2}^* \neq j$; Prob$[D = 2] = p_{jj}(1 - p_{jj})$

$D = 3$, if $s_t^* = s_{t+1}^* = s_{t+2}^* = j$ and $s_{t+3}^* \neq j$; Prob$[D = 2] = p_{jj}^2(1 - p_{jj})$

$\vdots$

The expected duration of volatility state *j* can be derived as

$$
\begin{aligned}
E(D) &= \sum_{j=1}^{\infty} j \,\text{Prob}[D = j] \\
&= 1 \text{ x Prob}[\, s_{t+1}^* \neq j \,|\, s_t^* = j \,] \\
&+ 2 \text{ x Prob}[\, s_{t+1}^* = j, s_{t+2}^* \neq j \,|\, s_t^* = j \,] \\
&+ 3 \text{ x Prob}[\, s_{t+1}^* = j, s_{t+2}^* = j, s_{t+3}^* \neq j \,|\, s_t^* = j \,] \\
&+ \ldots \\
&= 1 \text{ x } (1 - p_{jj}) + 2 \text{ x } p_{jj}(1 - p_{jj}) + 3 \text{ x } p_{jj}^2(1 - p_{jj}) + \ldots \\
&= 1 / (1 - p_{jj}) \text{ or } (1 - p_{jj})^{-1}
\end{aligned}
$$

Table 5. Parameter Estimates of Transition Probabilities

s_t^* = 1: Market 1 – low volatility, Market 2 – low volatility.
s_t^* = 2: Market 1 – low volatility, Market 2 – high volatility.
s_t^* = 3: Market 1 – high volatility, Market 2 – low volatility.
s_t^* = 4: Market 1 – high volatility, Market 2 – high volatility.

	Stock – Quasi-government Bond	Forward Exchange – Quasi-government Bond	Stock – Forward Exchange
$s_t^* = 1$	0.944	0.948	0.953
	(18 weeks)	(19 weeks)	(21 weeks)
$s_t^* = 2$	0.809	0.814	0.861
	(5 weeks)	(6 weeks)	(7 weeks)
$s_t^* = 3$	0.946	0.930	0.948
	(19 weeks)	(14 weeks)	(19 weeks)
$s_t^* = 4$	0.811	0.799	0.856
	(5 weeks)	(5 weeks)	(7 weeks)

Note: Figures in parentheses are measures of volatility-state persistence in number of weeks, which are calculated as $(1 – \text{transition probability})^{-1}$.

V. Conclusion

The analysis in this paper provides an understanding of the volatility linkages among three financial markets in Hong Kong, namely the stock market, the quasi-government bond market and the Hong Kong dollar forward exchange market. Such understanding is important to investment professionals from a risk diversification perspective as well as to policy makers who are concerned about financial stability issues.

As structural shifts in the conditional variance process are common in many financial data series, the regime switching ARCH (SWARCH) model clearly demonstrates its strength in identifying the presence of such shifts in the volatility processes of financial markets. Based on a bivariate SWARCH model, the analysis in this paper finds evidences of volatility co-movement among financial markets in Hong Kong, especially during crises like the Asian financial crisis. The expected duration for a pair of financial markets to jointly stay in a high-volatility state is between five and seven weeks. For a major shock similar to the Asian financial crisis, the duration for these financial markets to stay in a high-volatility state can be above six months. Overall, the analysis provides references to policy makers for gauging the possible duration of disruption in the financial system during a severe shock and in developing effective policies to deal with financial crises.

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