



A FRAMEWORK FOR STRESS TESTING BANKS' CREDIT RISK¹

Key Points:

- *This paper develops a framework for stress testing the credit exposures of Hong Kong's retail banks to macroeconomic shocks. It involves the construction of macroeconomic credit risk models, each consisting of a multiple regression model explaining the default rate of banks, and a set of autoregressive models explaining the macroeconomic environment estimated by the method of seemingly unrelated regression.*
- *Specifically, two macroeconomic credit risk models are built. One model is specified for the overall loan portfolios of banks and, to illustrate how the same framework can be applied for stress testing loans to different economic sectors, the other model is specified for the banks' mortgage exposures only.*
- *The empirical results suggest a significant relationship between the default rates of bank loans and key macroeconomic factors including Hong Kong's real GDP, real interest rates, real property prices and Mainland China's real GDP.*
- *Macro stress testing is then performed to assess the vulnerability and risk exposures of banks' overall loan portfolios and mortgage exposures. By using the framework, a Monte Carlo method is applied to estimate the distribution of possible credit losses conditional on an artificially introduced shock. Different shocks are individually introduced into the framework for the stress tests. The magnitudes of the shocks are specified according to those occurred during the Asian financial crisis.*
- *The result shows that even for the Value-at-Risk (VaR) at the confidence level of 90%, banks would continue to make a profit in most stressed scenarios, suggesting that the current credit risk of the banking sector is moderate. However, under the extreme case for the VaR at the confidence level of 99%, banks' credit loss would range from a maximum of 3.22% to a maximum of 5.56% of the portfolios, and if a confidence level of 99.9% is taken, it could range from a maximum of 6.08% to a maximum of 8.95%. These estimated maximum losses are very similar to what the market experienced one year after the Asian financial crisis shock. However, the probability of such losses and beyond is very low.*

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I. INTRODUCTION

Macro stress testing refers to a range of techniques used to assess the vulnerability of a financial system to “exceptional but plausible” macroeconomic shocks.² Increasingly, macro stress testing plays an important role in the macro-prudential analysis of public authorities. The main objective is to identify structural vulnerability and overall risk exposures in a financial system that could lead to systemic problems. In conjunction with stress testing to assess the vulnerability of the portfolios of individual institutions, macro stress testing forms the main part of system-wide analysis, which measures the risk exposure of a group of financial institutions to a specific stress scenario. It can also serve as a tool for cross-checking results obtained by financial institutions’ internal models.

In this paper, a macro stress testing framework is developed for testing the loan portfolios of retail banks in Hong Kong. It involves the construction of macroeconomic credit risk models, each consisting of a multiple regression model and a set of autoregressive models (for examining the relationship between the default rate of bank loans and different macroeconomic values based on historical data) estimated by the method of seemingly unrelated regression. Two macroeconomic credit risk models are built. One model is specified for the overall loan portfolios of banks and, to illustrate how the same framework can be applied for stress testing loans to different economic sectors, the other model is for the banks’ mortgage exposures only.

Macro stress testing is then performed to assess the vulnerability and risk exposures of banks’ overall loan portfolios and mortgage exposures. Adverse macroeconomic scenarios are taken and, using the framework, the possible combinations of stressed macroeconomic values are obtained from a Monte Carlo simulation. Based on this, distributions of possible default rates of bank loans under a specific shock can be generated. Value-at-Risk (VaR) is computed to evaluate how the stressed macroeconomic environment may affect the default probability of banks’ loan portfolios.³

² This follows the IMF definition. See Blaschke et al. (2001) and Sorge (2004).

³ VaR refers to the maximum amount of money that may be lost over a certain period at a specific confidence level.

II. ELEMENTS OF STRESS TEST AND THE COMMON METHODOLOGY

Macroeconomic stress tests involve two major elements. First, scenarios of extreme but plausible adverse macroeconomic conditions need to be devised.⁴ Secondly, the adverse macroeconomic scenarios need to be mapped onto the impact on banks' balance sheets. Through this, the robustness of banks can be evaluated.

For the first element, given that since an adverse macroeconomic scenario refers to a combination of adverse developments in several macroeconomic variables, it is important to ensure its internal consistency and that the specified values of the macroeconomic variables constitute a realistic mix. The conventional approach, as adopted by Froyland and Larsen (2002), Hoggarth and Whitley (2003), Mawdsley et al. (2004) and Bunn et al. (2005), is to devise scenarios that imitate historical episodes of tail events or to generate scenarios with the aid of a macro-econometric model.

After devising the scenarios, the impact on banks will be estimated. This usually requires first estimating an empirical model that relates a certain financial soundness indicator y to a number of macroeconomic variables x_1, \dots, x_M that the scenarios encompass:

$$y = f(x_1, \dots, x_M) + \varepsilon,$$

where ε is an error term capturing determinants of the indicator other than x_1, \dots, x_M . The values of x_1, \dots, x_M given by the scenarios will then be substituted into the estimated equation and the predicted values of y are computed under the assumption that $\varepsilon = 0$. These predicted values are (point) estimates of the expected values of y conditional on the occurrence of the scenarios. Changes in the predicted values of y as a result of the imposition of the scenarios are usually regarded as the estimated impacts. This approach suffers from two problems: first, once a scenario is chosen, how likely it is to occur is no longer an issue in the stress test;⁵ secondly, even if the predicted value of the soundness indicator is not significantly affected by the realisation of the adverse scenario, it is hard to conclude that the risk is low because a large deviation from the average may occur with a "tangible" probability.

By taking into account the possibility that ε is non-zero in the y equation and there is randomness in the behaviour of the macroeconomic variables with the various

⁴ The importance of the first element lies in the fact that relying on an improperly specified scenario would render the stress test useless as a way to uncover systemic risk. For example, if the specified scenarios have a negligible probability of occurring, the exercise will be irrelevant. On the other hand, if they are too mild to pose a challenge, the exercise will be unable to reveal the downside risk that the financial system is exposed to.

⁵ This treatment is criticised by Berkowitz (1999).

stochastic components being correlated, Wilson (1997a, 1997b) and Boss (2002) developed a stress-testing framework that examines default risk and the development of macroeconomic conditions. Their framework has several advantages over the conventional approach since it takes into account the probabilistic elements and explicitly considers the variation of ε and its correlation with the macroeconomic variables x_1, \dots, x_M . Boss (2002) and Virolainen (2004) applied this framework to conduct credit-risk stress tests for the corporate loan portfolio of Austrian and Finnish banks respectively.

III. THE FRAMEWORK

A framework for stress testing the credit exposure of Hong Kong's retail banks to macroeconomic shocks is developed based on Wilson (1997a, 1997b), Boss (2002), and Virolainen (2004). In essence, our framework comprises:

- (i) an empirical model with a system of equations on credit risk and macroeconomic dynamics, and
- (ii) a Monte Carlo simulation for generating distribution of possible default rates (or credit losses).

3.1 The system of empirical equations

Suppose there are J economic sectors to which banks lend.⁶ Let $p_{j,t}$ be the average default rate in sector j observed in period t , where $j = 1, \dots, J$. As $p_{j,t}$ is bound between zero and one, we use its logit-transformed value $y_{j,t}$ as the regressand. That is,

$$y_{j,t} = \ln \left(\frac{1 - p_{j,t}}{p_{j,t}} \right)$$

is applied to transform $p_{j,t}$ to $y_{j,t}$, hence $-\infty < y_{j,t} < +\infty$.⁷ Obviously, $p_{j,t}$ and $y_{j,t}$ are negatively related; a higher $y_{j,t}$ is associated with a better credit-risk status.

Let $\mathbf{y}_t = (y_{1,t}, \dots, y_{J,t})'$. We model it as depending linearly on its lags and on the current and lagged values of M macroeconomic variables:

$$\mathbf{y}_t = \mathbf{m} + \mathbf{A}_1 \mathbf{x}_t + \dots + \mathbf{A}_{1+s} \mathbf{x}_{t-s} + \mathbf{\Phi}_1 \mathbf{y}_{t-1} + \dots + \mathbf{\Phi}_k \mathbf{y}_{t-k} + \mathbf{v}_t, \quad (1)$$

⁶ Boss (2002) and Virolainen (2004) analyse loans to different sub-sectors of the corporate sector. However, there is no impediment in the framework to covering loans to the household sector as well.

⁷ This treatment represents a common practice (see, for example, Pain (2003), Boss (2002) and Virolainen (2004)). Alternative ways of transformation, such as the probit, have also been attempted, and similar results are obtained.

where \mathbf{x}_t is an $M \times 1$ vector of macroeconomic variables; \mathbf{m} is a $J \times 1$ vector of intercepts; $\mathbf{A}_1, \dots, \mathbf{A}_{1+s}$ are $J \times M$ and $\mathbf{\Phi}_1, \dots, \mathbf{\Phi}_k$ are $J \times J$ coefficient matrices; and \mathbf{v}_t is a $J \times 1$ vector of disturbances. The characterisation of equation (1) explicitly links the default behaviours in the J economic sectors to the macroeconomic conditions. In Wilson (1997a, 1997b), \mathbf{y}_t is assumed to depend only on \mathbf{x}_t . Similar to Virolainen (2004), our specification is more general, allowing the impact of a macroeconomic shock to be prolonged and defaults in different economic sectors to be correlated.⁸

Another part of the equation system in Wilson's framework is on the dynamics of the M macroeconomic variables. In his original specification, each of them follows an autoregressive (AR) process. We generalise it by adopting the following specification:

$$\mathbf{x}_t = \mathbf{n} + \mathbf{B}_1 \mathbf{x}_{t-1} + \dots + \mathbf{B}_p \mathbf{x}_{t-p} + \mathbf{\Theta}_1 \mathbf{y}_{t-1} + \dots + \mathbf{\Theta}_q \mathbf{y}_{t-q} + \boldsymbol{\varepsilon}_t, \quad (2)$$

where \mathbf{n} is an $M \times 1$ vector of intercepts; $\mathbf{B}_1, \dots, \mathbf{B}_p$ are $M \times M$ and $\mathbf{\Theta}_1, \dots, \mathbf{\Theta}_q$ are $M \times J$ coefficient matrices; and $\boldsymbol{\varepsilon}_t$ is an $M \times 1$ vector of disturbances. Our specification is similar to Virolainen (2004) and has two advantages over Wilson's. First, equation (2) embodies a more realistic dynamic process in which the macroeconomic variables are mutually dependent. Secondly, equation (2) explicitly models the feedback effects of bank performances on the economy by letting \mathbf{x}_t depend on $\mathbf{y}_{t-1}, \dots, \mathbf{y}_{t-q}$.⁹ Equations (1) and (2) together define a system of equations governing the joint evolution of the economic performance, the associated default rates, and their error terms.

In this system, we assume that \mathbf{v}_t and $\boldsymbol{\varepsilon}_t$ are serially uncorrelated and normally distributed with variance-covariance matrices $\boldsymbol{\Sigma}_v$ and $\boldsymbol{\Sigma}_\varepsilon$ respectively; \mathbf{v}_t and $\boldsymbol{\varepsilon}_t$ are correlated, with variance-covariance matrix $\boldsymbol{\Sigma}_{v,\varepsilon}$. In sum, the structure of the disturbances is as follows:

$$\mathbf{e}_t = \begin{pmatrix} \mathbf{v}_t \\ \boldsymbol{\varepsilon}_t \end{pmatrix} \sim N(\mathbf{0}, \boldsymbol{\Sigma}), \quad \boldsymbol{\Sigma} = \begin{pmatrix} \boldsymbol{\Sigma}_v & \boldsymbol{\Sigma}_{v,\varepsilon} \\ \boldsymbol{\Sigma}_{v,\varepsilon} & \boldsymbol{\Sigma}_\varepsilon \end{pmatrix}. \quad (3)$$

Allowing the off-diagonal elements of $\boldsymbol{\Sigma}_v$, $\boldsymbol{\Sigma}_\varepsilon$ and $\boldsymbol{\Sigma}_{v,\varepsilon}$ to be non-zero is desirable. First, influences stemming from factors affecting the dependent variables but not explicitly incorporated in equations (1) and (2) will not be omitted altogether. Secondly, the contemporaneous correlation between the two disturbances in Equations (1) and (2) can be

⁸ As pointed out by Sorge (2004), the impact of a macroeconomic shock may persist for a number of years. Therefore, a dynamic specification like equation (1) is more desirable.

⁹ See Hoggarth et al. (2005).

captured and the feedback effects of bank performances on the economy can be more accurately assessed.¹⁰

3.2 Monte Carlo simulations and stress tests

In our framework, stress tests are conducted by comparing the estimated frequency or probability distribution of credit losses of the stressed scenario, where an artificial adverse macroeconomic development is introduced, with that of the baseline scenario, where no artificial adverse shock takes place. Estimated frequency distributions of the horizon-end default rates for each sector corresponding to stressed and baseline scenarios are obtained separately from simulating a large number of future joint sector-specific default rates by applying a Monte Carlo method.

Let us first discuss the estimation of the baseline distribution. To simulate a vector of one-period-ahead values of joint sector-specific default rates, we first draw a vector of random variables \mathbf{r} from the multivariate normal distribution with mean being zero and variance-covariance matrix being the estimated Σ . The vector so drawn represents a realisation of the vector of disturbances \mathbf{e} . Given the current and past values of the M macroeconomic variables, the J default rates and the realisation \mathbf{r} , the associated one-period-ahead values $y_{j,t+1}$ and $x_{i,t+1}$ can be calculated based on the estimated equations (1) and (2). Similarly, the two-period-ahead values can be calculated with another independently drawn \mathbf{r} and the one-period-ahead values previously obtained. Repeating the same procedure yields a future path of the joint sector-specific default rates, given the time horizon. By simulating a large number of such paths, a frequency distribution of the horizon-end default rates (of the baseline scenario) for each of the J sectors can be constructed. These paths stem from different future evolutions of the macroeconomic environment and the innovations \mathbf{v}_t in equation (1). With specific assumptions or actual data on the loss given default (LGD), the associated distribution of possible credit losses can be estimated.

In constructing the distribution of possible credit losses for a stressed scenario, we introduce an artificial adverse macroeconomic development subject to which another set of paths of future joint sector-specific default rates is simulated. Consider first the simulation of the one-period-ahead default rates of a particular path. We introduce in the vector of innovations an artificial shock over a macroeconomic variable through replacing the corresponding element in \mathbf{r} by the assumed shock (normalised by the respective standard deviation), so \mathbf{r} becomes pseudo random. Nevertheless, the other macroeconomic variables would be accordingly affected, since the off-diagonal elements of $\Sigma_{\mathbf{e}}$ need not be zero. In other words, the artificially shocked macroeconomic variable

¹⁰ In other words, the two disturbances in Equations (1) and (2) of the same time period are allowed to correlate.

would not be the only variable that is affected adversely in the simulation because the shock would be transmitted to other variables through the impact on its disturbance to other disturbances.

The simulation of the two-period-ahead default rates requires drawing another \mathbf{r} . If the adverse development of the previously shocked macroeconomic variable is assumed to last, we continue to make the \mathbf{r} pseudo random. Otherwise, we simply let the \mathbf{r} to be random, as in the baseline simulations. The farther-ahead default rates can be simulated in the same manner. Based on this procedure, a path of future joint sector-specific default rates can be constructed, given the duration of the artificial shock. With a sufficiently large number of simulated paths, the distribution of credit losses for a stressed scenario can be estimated. Note that our equation system stated in Section 3.1 characterises both the dynamics of sector-specific default rates and the macroeconomic variables. In both baseline and stressed scenarios, a simulated future path of joint sector-specific default rates is partly governed by the simulated future paths of the macroeconomic variables. The reasonableness of the simulated mixes of macroeconomic variables is supported by the estimated relationships based on historical data.

Intuitively, the baseline simulations produce an estimated unconditional probability distribution of possible credit losses, without the information about the occurrence of a particular shock. In some simulations, a serious credit loss occurs because there can be adverse macroeconomic developments in the baseline simulations due to randomness. On the other hand, in stressed simulations, as the different future evolutions of the macroeconomic environment and the innovations \mathbf{v}_t that the simulated paths involved share the same artificial economic shocks, the estimated distribution is conditional on the occurrence of such shocks.¹¹ Hence, comparing the conditional loss distribution of the stressed scenario with the unconditional distribution of the baseline scenario provides information on the possible impact of adverse macroeconomic conditions triggered by the shock that we introduce.

A better understanding on the adopted stress-testing approach can be gained by comparing it to the conventional approach as described in Section 2. Consider for simplicity the aggregate case where borrowers of different sectors are not distinguished. Given a particular pre-selected macroeconomic scenario, in the conventional stress-testing approach, the impact is mapped out by substituting into equation (1) the values of the M macroeconomic variables given by the scenario. In constructing the scenario, with the aid of a macro-econometric model, a shock can be artificially introduced over a particular macroeconomic variable, which is the stress origin, and the responses of the other variables in the model can be computed assuming that all disturbances are zero -- the scenario is the combination of the obtained numerical values of the macroeconomic

¹¹ As mentioned earlier, the artificial shocks can be specified to last for one period or longer.

variables. Conceptually, the values so computed are, on average, what the responses would be. However, in our approach the responses of the other macroeconomic variables to the shock of the stress origin are probabilistic because the disturbances are not assumed to be zero in our Monte Carlo exercise. The effect of this probabilistic treatment is represented in the numerous simulated paths which associate with different r_s .¹² With this, the framework allows us to assess banks' vulnerability through the use of VaR statistics.

The above illustrates an essential feature of our stress-testing approach: the probabilistic components of the default rates and the macroeconomic variables are not ignored, but are used to produce information on responses that deviate from the average. This feature is important because in stress testing public authorities are concerned with "exceptional but plausible" shocks, which are usually accompanied by rather abnormal behaviour of the macroeconomic variables.

IV. THE MODEL AND ESTIMATION RESULTS

The equation system on default probability and macroeconomic dynamics is estimated by using retail banks' data covering the period from 1994 Q4 to 2006 Q1. The default rates for the overall loan portfolios and mortgage exposures of banks are chosen in this study.^{13,14} In particular, the default rate is specified to depend on the following macroeconomic variables:

- (i) real GDP growth of Hong Kong (g^{HK})
 - (ii) real GDP growth of Mainland China (g^{CN})
 - (iii) real interest rates in Hong Kong (r)¹⁵
 - (iv) real property prices in Hong Kong ($prop$).¹⁶
- The default rate is measured as a ratio of the amount of loans which have been overdue for more than three months to the total amount of loans. The data series on default rate is transformed by the logit formula to produce the y_t series. Results obtained from an augmented Dickey-Fuller test suggest that y_t is an I(1) process. Thus, we opt to model its first difference Δy_t .

¹² This means that, even in the baseline credit loss distribution, there could be certain simulated paths of default rates that accompany extreme movements in the macroeconomic variables.

¹³ The framework can also be applied for stress testing loans to other economic sectors.

¹⁴ The time series of classified loans of retail banks can be an alternative measure of default rates. However, such data only became available from 1997 Q1, which is too short for the estimation.

¹⁵ Real interest rates are calculated as $[(1+r_t)/(1+\pi_{t+1})]-1$, where r_t and π_{t+1} are the nominal interest rate in period t and the inflation rate in period $t+1$ respectively. We use the seasonally adjusted CPI to calculate the inflation rate.

¹⁶ The real rate of change of property prices is calculated as $[(1+prop_t)/(1+\pi_t)]-1$, where $prop_t$ is the change of nominal property prices in period t .

- GDP governs the ability of agents in the economy to service their debt. For loans used to finance economic activities in the domestic market, the GDP of Hong Kong should be an important factor influencing the ability to repay.
- We also incorporate the GDP of Mainland China because the Hong Kong and Chinese economies are closely integrated.
- The reason for incorporating interest rates as an explanatory variable is obvious: they directly affect the burden of the debt. We use the three-month HIBOR to represent nominal interest rates.
- We consider property prices relevant because real estate is the major item of collateral. If the collateral value declines, the incentive to continue servicing the debt will weaken. The property price index compiled by the Rating and Valuation Department is used to calculate the variations in property prices in Hong Kong.

The equation system, which consists of equations (1) to (3), is estimated by the seemingly unrelated regression (SUR) method. The four macroeconomic series stated above are $I(0)$, as suggested by the results of an augmented Dickey-Fuller test, so we do not use their first differences in the regression. The SUR estimation results are presented in Table 1. For the Δy_t equation, the results shown in the table are obtained by removing the insignificant variables from the more general specification in which g_t^{HK} , g_{t-1}^{HK} , g_t^{CN} , g_{t-1}^{CN} , r_t , r_{t-1} , $prop_t$, $prop_{t-1}$, Δy_{t-1} and Δy_{t-2} are incorporated as explanatory variables. Similarly, the results from the equations of the macroeconomic variables are also obtained by removing the insignificant variables from a more general specification.

As shown in Table 1, the signs of the coefficients of the macroeconomic variables in the Δy_t equation are all as expected. The results suggest that the default rate would become higher if real GDP growth in Hong Kong and the Mainland deteriorated, property prices in Hong Kong declined, and interest rates rose, and vice versa. The coefficient of the lagged default rate y_{t-2} is positive and significant, so there is positive autocorrelation in default rates, suggesting that a macroeconomic shock can produce a prolonged impact on the default rate. This leads us to analyse the development of the default rate over a time horizon that is longer than the duration of the artificial shock in order to reflect the long-term impact of the stress.

Table 1: SUR estimates for the equation system (sample period: 1994 Q4 to 2006 Q1)

Variable	Dependent Variable				
	Δy_t	g_t^{HK}	g_t^{CN}	r_t	$prop_t$
Intercept	-0.087** (0.037)	0.510** (0.220)	1.858*** (0.267)	-0.051 (0.080)	0.180 (0.731)
g_{t-1}^{HK}	0.034*** (0.008)	0.475*** (0.117)			
g_t^{CN}	0.032** (0.016)				
g_{t-1}^{CN}			0.198* (0.108)		
r_t	-0.024** (0.011)				
r_{t-1}				-0.173** (0.071)	
$prop_{t-1}$	0.005** (0.002)				0.629*** (0.104)
Δy_{t-2}	0.512*** (0.089)				
Adj. R^2	0.631	0.191	0.113	0.682	0.336
DW statistic	1.756	1.94	2.129	1.689	1.978
No. of obs.	43	64	64	56	51

Notes:

1. In the estimation, dummy variables are added respectively in the g^{HK} , g^{CN} and r equations to control for the effects of structural breaks.
2. Standard errors are in parentheses.
3. *, ** and *** indicate significance at the 10%, 5% and 1% levels respectively.

Data Sources: CEIC, Census & Statistics Department of Hong Kong, HKMA.

V. THE SIMULATION OF FUTURE CREDIT LOSSES AND STRESS-TESTING

We now proceed to simulate paths of future default rates based on the SUR estimates and to construct the accompanying distributions of credit losses.¹⁷ The time horizon of a path is one year. As most of the shocks last four quarters, taking the macroeconomic conditions in 2006 Q1 as the current environment, a simulated future path has the eight time points covering a two-year period from 2006 Q2 to 2008 Q1.

As mentioned earlier, in constructing the loss distribution for the baseline scenario, no artificial adverse shock is introduced. For the four stressed scenarios, different shocks arising from four different stress origins are considered:

- (i) reductions in Hong Kong's real GDP by 1.7%, 3.9%, 0.8% and 1.1% respectively in each of the four consecutive quarters starting from 2006 Q2;
- (ii) a fall in Mainland China's real GDP by 3% in only the first quarter (i.e. 2006 Q2);
- (iii) a rise of real interest rates by 300 basis points in the first quarter, followed by no change in the second and third quarters and another rise of 300 basis points in the fourth quarter; and
- (iv) reductions in real property prices by 4.4%, 14.5%, 10.8% and 16.9% respectively in each of the four consecutive quarters starting from 2006 Q2.

These are quarter-to-quarter changes and are supposed to separately from 2006 Q2 to 2007 Q1. Their magnitudes are in general similar to those during the Asian financial crisis.¹⁸ No further artificial shock is introduced for the subsequent quarters. For each of the baseline scenario and stressed scenarios, we simulate 10,000 future paths and use the simulated 10,000 default rates in 2008 Q1 to construct a frequency distribution of credit loss percentages.¹⁹

If no formal statistics are available for the loss given default (*LGD*), some studies assign a rough constant ratio based on market information to obtain the estimated credit loss. If no market information is available, a ratio of 0.5 may be assumed for the calculation of loss figures. In this paper, we assume the *LGD* will vary with property prices as properties are by far the most important collateral for lending. Property prices should therefore have an impact on how much banks can recover from their losses. For

¹⁷ A random vector of multivariate normal distribution can be obtained by first computing the Cholesky decomposition \mathbf{C} of the variance-covariance matrix $\mathbf{\Sigma}$, where \mathbf{C} is defined by $\mathbf{\Sigma} = \mathbf{C}\mathbf{C}'$. Pre-multiplying a random vector \mathbf{z} whose entries are independently drawn from the standardised normal distribution $N(0,1)$ by \mathbf{C}' gives \mathbf{r} .

¹⁸ Note that during the Asian financial crisis (from 1997 Q4 to 1998 Q3), real interest rates rose by 306 bps in the first quarter (i.e. 1997 Q4), but dropped by 90 bps and 86 bps in the second and third quarters respectively before rising again by 314 bps in the final quarter. Also, China's GDP in all quarters recorded positive growth. Our assumed shocks are therefore more severe than the actual situation.

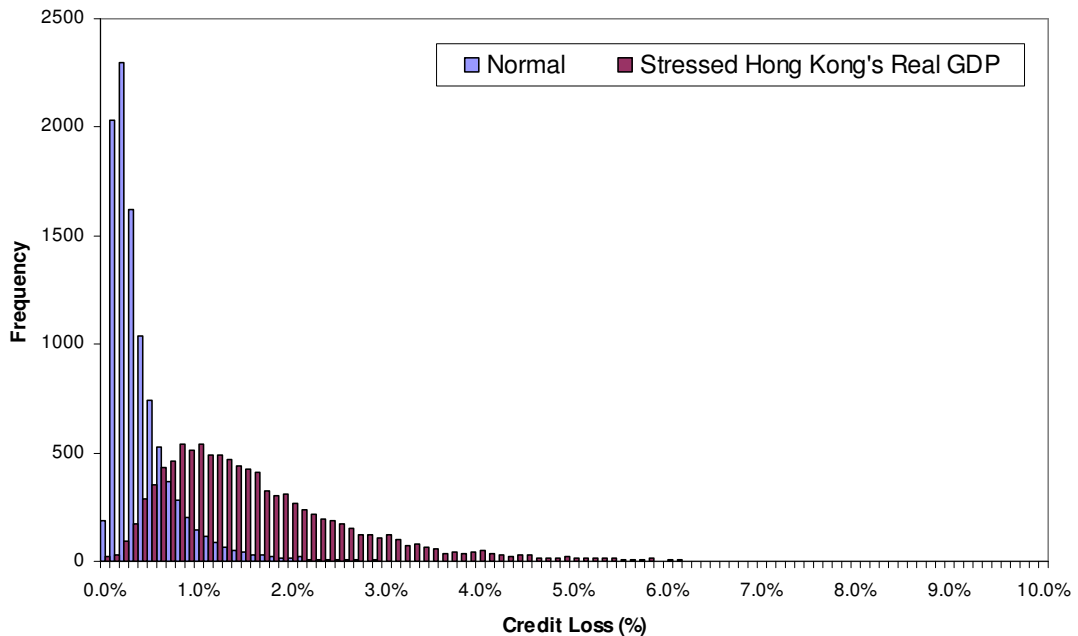
¹⁹ The percentage of credit loss is simply the product of the default rate and the *LGD*.

simplicity, we assume the *LGD* in 2006 Q1 to be 0.5 and the *LGD* in 2008 Q1 to be inversely proportional to the percentage change in the property price index (*PI*) around the initial level 0.5, as follows²⁰:

$$LGD_{2008Q1} = 0.5 - 0.5 \times \frac{PI_{2008Q1} - PI_{2006Q1}}{PI_{2006Q1}}.$$

The simulated frequency distributions of the baseline and stressed scenarios are depicted in Chart 1. Introducing a shock shifts the loss distribution to the right, representing an increase in the frequency of the higher credit loss percentages at the expense of the lower ones.

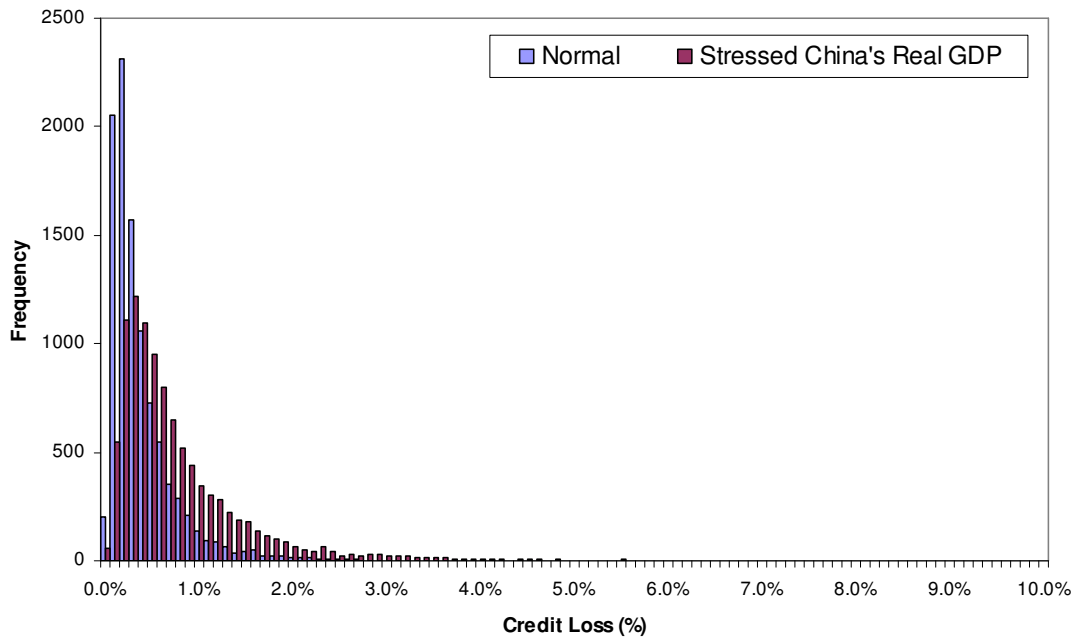
Chart 1a: A GDP shock: simulated frequency distributions of credit loss under baseline and stressed scenarios



Note: Each distribution is constructed with 10,000 simulated future paths of default rates.

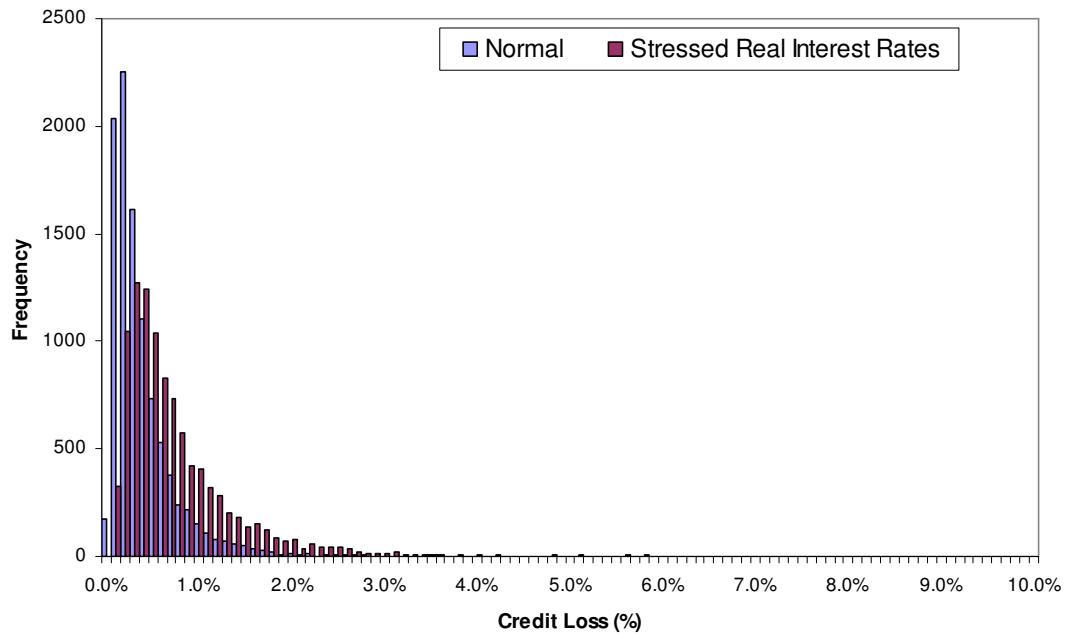
²⁰ This is indeed a very crude assumption.

Chart 1b: A China-GDP shock: simulated frequency distributions of credit loss under baseline and stressed scenarios



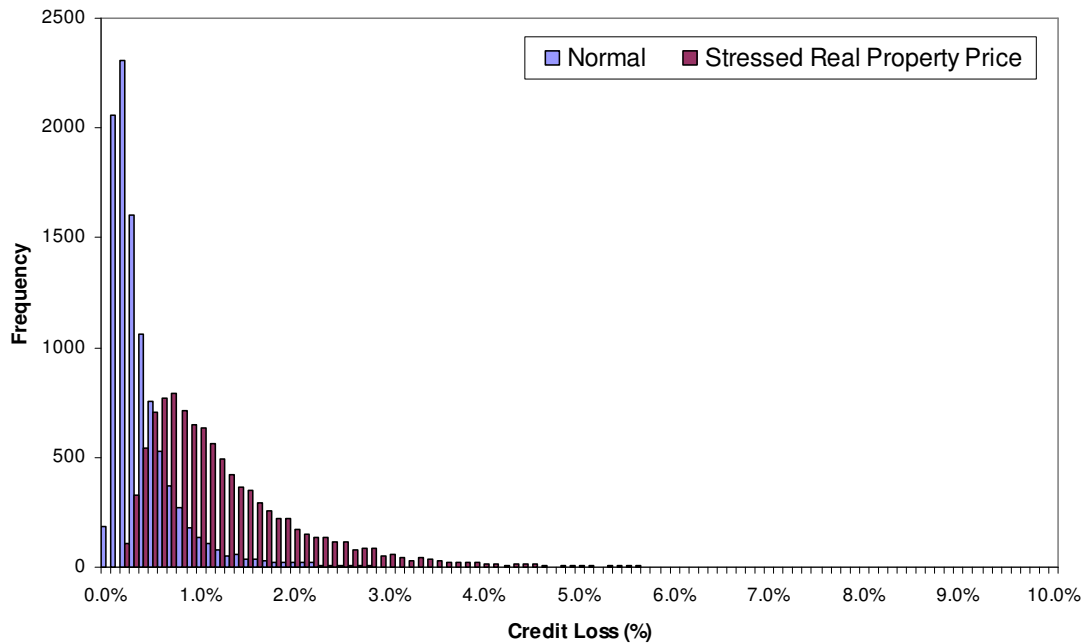
Note: Each distribution is constructed with 10,000 simulated future paths of default rates.

Chart 1c: An interest-rate shock: simulated frequency distributions of credit loss under baseline and stressed scenarios



Note: Each distribution is constructed with 10,000 simulated future paths of default rates.

Chart 1d: A property-price shock: simulated frequency distributions of credit loss under baseline and stressed scenarios



Note: Each distribution is constructed with 10,000 simulated future paths of default rates.

Salient statistics are presented in Table 2 to provide highlights of the distributions of credit losses for the baseline scenario and for the four stressed scenarios with different macroeconomic variables as the stress origin. In the baseline scenario, the percentage of credit loss that is expected to prevail in 2008 Q1 (or the mean of the credit loss distribution) is 0.34%. Introducing the artificial shocks substantially increases the expected percentage of credit loss. For example, it becomes 1.59% in the stressed scenario where Hong Kong's real GDP growth rate is shocked from 2006 Q2 to 2007 Q1.

However, our focus is on the more-than-average adverse responses of the other macroeconomic variables and the default behaviour. In particular, we are more interested in the tails of the credit loss distributions. Table 2 shows that even for the VaR at the confidence level of 90%, banks would continue to make a profit in most of the stressed scenarios, suggesting that the current credit risk of the banking sector is moderate. However, under the extreme case for the VaR at the confidence level of 99%, banks' credit loss with shocks from different origins would range from a maximum of 3.22% to a maximum of 5.56% of the portfolios, and if a confidence level of 99.9% is taken, it could range from a maximum of 6.08% to a maximum of 8.95%. The estimated maximum losses are very similar to what the market experienced one year after the Asian financial shock.²¹ However, the probability of such losses and beyond is very low.

²¹ In the event, the credit loss of banks is estimated to have risen from 1.4% before the Asian financial crisis to 6.0% one year after the shock. These rough estimates are based on an assumed LGD of 70%, and the actual default rates of overall loans at 2.01% in 1997 Q3 and 8.58% in 1998 Q4.

We can also map out the impact of credit losses on banks' profitability. For a given bank or the entire banking sector, the amount of credit losses is simply the product of the percentage of credit loss and the amount of loans and advances. Suppose the future level of operating profit before provisions was the same as the current level, if no default were to take place. After realising defaults, the level of operating profit before provisions falls by the amount of credit losses. Table 3 shows post-default levels of operating profit before provisions of a hypothetical bank corresponding to the credit loss percentages given in Table 2. The operating profit before provisions and the amount of loans and advances of the hypothetical bank are assumed to be HK\$3 billion and HK\$130 billion respectively. We can see that for the more extreme situations, the bank may incur a loss as a result of the materialisation of credit risk alone. Under the VaR at the 90% confidence level with the GDP shock, banks could incur a loss of HK\$882 million. The bank may also suffer a loss under shocks from other origins under the VaR at the 99% confidence level. However, the occurrence of such extreme scenarios resulting in the estimated maximum loss and beyond would have a very small probability of only 1%.

Table 2: The mean and VaR statistics of simulated credit loss distributions

Credit loss (%)	Baseline scenario	Stressed scenarios			
		GDP shock ^a	Property price shock ^b	Interest rate shock ^c	Mainland China GDP shock ^d
Mean	0.34	1.59	1.21	0.71	0.73
VaR at 90% CL ^e	0.76	2.99	2.30	1.48	1.56
VaR at 95% CL	1.05	3.77	2.88	1.94	2.12
VaR at 99% CL	1.91	5.56	4.54	3.22	3.73
VaR at 99.9% CL	3.13	8.95	8.29	6.08	6.66
VaR at 99.99% CL	4.38	12.56	10.92	9.15	9.38

Notes: a) Reductions in Hong Kong's real GDP by 1.7%, 3.9%, 0.8% and 1.1% respectively in each of the four consecutive quarters starting from 2006 Q2.

b) Reductions in real property prices by 4.4%, 14.5%, 10.8% and 16.9% respectively in each of the four consecutive quarters starting from 2006 Q2.

c) A rise of real interest rates by 300bps in the first quarter, followed by no change in the second and third quarters and another rise of 300 bps in the fourth quarter.

d) A fall in Mainland China's real GDP by 3.0% in only the first quarter (i.e. 2006 Q2).

e) CL denotes the confidence level

Table 3: Post-default operating profit of a hypothetical local bank ^{1,2} (in HK\$m)

Profit (HK\$m)	<u>Baseline</u> <u>scenario</u>	Stressed scenarios			
		GDP shock ^a	Property price shock ^b	Interest rate shock ^c	Mainland China GDP shock ^d
Mean	2,554	927	1,427	2,078	2,051
VaR at 90% CL ^e	2,013	-882	5	1,075	970
VaR at 95% CL	1,636	-1,900	-746	477	242
VaR at 99% CL	517	-4,226	-2,903	-1,182	-1,844
VaR at 99.9% CL	-1,066	-8,629	-7,774	-4,905	-5,661
VaR at 99.99% CL	-2,690	-13,332	-11,193	-8,900	-9,195

Notes: 1) The operating profit before provisions and the amount of loans and advances are assumed to be HK\$3 billion and HK\$130 billion respectively.

2) A positive figure indicates a profit while a negative figure indicates a loss.

3) For (a) to (e), see Table 2.

VI. A STRESS TEST FOR BANKS' MORTGAGE PORTFOLIO

The same framework can be applied for stress testing loans to different economic sectors. In this section, we apply the framework to analyse the default behaviour of residential mortgage loans (RMLs). This is of particular interest because banks in Hong Kong generally have a substantial exposure to this type of loan. For this exercise, the first difference of the logit-transformed default rate for RMLs Δy_t^{RML} is modelled as dependent on five macroeconomic variables: real GDP growth of Hong Kong (g^{HK}), the best lending rate in real terms (BLR), real property prices in Hong Kong ($prop$), real GDP growth of Mainland China and Hong Kong's unemployment rate.

Table 4 presents the SUR estimates for the equation system for RMLs. Similar to the treatment in Table 1, results in Table 4 are derived by removing the insignificant variables (including Mainland China's real GDP growth and Hong Kong's unemployment rate) from a more general specification. As expected, the performance of the RMLs depends negatively on the BLR and positively on Hong Kong's real GDP growth rate and changes in real property prices.²² Similar to the model for overall loans, the coefficient of the lagged dependent variable in the Δy_t^{RML} equation is positive and significant, so the impact of an economic shock on the credit risk associated with RMLs is likely to be prolonged.

²² The estimated coefficient for real GDP growth of Mainland China is insignificantly different from zero. This may reflect that, unlike the part of business credit of overall loan exposures, mortgage loans are more affected by domestic factors and are less directly affected by the China factor.

**Table 4: SUR estimates for the equation system for RMLs
(sample period: 1998 Q2 to 2006 Q1)**

Variable	Dependent variable			
	Δy_t^{RML}	g_t^{HK}	blr_t	$prop_t$
Intercept	0.014 (0.014)	0.530** (0.228)	-0.025 (0.065)	0.127 (0.728)
g_t^{HK}	0.011* (0.006)			
g_{t-1}^{HK}		0.452*** (0.124)		
blr_{t-1}	-0.029** (0.014)		0.189* (0.104)	
$prop_{t-1}$				0.541*** (0.106)
Δy_{t-1}^{RML}	0.562*** (0.072)			
Adj. R^2	0.842	0.190	0.363	0.336
DW statistic	2.191	1.892	2.051	1.819
No. of obs.	30	64	56	51

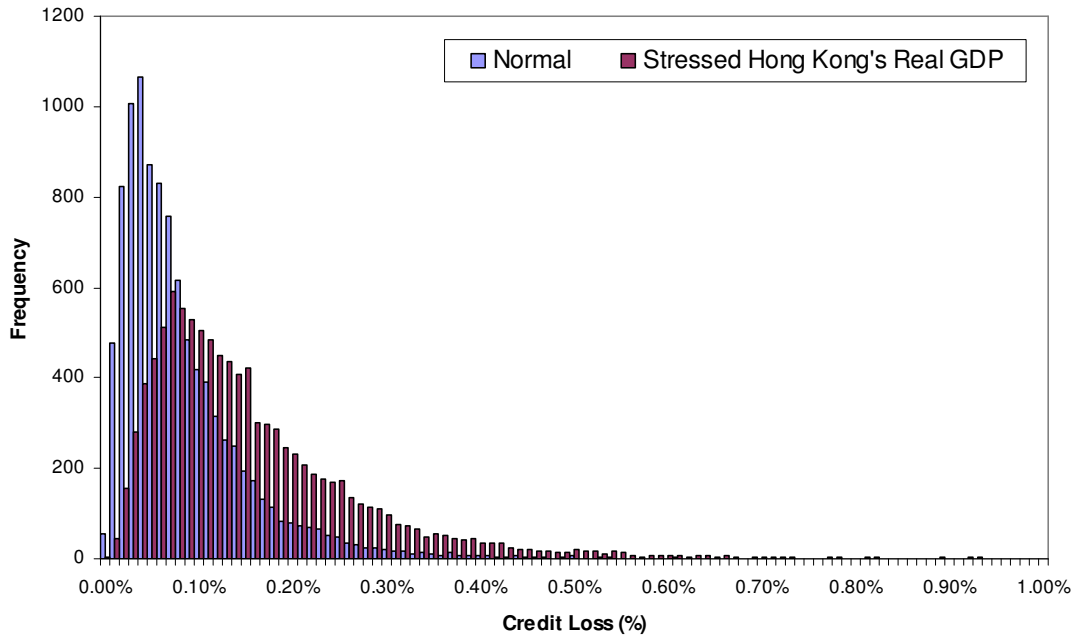
Notes:

1. In the estimation, dummy variables are added respectively in the g^{HK} and r equations to control for the effects of structural breaks.
2. Standard errors are in parentheses.
3. *, ** and *** indicate significance at the 10%, 5% and 1% levels respectively.

Data Sources: CEIC, Census & Statistics Department of Hong Kong, HKMA.

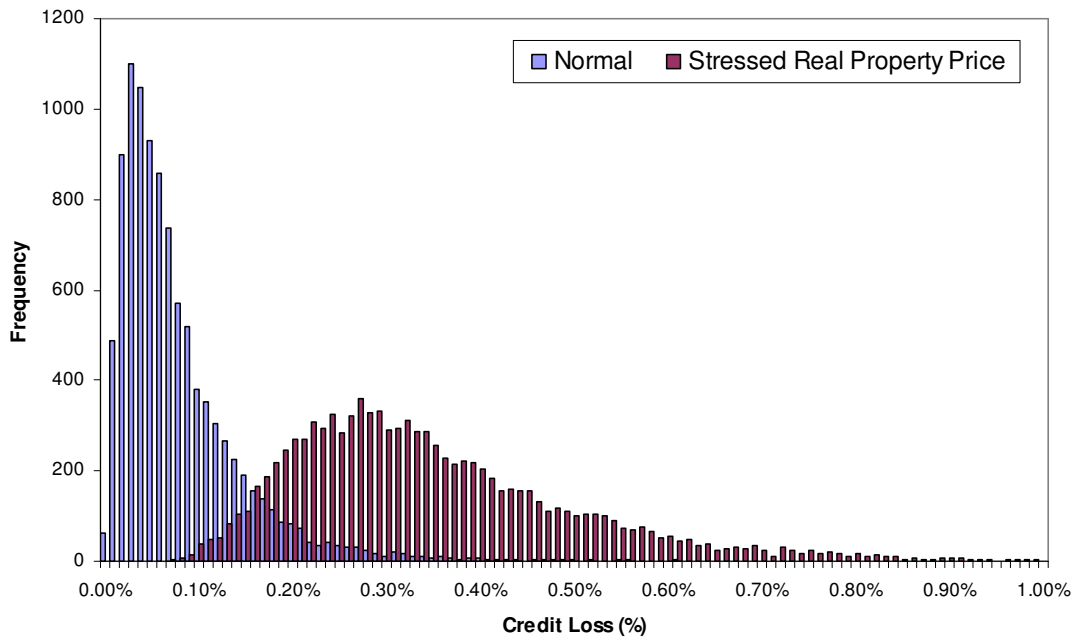
The credit loss is simulated over a one-year horizon after the three different shocks, originating separately from (1) real Hong Kong GDP, (2) real property prices, and (3) real interest rates, the magnitudes of which are similar to those during the Asian financial crisis. As in Chart 1, Chart 2 shows that the distribution of losses of the stressed scenarios shifts towards the right compared with the baseline scenarios, suggesting that the shocks have resulted in increases in the expected percentage of credit losses.

Chart 2a: A GDP shock: simulated frequency distributions of credit loss for RMLs under baseline and stressed scenarios



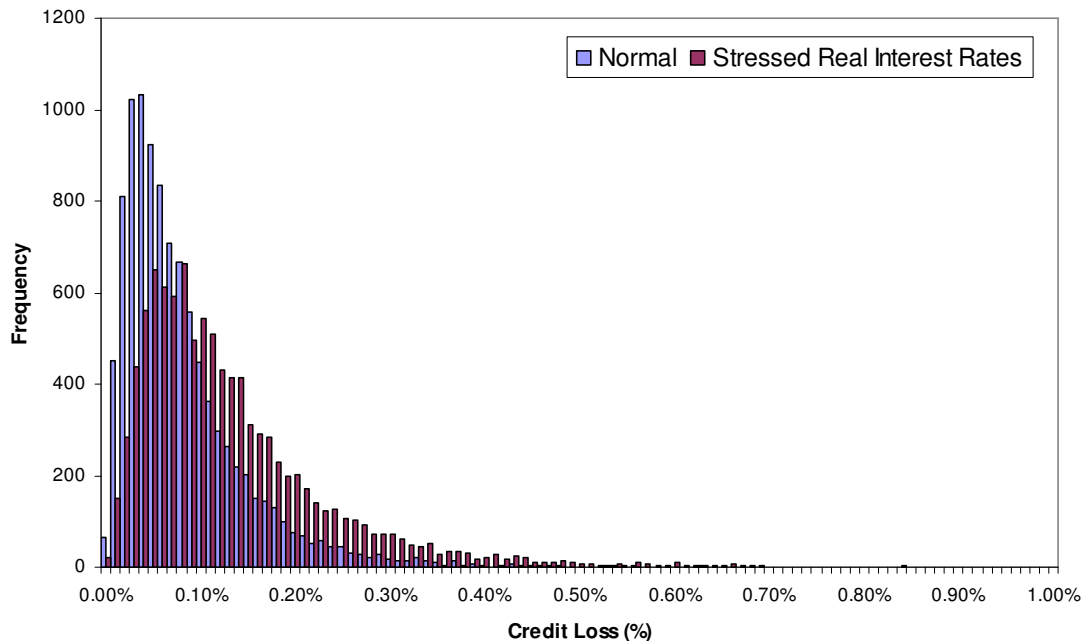
Note: Each distribution is constructed with 10,000 simulated future paths of default rates.

Chart 2b: A property-price shock: simulated frequency distributions of credit loss for RMLs under baseline and stressed scenarios



Note: Each distribution is constructed with 10,000 simulated future paths of default rates.

Chart 2c: An interest-rate shock: simulated frequency distributions of credit loss for RMLs under baseline and stressed scenarios



Note: Each distribution is constructed with 10,000 simulated future paths of default rates.

The simulation results also show that the impact on banks' profit would be moderate. For all the three shocks of different origins, even with a high confidence level of VaR measure, banks would continue to make a profit. As shown in Table 5, the expected credit losses (under the mean credit losses) for the given severe shock are moderate, ranging from 0.08% to 0.34% of the bank's total RMLs. Such credit loss may rise to a maximum of 1.12% at the 99.9% confidence level, which suggests that there is a probability of 0.1% for banks to suffer from a credit loss of 1.12% or more. Assuming that the hypothetical bank's outstanding loans for RMLs in 2006 Q1 is HK\$ 39 billion, the cut in profit is found to be at most HK\$436.8 million at the 99.9% confidence level, which amounts to 14.6% of total operating profit before provisions (see Table 6).²³ However, the occurrence of such adverse market conditions has a very low probability.

²³ It is assumed that the share of mortgage loans to the bank's total loans for use in Hong Kong is 30%, which is about the industry average. Note that this loss figure arises from only the bank's RML portfolio.

Table 5: The mean and VaR statistics of the simulated credit loss distributions for RMLs

Credit loss (%)	Baseline scenario	Stressed scenarios		
		GDP shock ^a	Property price shock ^b	Interest rate shock ^c
Mean	0.08	0.15	0.34	0.13
VaR at 90% CL ^d	0.16	0.29	0.55	0.25
VaR at 95% CL	0.21	0.38	0.64	0.32
VaR at 99% CL	0.35	0.58	0.83	0.50
VaR at 99.9% CL	0.53	0.91	1.12	0.84
VaR at 99.99% CL	0.69	1.07	1.41	1.33

Notes: a) Reductions in Hong Kong's real GDP by 1.7%, 3.9%, 0.8% and 1.1% respectively in each of the four consecutive quarters starting from 2006 Q2.
b) Reductions in real property prices by 4.4%, 14.5%, 10.8% and 16.9% respectively in each of the four consecutive quarters starting from 2006 Q2.
c) A rise of real interest rates by 300bps in the first quarter, followed by no change in the second and third quarters and another rise of 300 bps in the fourth quarter.
d) CL denotes the confidence level

Table 6: Post-default operating profit of a hypothetical local bank for RML^{1,2} (in HK\$m)

Profit (HK\$m)	Baseline scenario	Stressed scenarios		
		GDP shock ^a	Property price shock ^b	Interest rate shock ^c
Mean	2,970	2,941	2,866	2,951
VaR at 90% CL ^d	2,937	2,885	2,787	2,901
VaR at 95% CL	2,916	2,853	2,751	2,875
VaR at 99% CL	2,865	2,774	2,677	2,803
VaR at 99.9% CL	2,793	2,644	2,564	2,674
VaR at 99.99% CL	2,731	2,582	2,449	2,481

Notes: 1) The operating profit before provisions and the amount of loans for RML are assumed to be HK\$3 billion and HK\$39 billion respectively.
2) A positive figure indicates a profit while a negative figure indicates a loss.
3) For (a) to (d), see Table 5.

VII. CONCLUSIONS

This paper studied a macro stress testing framework for loan portfolios of banks in Hong Kong. Two macroeconomic credit risk models, each comprising a multiple regression model explaining the default probability and a set of autoregressive models describing the macroeconomic environment, were constructed for the overall loan portfolios and mortgage exposures of banks respectively. The analysis suggests a significant relationship between the default rates of bank loans and key macroeconomic factors, including Hong Kong's real GDP, real interest rates, real property prices and Mainland China's real GDP.

Macro stress testing is then performed to assess the vulnerability and risk exposures of banks' overall loan portfolios and mortgage exposures. By using the framework, a Monte Carlo method is applied to estimate the distribution of possible credit losses conditional on an artificially introduced shock. Different shocks, the magnitude of which are specified according to those occurring during the Asian financial crisis, are individually introduced into the framework for the stress tests. The results show that even for the VaR at the confidence level of 90%, banks would continue to make a profit in most of the stressed scenarios, suggesting that the current credit risk of the banking sector is moderate. Under extreme cases for the VaR at the confidence level of 99%, banks could incur material losses. However, the probability of the occurrence of such events is extremely low.

Using a hypothetical bank as an example, this paper illustrates how estimates obtained from aggregate default-rate data can be applied to stress test individual banks. The framework can also be applied in a more comprehensive manner to assess the vulnerability of individual (or groups of) banks by using bank level (or group level) data to obtain bank-specific (or group-specific) estimates for the macro credit risk model and VaR statistics.

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