November 2004

RESIDENTIAL MORTGAGE DEFAULT RISK IN HONG KONG

Key points:

- *The sharp fall of property prices after the Asian financial crisis has led many residential mortgage holders in Hong Kong to experience negative equity. Among other factors, this study looks at the impact of negative equity on the probability of default on mortgage loans, which is an important issue in view of the fact that residential mortgage lending represents a significant component of bank assets.*

- *The empirical analysis confirms the role of current loan-to-value ratio (CLTV) as a major determinant for mortgage default decisions. It also finds that the default probability is positively correlated with the level of interest rates and the unemployment rate, and negatively correlated with financial market sentiment.*

- *Under a hypothetical scenario that the maximum 70% LTV ratio guideline on residential mortgages were relaxed to 90% some time before 1997, the potential amount of default among the negative equity loans are estimated to be significantly higher than otherwise.*

- *Given the importance of the CLTV for defaults, this study lends strong support to the prudential policy of encouraging the adoption of a maximum 70% LTV ratio in residential mortgage lending.*

Prepared by : Jim Wong, Laurence Fung, Tom Fong and Angela Sze
Research Department
Hong Kong Monetary Authority

I. INTRODUCTION

The sharp fall in property prices following the Asian financial crisis has led many residential mortgage holders in Hong Kong to experience negative equity. At the end of September 2004, there were about 25,400 loans with a market value lower than the outstanding loan amount. The total value of these loans was HK$43 billion. The rate of mortgage delinquency reached a peak of 1.43% in April 2001. While it has improved since the second half of 2001, the delinquency rate in September 2004, at 0.47%, is still higher than 0.29% in June 1998 when data were first collected.[1] Given that residential mortgage lending represents a significant component of bank assets, how borrowers' decisions to default are affected by the negative equity position of their mortgages is of interest to policymakers.[2]

This study utilises micro-level mortgage loan data to examine the determinants of residential mortgage default risk in Hong Kong, and the effect of changes in these determinants, in particular the current loan-to-value ratio (CLTV), on default probabilities. A preliminary attempt is also made to assess the impact of macroeconomic variables on default probability. Overall, the results suggest that the CLTV ratio is central to mortgage default decisions. The study finds that default probability is positively correlated with the CLTV ratio, as well as with interest rates and the unemployment rate, and negatively correlated with changes in stock prices.

The remainder of the paper is organised as follows. Section II provides a brief review of the theoretical framework in explaining mortgage default behaviour. Section III discusses the specification of the logit model and the data for empirical estimations. The estimation results are summarised in Section IV. Simulations to estimate the impact of the variation in CLTV ratio on default probabilities are given in Section V. Section VI introduces two macroeconomic variables, the unemployment rate and changes of stock prices, into the model. Section VII simulates how a relaxation of the maximum 70% loan-to-value ratio guideline on mortgage lending may affect banks' asset quality. Concluding remarks are provided in the final section.

[1] The improvement is smaller if rescheduled loans are taken into account.

[2] "Decision to default" is a widely used term in literature. In practice, however, such defaults are best seen as arising from the financial hardship of borrowers.

II. THEORETICAL BACKGROUND AND LITERATURE REVIEW

There are two alternative views relating of home mortgage default behavior (Jackson and Kasserman, 1980). The *equity theory of default* holds that borrowers base their default decisions on a rational comparison of financial costs and returns involved in continuing or terminating mortgage payments.[3] An alternative is the *ability-to-pay theory of default* (the cash flow approach), according to which mortgagors refrain from loan default as long as income flows are sufficient to meet the periodic payment without undue financial burden. Under the equity theory, the CLTV ratio, which measures the equity position of the borrower, is considered to be the most important factor impacting on default decisions. By contrast, under the ability-to-pay model, the current debt servicing ratio (CDSR), defined as the monthly repayment obligations as a percentage of current monthly income, which captures the repayment capability of the borrower, plays a critical role in accounting for defaults.

More recently, research has attempted to incorporate trigger events, such as divorce, loss of a job, and accident or sudden death, in influencing default behaviour (Riddiough, 1991). In the simple model, some defaults may be driven by a sudden drop or loss of income caused by unemployment, job shift or a sudden increase in expenses such as medical fees. Furthermore, there was also empirical evidence that transaction costs were present in default decisions (Vandell, 1990 and 1992; Riddiough and Vandell, 1993). For instance, borrowers may consider the value of their reputation and credit rating when deciding on whether to default or not. A final issue relates to the lender's influence on default decisions. Workout plans helping borrowers who are faced with financial hardships to overcome payment difficulties have long provided an alternative to default. Upon consideration of the financial health of the borrower, the lender may respond in different ways to the threat of a possible default, such as loan restructuring, mortgage recourse, and the adoption of extended repayment plan or refinancing.[4] In Hong Kong's case, post-foreclosure debt collections and possible initiation of a bankruptcy petition by creditors are believed to be the major deterrent to default. Transaction costs and lender's influence are clearly part of the reasons why a borrower does not default when the value of the property falls below the outstanding amount of the mortgage loan.

Earlier empirical work has not come to firm conclusions regarding the relative importance of equity and affordability in mortgage default behaviour.

---

[3] If borrowers attempt to maximise the equity position in the mortgaged property at each point of time, they will cease to continue payments when and if the market value of the mortgaged property at time t declines sufficiently to equal the outstanding mortgage loan balance at time t.

[4] Loan restructuring has helped keep the mortgage delinquency rate in Hong Kong at a relatively low level in more recent years (see Chart 1 in Section 3.2).

While most of the literature finds the equity position to be the primary determinant in mortgage default decisions, some studies argue that non-equity effects, such as the source of income, are more significant. The importance of loan-to-value ratio can be overstated if other variables are excluded from the empirical specification.

In general, there are two approaches taken in the empirical literature on mortgage defaults. One approach is to relate individual mortgage defaults to loan and borrower characteristics as well as macroeconomic variables. The alternative is to relate aggregate measures of default incidence to macroeconomic variables. While most previous studies apply individual mortgage data for empirical investigations of mortgage defaults, there exists a limited literature on empirical analysis using aggregate data.

There are several studies that look into the residential mortgage market in Hong Kong. However, these studies concentrate mainly on explaining the characteristics of the mortgage terms such as mortgage tenors and variable payments, fixed versus floating rate loans and mortgage rates.[5] Few have focused on the default risk of mortgage loans.

III. Methodology and Data

3.1 Model Specification

Research on mortgage default or prepayment behaviour using micro-level data is typically based on techniques for survival analyses and duration modelling. An alternative approach, where survival time is less an issue, is to estimate binary choice models for a particular study period. Following many previous studies, this paper applies the logit model to explain mortgage defaults, which is a binary (0/1) dependent variable.[6]

a) The logistic function

In general, if the default probability ($P(Y)$) is a linear function f of a vector of explanatory variables x, where x includes loan-related and non-loan-related variables, under the logistic distribution, the default probability can be specified as:

---

[5] See He and Liu (2002), Chiang et al. (2002) and Chow and Liu (2003).

[6] See Campbell and Dietrich (1983), Vandell and Thibodeau (1985), Gardner and Mills (1989), Capozza, et al (1997), Goldberg and Capone (1998) and Archer, et al (2002). Other studies use the logit model to predict mortgage prepayment risk, for instance, LaCour-Little (1999). For a review of logit model, see Horowitz and Savin (2001).

$$P(Y = default) = \frac{e^{f(x)}}{1 + e^{f(x)}} \tag{1}$$

and $$f(x) = c + \beta_1 X_1 + \beta_2 X_2 + \ldots + \beta_n X_n$$

where c is the constant term, X_i is the explanatory variable and β_i is the coefficient.

b) Dependent variable

The dependent variable is the default status of a loan. A mortgage loan is defined as a default case in this study if it is overdue for more than 90 days.[7] The HKMA defines delinquency to be loans overdue for more than three months, and the Hong Kong Mortgage Corporation (HKMC) uses 90 days as the benchmark. Based on this definition of default, the dependent variable used in the logit model is equal to 1 if the loan becomes overdue for more than 90 days in the study period and 0 otherwise.

c) Explanatory variables

Both loan-related and non-loan-related factors are used as explanatory variables.[8] Reflecting the structure of Hong Kong's mortgage market and data availability, the following explanatory variables are included in the model (see Table 1).[9]

As for loan-related factors, the inclusion of the current loan-to-value ratio and current debt servicing ratio has been discussed in the preceding section. Other loan-related factors include the loan-to-value ratio and the debt servicing ratio at origination. One view is that as banks only offer loans with a high LTV ratio and DSR at loan origination to mortgagors with good credit standing and payment ability (such as having a stable job), such loans should correspond to lower default risks.

---

7 Loans which are overdue for more than 90 days are more likely to be finally defaulted than those that are overdue for a shorter duration. This is because the third missed payment is unlikely to be due to negligence and may thus reflect severe financial stresses of the borrower. Furthermore, with three missed payments and a fourth payment due, it becomes more difficult for the borrower to raise enough funds to settle the overdue amount. According to data from HKMC, among the 214 loans which were overdue for more than 90 days during the period from February 1999 to September 2003, 99 were written off, 23 were fully prepaid, while 92 loans were still in the HKMC's portfolio at the end of the period. Assuming about half of the loans which were still outstanding would be written off at the end, the write-off ratio would be as high as 60-70%. In some states of the US, state property laws permit initiation of foreclosure processes after a delinquency of 90 days.

8 The unemployment rate and changes in the HSI are introduced in Section VI to capture the effect of macroeconomic conditions.

9 To address the effect of trigger events, transaction costs and lenders' influence, a micro-behavioural mortgage payment database is required to gather detailed information when mortgage termination occurs. Due to the absence of these data, the effects of these factors have not been examined in this study.

The alternative view is that as less wealthy mortgagors tend to borrow a larger amount of loan and at a higher DSR at origination, a higher LTV ratio or DSR should point to higher default risks. The signs of these two variables are thus ambiguous.

Non-loan-related factors include seasoning variables and property-related variables such as the property area, current unit property price and the age of the property.[10] The seasoning of the mortgage is expected to have a negative sign, as the longer one has served the mortgage, the less likely one will default. The signs of the other property-related variables could be positive or negative. Two explanatory variables, the CLTV ratio and the age of the property, are included in the model in squared terms as well to capture the potential non-linearity effect of these variables on default probabilities. They have negative signs in most previous studies (in contrast to the signs of the original variables), suggesting the existence of non-linearity.

Table 1. Explanatory Variables for the Logit Model

	Abbreviation	Expected Sign[4]
Loan-related Factors		
- Loan-to-value ratio at origination (%)	OLTV	+/-
- Current loan-to-value ratio (%)	CLTV	+
- Current loan-to-value ratio squared	CLTVSQ	-
- Debt servicing ratio at origination (%)	ODSR	+/-
- Current debt servicing ratio (%)[1]	CDSR	+
- Mortgage rate (%)[2]	Mortgage	+
Non-Loan-related Factors		
Seasoning		
- Expected seasoning at origination (months)	Oseason	+/-
- Seasoning up to the study period (months)	Season	-
Property		
- Property area (sq. ft.)	Garea	+/-
- Current unit property price (HK$)[3]	Price	+/-
- Age of property (months)	Oage	+/-
- Age of property squared	Oagesq	+/-

Notes:
1. Current debt servicing ratio is derived as the payment in the current month divided by the estimated income in the current month. Monthly income at origination is estimated by mortgage payment for the first month divided by the debt servicing ratio at origination. Income in the current month is derived by adjusting the estimated monthly income at origination by the nominal wage index.
2. The mortgage rate variable is given by BLR pluses mortgage rate spreads.
3. Defined as the current price of the property per sq. ft.
4. Expected signs indicated are based on theoretical deliberations and previous empirical findings.

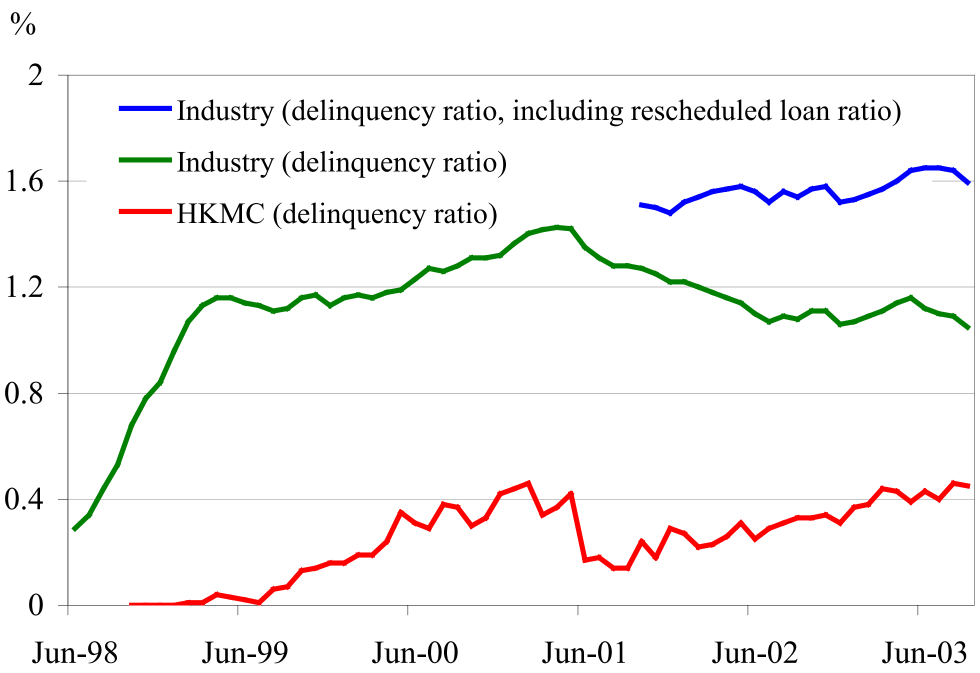
[10] Seasoning variables measure how long the mortgage is expected to be served or has been served. The current unit property price is defined as the current price of the property in question per square foot.

3.2 Sources of Data and Data Characteristics

Micro-level loan data are obtained from the HKMC. The HKMC purchased a total of 19,176 mortgage loans from authorized institutions (AIs) during the period between its incorporation in March 1997 and September 2003. As at the end of 2003, the loan portfolio of HKMC totalled HK$9.8 billion, equivalent to 1.9% of all mortgages extended by AIs in Hong Kong. There exist two types of data for each loan: information at origination and the dynamic record. Information at origination includes those loan- and property-related data listed in Table 1. The dynamic record includes data of payment history, CLTV ratio over time, mortgage rate spreads over time and the delinquency status.

It should be noted that HKMC's mortgage portfolio appears to be of better quality compared with the industry average, which is reflected by the fact the delinquency rate has consistently been lower than that of the industry (see Chart 1). As the current study utilises only loan data from the HKMC, inference regarding the overall market drawn from findings of this study should be made with caution. In particular, the default probabilities estimated in this paper are likely to be lower than the industry average.

Chart 1. Mortgage Delinquency Rate in Hong Kong



Note: Information of rescheduled loans of HKMC's portfolio is not available.
Sources: HKMA and HKMC Monthly Mortgage Portfolio Statistics.

3.3 Sample Periods

The study covers the period from July 2000 to September 2003. As the HKMC acquired loans from different AIs throughout the period and market conditions have been changing all the time, in order to capture more comprehensively information in the portfolio, "snapshots" are taken in January and July of each year to examine loan delinquencies. Data in a selected month by themselves are utilised to examine the determinants of default in that particular month. A loan is considered as a default case if it is overdue for more than 90 days during that month. Table 2 shows the total number of loan cases and the number of loans which were defaulted in each of the samples.

Table 2. Number of Loan Cases and Delinquency Rate

No. of Loans	Jul 2000	Jan 2001	Jul 2001	Jan 2002	Jul 2002	Jan 2003	Jul 2003	Sep 2003
Total	6,622	8,199	7,256	6,320	5,698	8,861	9,317	9,087
Delinquency								
> 90 days	17	30	34	38	44	50	62	68
% of total	*0.3*	*0.4*	*0.5*	*0.6*	*0.8*	*0.6*	*0.7*	*0.8*

IV. THE MODEL AND ESTIMATION RESULTS

4.1 The Model

In the initial process, models specified to have various combinations of the explanatory variables listed in Table 1 are examined. We start with models focusing on the CLTV ratio and CDSR. The inclusion of the CLTV ratio is based on the equity theory while the CDSR is used to test the "ability-to-pay" hypothesis. All other variables, in different combinations, are also included in the model specification.

Contrary to expectation, the models with CDSR as one of the explanatory variables are unsatisfactory (see Annex A). Specifically, the estimated coefficients of CDSR are either statistically insignificant in some snapshot months, or are sometimes unexpectedly negative. This could be due to the data quality of the derived CDSR.[11] Moreover, as some of the mortgagors may have other debt

[11] Due to the absence of actual current income data, income data are all proxies derived by the debt servicing ratio at origination with adjustments made by changes in the nominal wage index (see Note 1 of Table 1).

obligations, such as car loans and other consumer loans, the data of CDSR derived from the mortgage records may not reflect their complete payment burden. Furthermore, possible reductions in salaries, changes of job, or layoff of the borrower since the origination of the loan may also affect the accuracy of the data significantly.[12] The regression findings in Annex A should thus not be interpreted as suggesting that CDSR is not a factor determining default risk.

In view of this, mortgage rates are used to proxy CDSR. These current mortgage rates differ quite significantly among customers even for the same snapshot, as different spreads are charged on customers of different credit worthiness. For example, the mortgage spreads charged on July 2003 ranged from 5 percentage points to –2.65 percentage points and have a median of –1.75 percentage points. As shown in Annex B, this modification leads to improved results. The estimated coefficients for the CLTV ratio and the mortgage rate are both statistically significant and have an expected positive sign. On the other hand, most non-loan factors (seasoning and property variables) are statistically insignificant, and they are therefore dropped from the subsequent statistical analysis. Note that the variables of squared property age and squared CLTV are statistically insignificant, suggesting that in Hong Kong's case non-linearity in these two variables is not an issue.

4.2 Estimation Results

Logistic regressions are then performed on models with the CLTV ratio and the mortgage rate as core variables together with different combinations of other variables. The model specifications which yield the best results are adopted and further analysed. The estimation results of the standard model are given in Table 3.

[12] These trigger events have not been examined in the study due to the absence of relevant micro-level data.

Table 3. Estimation Results

Variables	Jul 00	Jan 01	Jul 01	Jan 02	Jul 02	Jan 03	Jul 03	Sep 03	Jul 00 to Sep 03[A]	Jul 00 to Sep 03[B]	Jul 00 to Sep 03[C]
Price	0.28 (0.20)	0.21 (0.24)	0.06 (0.82)	-0.10 (0.51)	-0.28 (0.10)	-0.39 (0.02)	-0.48 (0.00)	-0.22 (0.05)	-0.24 (0.00)	-0.24 (0.08)	-0.30 (0.00)
CLTV	0.05 (0.00)	0.04 (0.00)	0.04 (0.00)	0.02 (0.00)	0.02 (0.00)	0.02 (0.00)	0.01 (0.00)	0.01 (0.00)	0.02 (0.00)	0.02 (0.00)	0.02 (0.00)
Mortgage	1.23 (0.00)	1.13 (0.00)	1.02 (0.00)	0.90 (0.00)	0.73 (0.00)	0.89 (0.00)	0.77 (0.00)	0.63 (0.00)	0.37 (0.00)	0.37 (0.00)	0.39 (0.00)
Unemployment	---	---	---	---	---	---	---	---	0.49 (0.00)	0.49 (0.00)	0.66 (0.00)
Percentage Change of Hang Seng Index	---	---	---	---	---	---	---	---	-0.15 (0.00)	-0.15 (0.00)	-0.13 (0.00)
Constant	-23.93 (0.00)	-20.96 (0.00)	-16.53 (0.00)	-11.78 (0.00)	-9.35 (0.00)	-10.32 (0.00)	-8.64 (0.00)	-8.61 (0.00)	-12.75 (0.00)	-12.75 (0.00)	-13.70 (0.00)
Wald Test	28.78 (0.00)	68.78 (0.00)	109.67 (0.00)	109.38 (0.00)	93.28 (0.00)	162.66 (0.00)	152.46 (0.00)	159.75 (0.00)	708.45 (0.00)	330.63 (0.00)	676.58 (0.00)
Pseudo R^2	0.23	0.26	0.25	0.21	0.17	0.24	0.20	0.16	0.18	0.18	0.18
Log- Pseudo Likelihood	-88.80	-128.00	-155.30	-177.40	-203.90	-219.20	-281.60	-325.60	-1646.20	-1646.20	-1598.10
Goodness-of-fit Test	12658 (0.00)	8074 (0.00)	3246 (1.00)	3310 (1.00)	3454 (1.00)	3531 (1.00)	5643 (1.00)	6071 (1.00)	50754 (1.00)	50754 (1.00)	N.A.

Notes: 1. For a discussion of the models with macroeconomic variables, please see Section VI. The estimation result A refers to the regression using data without adjustment. The estimation results B and C refer to the regressions using variance-adjusted and weight-adjusted methods respectively. For the weight-adjusted method, the goodness-of-fit test statistic is not available.
2 Numbers in parentheses are p-values.

As can be seen from the table, the estimated coefficients for CLTV ratio and the mortgage rates are both statistically significant and have an expected positive sign in all the eight snapshot months. The results suggest the higher the CLTV ratio of a loan, the greater is the default probability, and the higher the mortgage rate, which implies a relatively heavier payment burden for the borrower, the greater is the likelihood of default. This lends support to both the “equity theory” and the “ability-to-pay” approaches of explaining default decisions. The estimated coefficients of current unit property price variable are negative in most of the snapshot periods. This suggests that mortgage loans on properties at the luxury end of the market are less likely to experience default.

The estimated parameters are not easily interpretable, and, in particular, cannot be used in the same way as the parameters in linear regression. As shown in Equation (1), the default probability is a non-linear function of the independent variables and there is no simple way to express the effect on the default probability of changing the independent variables. One way to express the effect is to derive the relationship of default probability and the level of CLTV ratio by holding the other variables at their mean levels. This is discussed in greater detail in Section V.

4.3 Diagnostic Checks

The Wald test statistics test the null hypothesis that all the coefficients in the model are zero are highly significant, indicating that all estimated coefficients are statistically different from zero. The Pseudo R^2 statistics range from 0.16 to 0.26, which are low but common in micro-level analyses.[13, 14] The Pearson chi-squared (χ^2) goodness-of-fit tests indicate that the selected model does not differ from the theoretical distribution for most of the selected months. Results for the goodness-of-fit test are satisfactory in general. There are concerns about the multicollinearity between the variables CLTV ratio and mortgage rate. As the correlation coefficient of the two variables is estimated to be –0.05, the issue of multicollinearity between the two variables does not appear to be a problem.

[13] The Pseudo R^2 is McFadden's (1974) likelihood ratio index. It equals to 1 - (L_{UR} / L_0), where L_{UR} is the log-likelihood function for the estimated model with all coefficients present, the L_0 is the log-likelihood function with an intercept only (under the null hypothesis that all coefficients are zero in the restricted model). If all coefficients are zero, then the Pseudo R^2 equals to 0.

[14] For the case of a dischotomous dependent variable the upper limit for Pseudo R^2 is likely to be substantially less than one, see Christensen (1997).

V. DEFAULT PROBABILITY AND THE LEVEL OF CLTV

With the estimated results, the relationship between default probability and the level of CLTV ratio, holding other explanatory variables at their mean levels, can be derived based on Equation (2).[15, 16]

$$\hat{P}(Y = default)\Big|_{X_2=\overline{X}_2,\ X_3=\overline{X}_3,\ \ldots,\ X_n=\overline{X}_n} = \frac{e^{\ln\left(\frac{\overline{P}}{1-\overline{P}}\right)+\hat{\beta}_1(X_1-\overline{X}_1)}}{1+e^{\ln\left(\frac{\overline{P}}{1-\overline{P}}\right)+\hat{\beta}_1(X_1-\overline{X}_1)}} \qquad (2)$$

where $\overline{P}$ is the average default probability, $\overline{X}_1$ is the mean level of the CLTV ratio and $\hat{\beta}_1$ is the estimated coefficient for the ratio.

By holding all other variables at their mean levels for all months, the default probability of loans at different levels of CLTV (up to the upper end of the actual CLTV level) for respective months is derived and presented in Chart 2.[17, 18] An enlarged graphical exhibition of simulations up to the CLTV level of 150% is given in Chart 3 for detailed inter-period comparison.[19]

---

15 See the derivation of the formula in Annex C.

16 Another common way to see the relationship is to derive the marginal effect of the j^{th} explanatory variable on the default probabilities by the following formula:

$$\frac{\partial P(Y=default)}{\partial X_j} = Z\ \hat{\beta}_j \qquad \text{where } Z=\frac{e^{\hat{c}+\hat{\beta}_1\overline{X}_1+\hat{\beta}_2\overline{X}_2+\ldots+\hat{\beta}_n\overline{X}_n}}{(1+e^{\hat{c}+\hat{\beta}_1\overline{X}_1+\hat{\beta}_2\overline{X}_2+\ldots+\hat{\beta}_n\overline{X}_n})^2}$$

17 Mean values of the explanatory variables in different periods are as follows:

	Jul 2000	Jan 2001	Jul 2001	Jan 2002	Jul 2002	Jan 2003	Jul 2003	Sep 2003	All
Price (HK$)	3,751	3,378	3,363	3,245	3,374	3,222	3,094	3,156	3,301
CLTV(%)	73	80	81	86	85	90	93	92	84
Mortgage (%)	9.6	8.8	6.1	4.3	4.2	3.7	3.6	3.6	5.3

18 The chart covers CLTV levels up to the upper end of the actual CLTV ratio in the respective snapshot months, as below:

	Jul 2000	Jan 2001	Jul 2001	Jan 2002	Jul 2002	Jan 2003	Jul 2003	Sep 2003
CLTV(%)	195	222	214	229	255	319	306	303

19 The CLTV ratios are mostly below 150%.

Chart 2. Default Probability and CLTV Ratio by Snapshot Month
(With other explanatory variables held at mean levels)

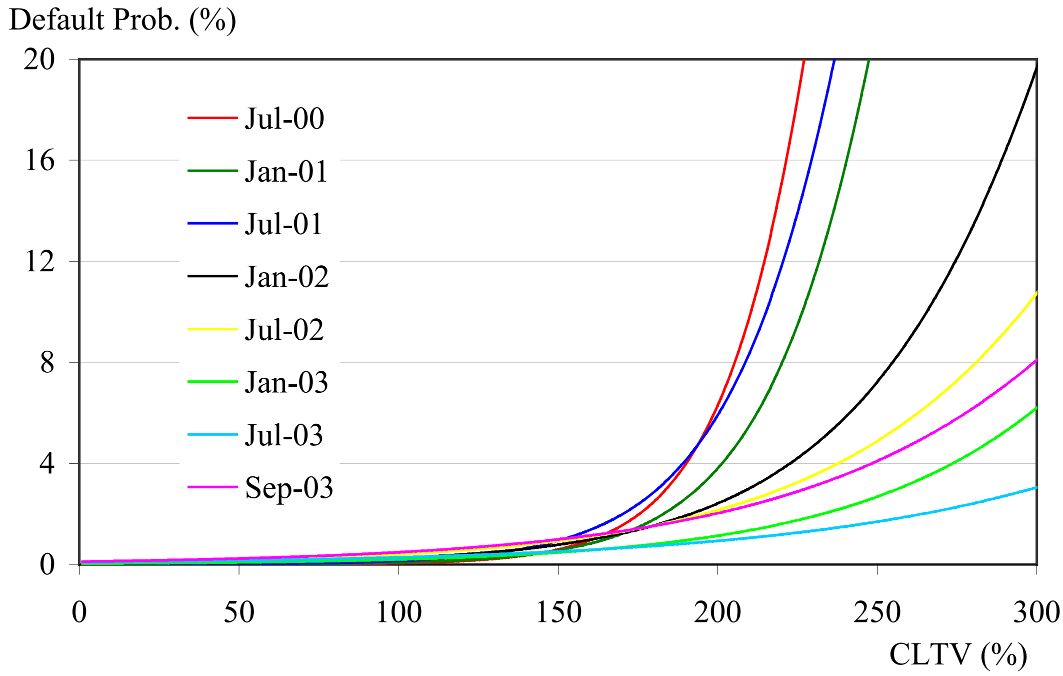
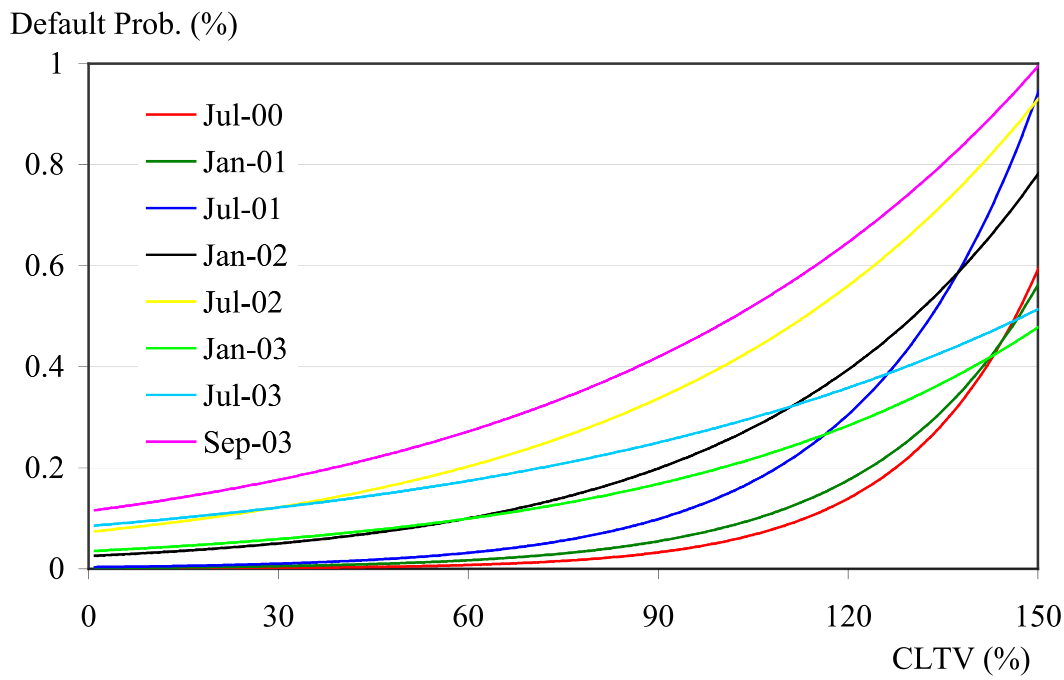


Chart 3. Default Probability and CLTV Ratio
(An enlarged graphical exhibition of Chart 2)



The estimated default probabilities at selected CLTV levels are listed in Table 4. It is found that they differ significantly at the same level of CLTV for different months. For instance, when the level of CLTV is 150%, the expected probability of default ranges from 0.48% (for January 2003) to 0.99% (for September 2003). The diversity in results may be due to the variations of macroeconomic conditions in different months.

Table 4. Estimated Default Probability at Different CLTV Levels

	Default Probability (%)							
CLTV Level (%)	Jul 2000	Jan 2001	Jul 2001	Jan 2002	Jul 2002	Jan 2003	Jul 2003	Sep 2003
50	0.00	0.01	0.02	0.08	0.17	0.08	0.15	0.24
75	0.02	0.03	0.06	0.14	0.26	0.13	0.21	0.34
100	0.05	0.08	0.14	0.25	0.40	0.20	0.28	0.48
125	0.18	0.21	0.37	0.44	0.61	0.31	0.38	0.69
150	0.59	0.56	0.94	0.78	0.93	0.48	0.51	0.99
175	1.96	1.47	2.38	1.38	1.41	0.74	0.69	1.42
200	6.29	3.80	5.91	2.42	2.15	1.14	0.93	2.03

VI. ESTIMATED DEFAULT PROBABILITY AND MACRO VARIABLES

In some studies, data on residential mortgages in different regions were matched with economic variables in the corresponding regions to assess the role of macroeconomic conditions.[20] This approach is not feasible in the present case since there is no "regional" variation in macroeconomic conditions in Hong Kong. While not resorting to more complex models, in order to capture the effect of changes in economic conditions on default probability, a preliminary attempt is made by pooling all loan data of the eight time-series observations to form one cross-sectional data set. Data of each loan in a specific month are then matched with the prevailing macroeconomic conditions in that month.[21] It should be noted that by so doing, the

[20] For instance, see Campbell and Dietrich (1983), Cunningham and Capone (1990), Lawrence and Smith (1992).

[21] The unemployment rates used in the analysis are given below. Based on the definition of default, the unemployment rate used in the estimation should lead the dependent variable by three months. For example, for the snapshot month of July 2000, the unemployment rate of April 2000 is used.

	Apr 2000	Oct 2000	Apr 2001	Oct 2001	Apr 2002	Oct 2002	Apr 2003	Jun 2003
%	5.4	4.8	4.5	5.7	7.1	7.4	7.8	8.5

Source: CEIC.

same loan, as far as it continues to be in HKMC's portfolio, is treated as different observations in the various months.

Given that major characteristics of the loan – the CLTV ratio and mortgage rate – would have changed tangibly in the six-month intervals, pooling the data together may be in general acceptable (see Loh and Tan, 2002). However, the results should be interpreted with caution. To the extent that some characteristics specific to an individual loan may have remained the same throughout the period, pooling the loan data together may result in using repeated observations in the sample and could cause biases in the statistical analysis. For instance, the true variance would be underestimated, so it may wrongly reject the null hypothesis (Type I errors) in parameter testing (see Neuhaus, 1992; Williams, 2000; Cho and Kim, 2002). A conventional method to deal with repeated observations is to consider an unbiased variance estimation which adjusts the variance for the intra-cluster correlation. This method avoids Type I errors in hypothesis testing. Another method is to introduce sampling weights – weights are given to specific loans in order to make adjustments for the relative frequencies that these loans are included due to the sampling design.[22] In this study, logistic regressions are performed using both methods to assess the possible biases.

In addition to the interest rate variable, which is already included in the model, the unemployment rate and the change in the Hang Seng Index (HSI) are selected as proxies for macroeconomic conditions. The former is intended to reflect the stress in the labour market, and the latter is chosen to represent the general financial market sentiment.[23]

The estimation results for regressions using unadjusted, variance-adjusted, and weight-adjusted methods are given in the last three columns of Table 3. All estimated coefficients are statistically significant and are with expected signs. All specification tests, including the Wald test, the Pearson χ^2 goodness-of-fit test and the Pseudo R^2 statistic, are satisfactory. The positive sign for the estimated coefficient of unemployment rate is in line with the expectation that the higher the unemployment rate, the greater the default probability. The negative sign for the percentage change in HSI is also consistent with general belief that when market conditions are buoyant, there is less incentive to default. As expected, Models A and B have the same estimated coefficients but different standard errors because Model A uses the traditional variance estimators with full scores but Model B calculates the

[22] The weight attached is the inverse of the frequency that a particular loan appears in the sample. This is particularly applicable for cases that the attributes of repeated observations are constant.

[23] Based on the definition of default, the percentage change in HSI used in the estimation should lead the dependent variable by three months.

variance estimators by using grouped scores.[24] Empirical results show that there is no change in the significance of the coefficients between Models A and B. At the same time, the estimated coefficients of Model C are similar in magnitude to that of Models A and B. All these imply that the assumption of independence among repeated observations may not be too strong.[25] For simplicity, the analysis in the following sections is based on the set of estimated coefficients in Model B, which is estimated by using the variance-adjusted method.

Equation (2) computes the effects of changes in the CLTV on the default probability, under the assumption that all other explanatory variables are at their mean levels. Such effects are derived and summarised in Chart 4. To illustrate how labour market conditions may affect default probability, Chart 5 shows the simulated default probability in relation to the CLTV ratio when the unemployment rate is set at 8.5% and 4.5%, as well as its mean level (6.5%). The estimated default probability would be 2.0% at the CLTV level of 200%, when the unemployment rate is at its mean level (6.5%). With a higher unemployment rate, the default probability curve is higher. When unemployment rates are at 8.5% and 4.5%, the estimated default probabilities are 5.3% and 0.8% respectively. Similar comparisons, holding other variables at their mean levels, with regard to the relationship between default probability and the CLTV ratio at different levels of mortgage rate or percentage change of HSI are given in Charts 6 and 7 respectively. In general, a higher mortgage rate or a lower percentage change of HSI tends to raise the default probability at a given CLTV level. Illustrations showing how estimated default probability changes at selected CLTV levels with varying macroeconomic conditions are given in Table 5.

---

[24] Scores are the first partial derivatives of the log-likelihood function with respect to the model parameters. Full scores include all individual observations (regardless of whether they are repeated observations) in the computation, while for grouped scores, repeated observations are grouped as specific independent observations in the calculation.

[25] Various studies have shown that the estimated variances of coefficients are biased because of the correlation of repeated observations, but the values of estimated coefficients remain unbiased (see Cirillo and others, 1996; Cho and Kim, 2002). The estimated results of this study are in line with these studies.

Chart 4. Default Probability and CLTV Ratio
(All Other Variables at Mean Levels)

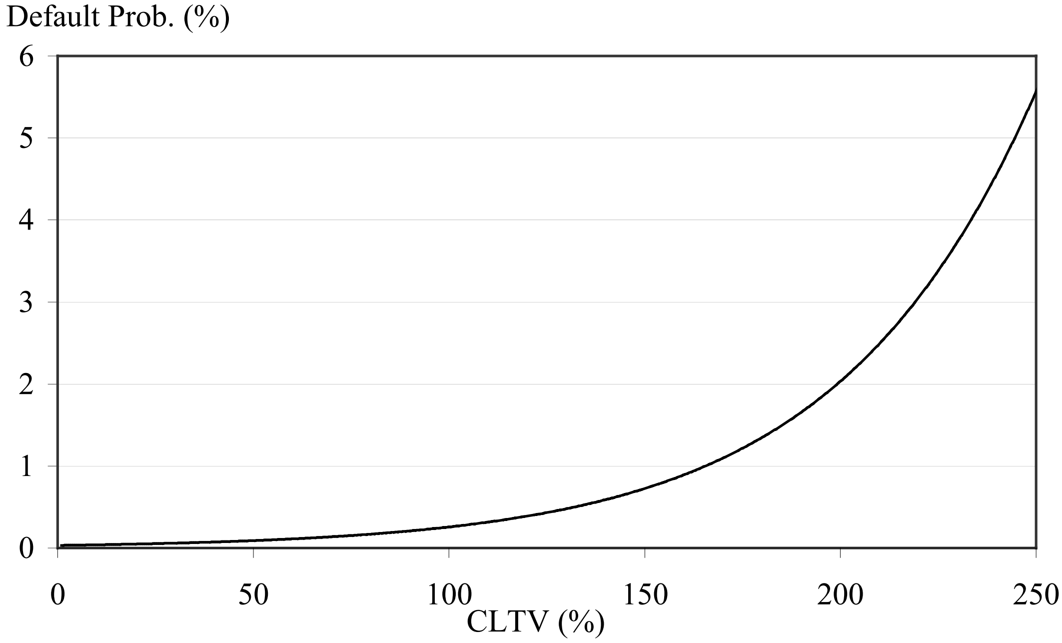


Chart 5. Default Probability and CLTV Ratio
4at Different Unemployment Rates

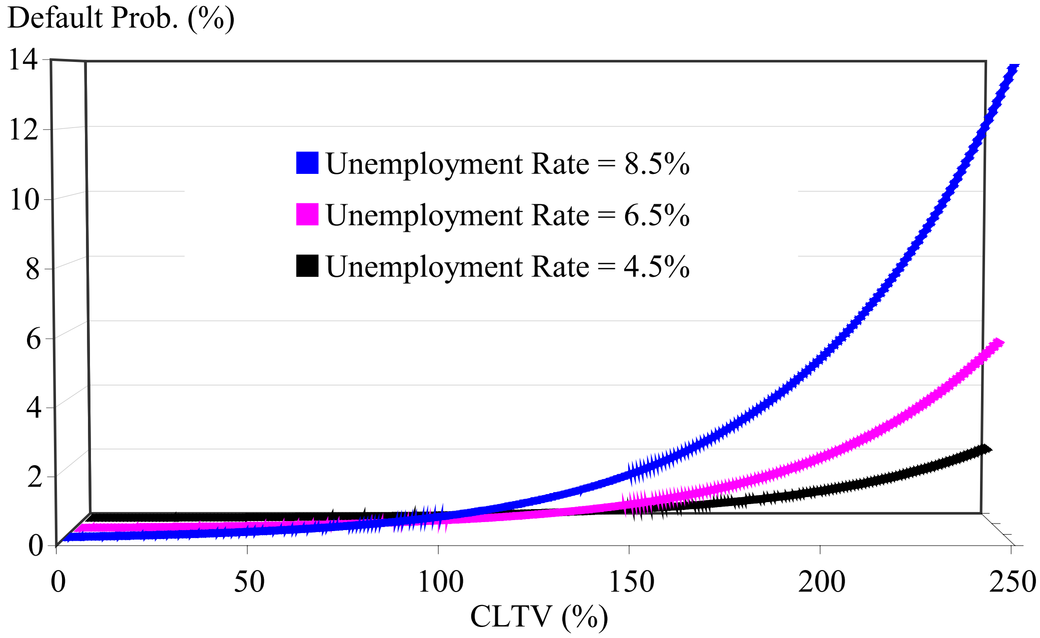


Chart 6. Default Probability and CLTV Ratio at Different Mortgage Rates

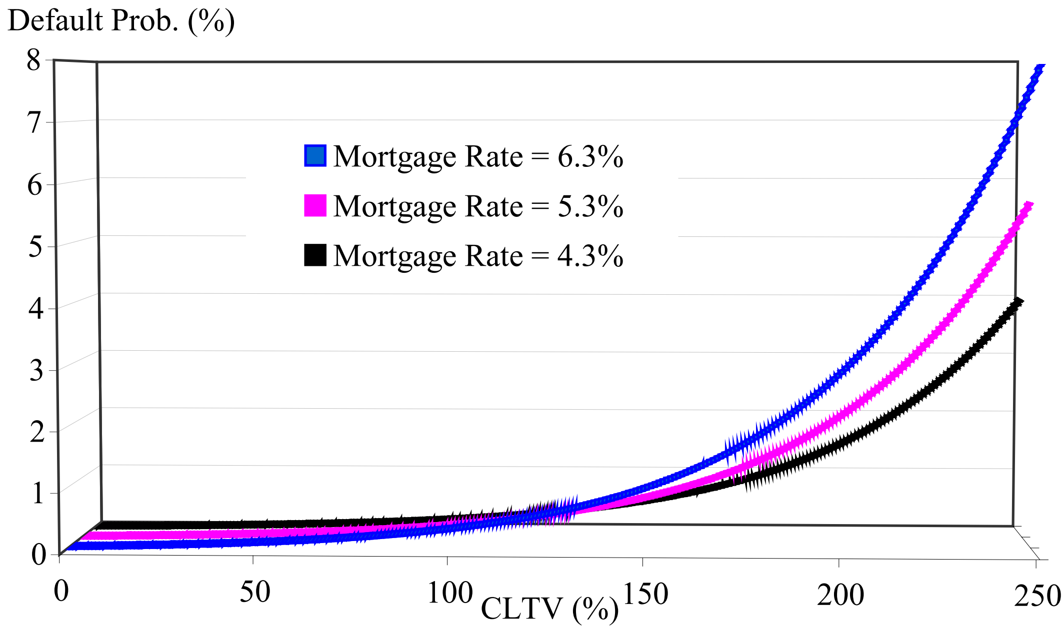


Chart 7. Default Probability and CLTV Ratio at Different Changes of Hang Seng Index

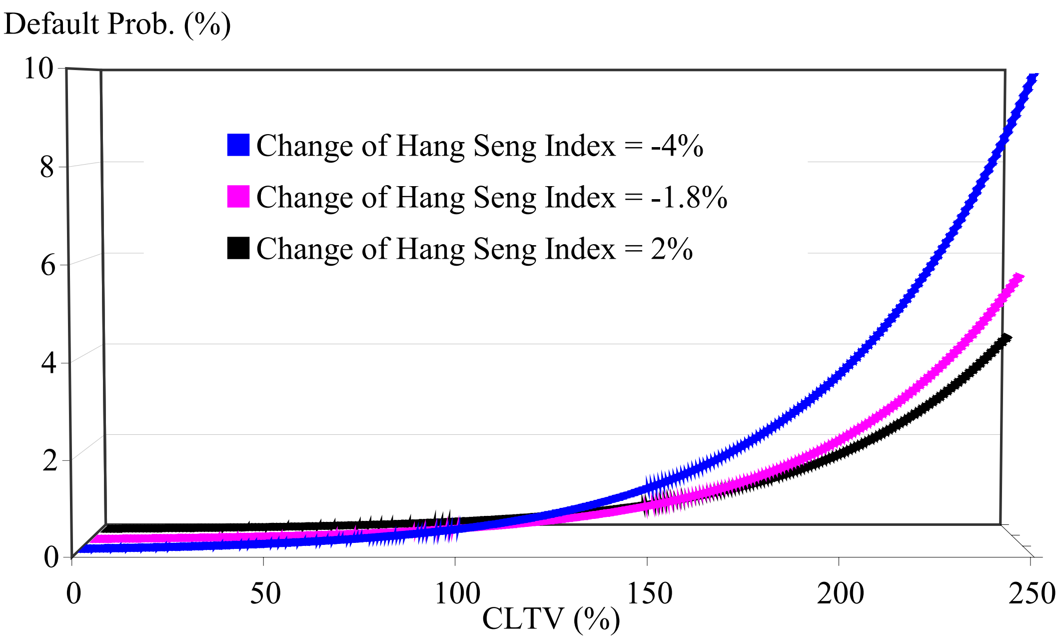


Table 5. Estimated Default Probability (%)
at Different CLTV Levels under Varying Macroeconomic Conditions

CLTV Level (%)	All other explanatory variables at mean levels								
	Unemployment Rate			Mortgage Rate			Change of HSI		
	8.5%	6.5%	4.5%	6.3%	5.3%	4.3%	-4.0%	-1.8%*	2.0%
50	0.24	0.09	0.03	0.13	0.09	0.06	0.17	0.09	0.07
75	0.41	0.15	0.06	0.22	0.15	0.11	0.28	0.15	0.11
100	0.69	0.26	0.10	0.37	0.26	0.18	0.47	0.26	0.19
125	1.15	0.43	0.16	0.62	0.43	0.30	0.79	0.43	0.32
150	1.92	0.73	0.27	1.05	0.73	0.50	1.32	0.73	0.54
175	3.20	1.22	0.46	1.75	1.22	0.84	2.20	1.22	0.90
200	5.27	2.03	0.77	2.91	2.03	1.41	3.66	2.03	1.51
225	8.58	3.38	1.29	4.80	3.38	2.36	6.01	3.38	2.52
250	13.65	5.57	2.15	7.80	5.57	3.91	9.73	5.57	4.18

Note:* The mean level of the variable in question.

VII. THE 70% LOAN-TO-VALUE RATIO AND ASSET QUALITY

The quarterly survey on residential mortgage loans in negative equity provides statistics on the average CLTV level since March 2002 for residential mortgages which are in negative equity. As of September 2004, the average CLTV level is estimated at 121%. To assess how a relaxation of the maximum 70% loan-to-value ratio guideline on property lending may affect banks' asset quality, we consider a hypothetical scenario under which the guideline was relaxed to 90% some time before 1997. We further assume that all banks would aggressively exploit this relaxation to expand their business by extending mortgage loans to cover 90% of the property values.[26] We then compare the estimated potential amount of defaulted loans based on the actual average CLTV level and the simulated CLTV level under the hypothetical scenario. The difference will measure the impact of a relaxation of the guideline.

Using the negative equity loan position in September 2004 as an example, the impact is simulated and presented in Table 6. With the sharp fall in property prices since late 1997, the average CLTV of negative equity loans under the hypothetical scenario would be about 163%, significantly higher than the actual

[26] This assumption is made to assess the maximum effect. However, in reality, this is unlikely to happen, as banks will decide on the maximum loan amount based on their assessment on the credit worthiness of the borrowers and the debt servicing ratio. This is evidenced by the fact that the actual LTV ratio for new loans made around 1997 was on average below 60%, far lower than the maximum ratio of 70% permitted under the guideline.

CLTV level reported by the mortgage survey. At this level of CLTV ratio, the default probability of these negative equity loans, as derived from our model developed in Section VI based on the pooled data of July 2000 to September 2003, would have been 0.95%, which is twice the actual level of 0.45%. Correspondingly, the potential amount of loans in default is estimated to have risen from HK$0.2 billion to HK$0.4 billion, an increase of HK$0.2 billion. These are conservative estimates as the delinquency rate of HKMC's loan portfolio is only two-fifths that of the industry. If the estimated probability of default is adjusted proportionally according to the ratio of actual delinquency rate of the industry to that of HKMC, the estimated increase in the potential amount of defaulted loans would be more than twice this amount.[27]

Table 6. Estimated Loan Defaults with and without a Relaxation of the Maximum LTV Ratio Guideline

	Actual Policy of Maximum LTV	Hypothetical Maximum LTV
Maximum LTV Ratio Guideline	70%	90%
Average CLTV Level (%)	127	163
Default Probability (%)	0.45	0.95
Estimated Amount of Default Loans (HKD billion)	0.2	0.4

Note: Mortgage loans in negative equity amounted to HK$43 billion as of September 2004.

VIII. CONCLUSION

The above analysis of mortgage default probability in Hong Kong confirms the importance of the CLTV ratio as a determinant of mortgage default decisions, with default probability of a mortgage loan positively correlated with its CLTV level. While this relation holds consistently well for the study period, its precise shape is found to vary over time with the prevailing market conditions. The mortgage rate, which serves as a proxy for the payment burden of borrowers, is also positively correlated with mortgage default risks. These results provide support for both the "equity theory" and the "ability-to-pay" approaches of explaining mortgage default.

[27] There are also other caveats. On the one hand, the impact can be underestimated as loans which were originally in positive equity region could have fallen into negative equity region if the loans were initially originated at a CLTV level of 90% under the hypothetical scenario. On the other hand, as pointed out in footnote 25, in reality, it is unlikely that banks would be so aggressive to fully exploit the hypothetical relaxation.

A preliminary attempt to introduce macroeconomic variables into the model by pooling data of different months into one single cross-sectional data set reveals that, in addition to interest rates, both labour and stock market conditions have a significant impact on default probability. While default probability is positively correlated with the unemployment rate, it is negatively correlated with changes in the HSI.

With the CLTV level found to be central to mortgage default decisions, this study lends strong support to the prudential policy of encouraging the adoption of a maximum 70% LTV ratio in residential mortgage lending.

References

Archer, W. R., Peter J. Elmer, David M. Harrison and David C. Ling (2002): "Determinants of Multifamily Mortgage Default", *Real Estate Economics* 30 (3), 445-473.

Campbell, T. and J. Dietrich (1983): "The Determinants of Default on Conventional Residential Mortgages", *Journal of Finance* 38 (5), 1569-1581.

Capozza, D. R., Dick Kazarian and Thomas A. Thomson (1997): "Mortgage Default in Local Markets", *Real Estate Economics* 25 (4), 631-655.

Christensen, R. (1997): "Log-linear Models and Logistic Regression", 2nd ed. Springer-Verlag, New York.

Chiang, R. C., Y. F. Chow and M. Liu (2002): "Residential Mortgage Lending and Borrower Risk: The Relationship between Mortgage rates and Individual Characteristics", *Journal of Real Estate Finance and Economics* 25 (1), 5-32.

Cho, Hye-Jin and Kang-Soo Kim. (2002): "Analysis of Heteroscedasticity and Correlation of Repeated Observations in Stated Preference (SP) Data", *KSCE Journal of Civil Engineering* 6 (2).

Chow, Y. F. and M. Liu (2003): "The Value of the Variable Tenor Mortgage Feature in Hong Kong", *Pacific-Basin Finance Journal* 11, 61-80.

Cirillo, C., A. Daly and K. Lindveld (1996): "Eliminating Bias due to the Repeated Measurements Problem in SP Data", *Proceedings of the 24th European Transport Forum (PTRC)*, Association for European Transport, Brunel University, England.

Cunningham, D. F. and C. A. Capone (1990): "The Relative Termination Experience of Adjustable to Fixed-Rate Mortgages", *The Journal of Finance* 45 (5), 1687-1703.

Gardner, M. J. and Dixie L. Mills (1989): "Evaluating the Likelihood of Default on Delinquent Loans", *Financial Management* (Winter), 55-63.

Goldberg, L. and Charles A. Capone, Jr. (1998): "Multifamily Mortgage Credit Risk: Lessons From Recent History", *Journal of Policy Development and Research* 4 (1), 93-113.

He, J. and M. Liu (2002): "Swapping Default Risk for Interest Rate Risk: The Rise of Fixed Rate Mortgage Loans in Hong Kong", *Research in Banking and Finance* 2, 167-178.

Horowitz, J. L. and N. E. Savin (2001): "Binary Response Models: Logits, Probits and Semiparametrics", *Journal of Economic Perspectives* 15 (4), 43-56.

Jackson, J. and D. Kasserman (1980): "Default Risk on Home Mortgage Loans: A Test of Competing Hypotheses", *Journal of Risk and Insurance* 3, 678-690.

LaCour-Little, M. (1999): “Another Look at the Role of Borrower Characteristics in Predicting Mortgage Prepayments”, *Journal of Housing Research* 10 (1), 45-60.

Lawrence, E. C. and L. D. Smith (1992): “An Analysis of Default Risk in Mobile Home Credit”, *Journal of Banking and Finance* 16, 299-312.

Loh, Lye Chye and Tin Hoe Tan (2002): “Asset Write-Offs – Managerial Incentives and Macroeconomic Factors”, *ABACUS* 38 (1), 134-151.

McFadden, D. (1974): “The Measurement of Urban Travel Demand”, *Journal of Public Economics* 3, 303-328.

Neuhaus, J. M. (1992): “Statistical Methods for Longitudinal and Clustered Designs with binary Responses”, *Statistical Methods in Medical Research* 1, 249-273.

Riddiough, T. J. (1991): “Equilibrium Mortgage Default Pricing with Non-Optimal Borrower Behavior”, *University of Wisconsin* Ph.D. diss.

Riddiough, T. J. and K. Vandell (1993): “Implied Foreclosure Probability and Loss Recoveries in the Application of the Contingent-Claims Approach to Pricing Risky Commercial Mortgage Debt: A Comment”, *University of Cincinnati Working Paper Series*.

Vandell, K. (1990): “Predicting Commercial Mortgage Foreclosure Experience”, *Salomon Brothers, Bond Market Research*.

Vandell, K. (1992): “Predicting Commercial Mortgage Foreclosure Experience”, *Journal of the American Real Estate and Urban Economic Association* 20 (1), 55-88.

Vandell, K. and T. Thibodeau (1985): “Estimation of Mortgage Defaults Using Disaggregate Loan History Data”, *AREUEA Journal* 13 (3), 292-316.

Williams, R. L. (2000): “A Note on Robust Variance Estimation for Cluster-Correlated Data”, *Biometrics* 56, 645-646.

Annex A

Estimation Results for Initial Model Specification with the CLTV Ratio and CDSR as Core Variables

Variables	Jul 00	Jan 01	Jul 01	Jan 02	Jul 02	Jan 03	Jul 03	Sep 03
OLTV	-0.03	-0.01	0.01	-0.02	-0.03	-0.03	-0.03	-0.01
	(0.47)	(0.37)	(0.45)	(0.28)	(0.05)	(0.01)	(0.01)	(0.33)
CLTV	0.02	0.04	0.15	0.19	0.15	0.09	0.06	0.04
	(0.61)	(0.12)	(0.10)	(0.00)	(0.00)	(0.00)	(0.00)	(0.01)
CLTVSQ	0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00
	(0.33)	(0.80)	(0.22)	(0.01)	(0.01)	(0.01)	(0.03)	(0.24)
ODSR	0.14	0.00	-0.14	-0.12	-0.07	-0.02	0.01	0.02
	(0.00)	(1.00)	(0.00)	(0.00)	(0.00)	(0.25)	(0.47)	(0.35)
CDSR	-0.14	0.01	0.13	0.14	0.10	-0.03	-0.05	-0.04
	(0.00)	(0.90)	(0.00)	(0.00)	(0.00)	(0.18)	(0.00)	(0.01)
Oseason	-0.01	-0.00	0.00	-0.00	0.00	-0.00	0.00	0.00
	(0.40)	(0.68)	(0.97)	(0.81)	(0.31)	(0.71)	(0.95)	(0.68)
Season	0.03	0.02	0.01	0.01	0.02	0.01	0.01	-0.01
	(0.01)	(0.20)	(0.60)	(0.59)	(0.28)	(0.16)	(0.58)	(0.55)
Garea	-0.00	-0.00	-0.00	0.00	-0.00	0.00	-0.00	-0.00
	(0.96)	(0.35)	(0.54)	(0.55)	(0.81)	(0.49)	(0.76)	(0.29)
Price	-0.05	0.11	-0.02	0.11	-0.03	-0.15	-0.24	-0.07
	(0.88)	(0.64)	(0.96)	(0.64)	(0.91)	(0.47)	(0.18)	(0.64)
Oage	0.01	-0.01	-0.01	0.00	0.00	0.00	0.01	0.01
	(0.40)	(0.23)	(0.07)	(0.75)	(0.58)	(0.59)	(0.38)	(0.15)
Oagesq	-0.00	0.00	0.00	-0.00	-0.00	-0.00	-0.00	-0.00
	(0.88)	(0.11)	(0.24)	(0.87)	(0.99)	(0.68)	(0.74)	(0.49)
Constant	-8.73	-9.21	-14.32	-17.42	-14.77	-9.76	-7.36	-7.29
	(0.00)	(0.00)	(0.01)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Wald Test	88.36	108.70	79.42	110.42	109.92	103.98	104.98	124.50
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Pseudo R^2	0.19	0.11	0.22	0.22	0.18	0.20	0.17	0.14
Log- Pseudo Likelihood	-93.60	-160.70	-149.70	-172.40	-196.40	-224.70	-286.50	-326.40
Goodness-of-fit Test	5037	6597	58876	4997	4786	5629	8444	7030
	(1.00)	(1.00)	(0.00)	(1.00)	(1.00)	(1.00)	(0.93)	(1.00)

Notes: Numbers in parentheses are p-values.

Annex B

Estimation Results for Initial Model Specification with CLTV Ratio and Mortgage Rate (as a Proxy to CDSR) as Core Variables

Variables	Jul 00	Jan 01	Jul 01	Jan 02	Jul 02	Jan 03	Jul 03	Sep 03
OLTV	-0.05 (0.27)	-0.01 (0.26)	0.02 (0.13)	-0.01 (0.52)	-0.03 (0.07)	-0.02 (0.08)	-0.01 (0.22)	0.01 (0.66)
CLTV	0.06 (0.27)	0.07 (0.01)	0.12 (0.08)	0.13 (0.00)	0.13 (0.00)	0.07 (0.00)	0.04 (0.03)	0.02 (0.29)
CLTVSQ	0.00 (0.90)	-0.00 (0.32)	-0.00 (0.23)	-0.00 (0.03)	-0.00 (0.02)	-0.00 (0.03)	-0.00 (0.18)	-0.00 (0.92)
ODSR	0.02 (0.54)	0.04 (0.12)	-0.02 (0.46)	-0.01 (0.72)	0.01 (0.68)	-0.03 (0.04)	-0.02 (0.08)	-0.01 (0.33)
Mortgage	1.48 (0.00)	1.28 (0.00)	1.08 (0.00)	0.91 (0.00)	0.67 (0.00)	0.84 (0.00)	0.71 (0.00)	0.61 (0.00)
Oseason	0.00 (0.93)	-0.00 (0.31)	-0.00 (0.25)	-0.01 (0.23)	0.00 (0.89)	-0.00 (0.86)	0.00 (0.84)	0.00 (0.44)
Season	0.05 (0.00)	0.03 (0.00)	0.01 (0.54)	0.00 (0.91)	0.00 (0.75)	0.02 (0.13)	0.01 (0.55)	-0.01 (0.49)
Garea	-0.00 (0.41)	-0.00 (0.12)	-0.00 (0.30)	0.00 (0.48)	-0.00 (0.80)	0.00 (0.18)	0.00 (0.84)	-0.00 (0.34)
Price	0.25 (0.25)	0.28 (0.07)	0.09 (0.81)	0.03 (0.88)	-0.02 (0.92)	-0.17 (0.38)	-0.03 (0.06)	-0.13 (0.35)
Oage	0.01 (0.54)	-0.01 (0.24)	-0.01 (0.42)	0.00 (0.93)	0.00 (0.60)	0.00 (0.73)	0.01 (0.32)	0.01 (0.11)
Oagesq	0.00 (0.94)	0.00 (0.07)	0.00 (0.92)	-0.00 (0.95)	-0.00 (0.98)	-0.00 (0.91)	-0.00 (0.62)	-0.00 (0.38)
Constant	-27.00 (0.00)	-24.60 (0.00)	-20.80 (0.00)	-17.06 (0.00)	-15.53 (0.00)	-13.29 (0.00)	-9.85 (0.00)	-9.34 (0.00)
Wald Test	91.66 (0.00)	94.89 (0.00)	122.21 (0.00)	121.53 (0.00)	141.42 (0.00)	157.70 (0.00)	161.76 (0.00)	203.23 (0.00)
Pseudo R^2	0.32	0.29	0.28	0.23	0.20	0.27	0.21	0.18
Log- Pseudo Likelihood	-79.20	-122.00	-150.20	-172.20	-195.60	-211.80	-277.20	-320.50
Goodness-of-fit Test	5829 (0.17)	7816 (0.00)	3912 (1.00)	3445 (1.00)	4655 (1.00)	4047 (1.00)	6699 (1.00)	6002 (1.00)

Notes: Numbers in parentheses are p-values.

Annex C

The Derivation of the Relationship between Default Probability and the CLTV Level

This annex presents the steps, based on the estimated logit model, for deriving the relationship between a particular variable X_1 (i.e. CLTV ratio in the paper) and the default probability, by holding other independent variables at their mean levels.

Consider a logit model with three independent variables:

$$P = \frac{e^{c+\beta_1 X_1+\beta_2 X_2+\beta_3 X_3}}{1+e^{c+\beta_1 X_1+\beta_2 X_2+\beta_3 X_3}}$$

The fitted model is: $$\hat{P} = \frac{e^{\hat{c}+\hat{\beta}_1 X_1+\hat{\beta}_2 X_2+\hat{\beta}_3 X_3}}{1+e^{\hat{c}+\hat{\beta}_1 X_1+\hat{\beta}_2 X_2+\hat{\beta}_3 X_3}}$$

As $$\overline{P} = \frac{e^{\hat{c}+\hat{\beta}_1 \overline{X}_1+\hat{\beta}_2 \overline{X}_2+\hat{\beta}_3 \overline{X}_3}}{1+e^{\hat{c}+\hat{\beta}_1 \overline{X}_1+\hat{\beta}_2 \overline{X}_2+\hat{\beta}_3 \overline{X}_3}}$$

So $$\hat{c} = \ln\left(\frac{\overline{P}}{1-\overline{P}}\right)-\hat{\beta}_1 \overline{X}_1-\hat{\beta}_2 \overline{X}_2-\hat{\beta}_3 \overline{X}_3$$

$$\begin{aligned}
\hat{P} &= \frac{e^{\hat{c}+\hat{\beta}_1 X_1+\hat{\beta}_2 X_2+\hat{\beta}_3 X_3}}{1+e^{\hat{c}+\hat{\beta}_1 X_1+\hat{\beta}_2 X_2+\hat{\beta}_3 X_3}} \\
&= \frac{e^{\ln\left(\frac{\overline{P}}{1-\overline{P}}\right)-\hat{\beta}_1 \overline{X}_1-\hat{\beta}_2 \overline{X}_2-\hat{\beta}_3 \overline{X}_3+\hat{\beta}_1 X_1+\hat{\beta}_2 X_2+\hat{\beta}_3 X_3}}{1+e^{\ln\left(\frac{\overline{P}}{1-\overline{P}}\right)-\hat{\beta}_1 \overline{X}_1-\hat{\beta}_2 \overline{X}_2-\hat{\beta}_3 \overline{X}_3+\hat{\beta}_1 X_1+\hat{\beta}_2 X_2+\hat{\beta}_3 X_3}} \\
&= \frac{e^{\ln\left(\frac{\overline{P}}{1-\overline{P}}\right)+\hat{\beta}_1 (X_1-\overline{X}_1)+\hat{\beta}_2 (X_2-\overline{X}_2)+\hat{\beta}_3 (X_3-\overline{X}_3)}}{1+e^{\ln\left(\frac{\overline{P}}{1-\overline{P}}\right)+\hat{\beta}_1 (X_1-\overline{X}_1)+\hat{\beta}_2 (X_2-\overline{X}_2)+\hat{\beta}_3 (X_3-\overline{X}_3)}}
\end{aligned}$$

By holding other independent variables at their mean levels, i.e. $X_2 = \overline{X}_2$ and $X_3 = \overline{X}_3$, the above formula is reduced to:

$$\hat{P}(Y = default)\Big|_{X_2=\overline{X}_2,\ X_3=\overline{X}_3} = \frac{e^{\ln\left(\frac{\overline{P}}{1-\overline{P}}\right)+\hat{\beta}_1(X_1-\overline{X}_1)}}{1+e^{\ln\left(\frac{\overline{P}}{1-\overline{P}}\right)+\hat{\beta}_1(X_1-\overline{X}_1)}}$$

The model can be extended to include n independent variables. By holding all other independent variables at their mean levels, i.e. $X_2 = \overline{X}_2$, $X_3 = \overline{X}_3$, …, $X_n = \overline{X}_n$, the default/CLTV probability formula becomes:

$$\hat{P}(Y = default)\Big|_{X_2=\overline{X}_2,\ X_3=\overline{X}_3,\ \ldots,\ X_n=\overline{X}_n} = \frac{e^{\ln\left(\frac{\overline{P}}{1-\overline{P}}\right)+\hat{\beta}_1(X_1-\overline{X}_1)}}{1+e^{\ln\left(\frac{\overline{P}}{1-\overline{P}}\right)+\hat{\beta}_1(X_1-\overline{X}_1)}}$$

Furthermore, to consider different scenarios, one may alter the level of any of the other variables from their means. For example, when $X_2 - \overline{X}_2 = 2$ (the unemployment rate is set at 2% higher than its mean level to see how unemployment rate changes may shift the default/CLTV probability curve), $X_3 = \overline{X}_3$, the above equation becomes:

$$\hat{P}(Y = default)\Big|_{X_2-\overline{X}_2=2,\ X_3=\overline{X}_3} = \frac{e^{\ln\left(\frac{\overline{P}}{1-\overline{P}}\right)+\hat{\beta}_1(X_1-\overline{X}_1)+2\hat{\beta}_2}}{1+e^{\ln\left(\frac{\overline{P}}{1-\overline{P}}\right)+\hat{\beta}_1(X_1-\overline{X}_1)+2\hat{\beta}_2}}$$

Based on the above equation, the CLTV ratio/default probability relationship, with $X_2 - \overline{X}_2 = 2$ and other independent variables (except X_1) at their mean levels, can be derived. The formula can be extended for the model with n independent variables.