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TESTING FOR BUBBLES IN THE HONG KONG STOCK MARKET

Key Points :

- *Despite the global low inflation environment, large swings in equity markets have been common in both industrialised and developing economies in recent years. In many countries, bursts of asset price bubbles have been followed by sharp contractions in real economic activity. The significant repercussion of financial market fluctuations has triggered discussions on how bubbles can be identified from asset price movements and the appropriate policy response.*

- *Using the Hong Kong stock market as an example, this study addresses the identification issue by exploring whether the market has been subject to equity price bubbles. Unlike other studies on Hong Kong which focus on a single empirical method, three different approaches are used in this study. Perhaps not surprisingly, these give rise to different results.*

- *The mixed results on the identification issue suggest that the lack of monetary tools to handle asset price bubbles in Hong Kong should not be regarded as a limitation as there can be significant risks associated with policy responses against asset bubble formation. To reduce the impact on real economy of asset bubble bursting, policy makers should focus more on fundamental measures that help strengthen the resilience of financial institutions and market infrastructure (such as the payment systems), and enhance market transparency to reduce information asymmetry.*

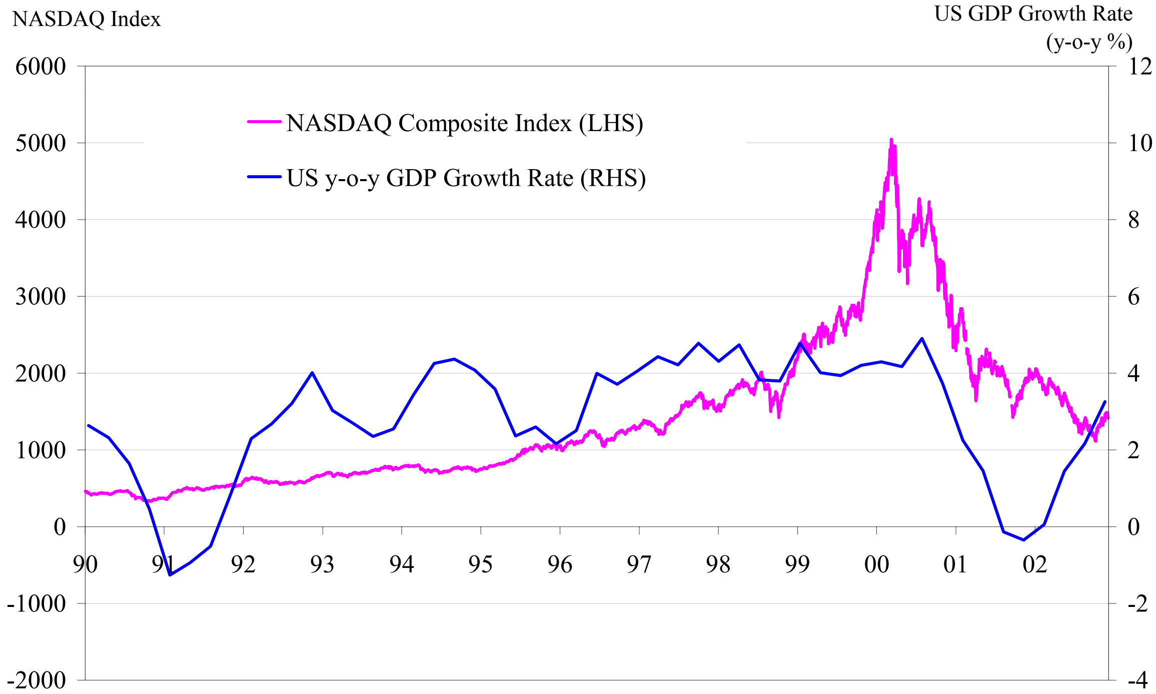
Prepared by : Ip-wing Yu and Angela Sze
Market Research Division
Research Department
Hong Kong Monetary Authority

I. INTRODUCTION

In recent years, inflation rates have fallen to low levels in response to the growing emphasis attached to price stability by central banks. Price stability, however, is not necessarily accompanied by financial stability. Despite a common belief that low inflation promotes financial stability, asset market imbalances can build up in periods of stable prices. In particular, the success of policymakers in keeping prices stable can, by anchoring price expectations, induce greater stickiness in price and wages and hence mask, at least for a while, the inflationary pressures normally associated with unsustainable growth in aggregate demand. At times, this has led to a surge in asset prices, driven mainly by investors' optimistic assessments of the asset's future return alongside the arrival of the "new" economy.

In the past two decades, the build-up of imbalances in asset markets, reflected in the increased volatility in asset prices, has led to occasional incidents of financial market instability. Since 1980s, boom-bust cycles in the prices of equity and real estates have been observed in a number of industrialised and developing economies (Borio, Kennedy and Prowse (1994), Borio and Lowe (2002)). In many of these cases, the burst of asset price bubbles has been followed by a significant contraction in real economic activity. For example, many economists attribute the 2001 recession in the United States to the sharp reduction in capital spending after the burst of technology stock bubble in 2000. (Chart 1)

Chart 1. The US Recession in 2001 and the Recent Movement in NASDAQ Composite Index



The significant repercussion of financial market fluctuations on the economy has in turn triggered discussions on the role of central bank policy in handling asset price bubbles. Views on this issue are diverse. Some argue that a central bank should implement monetary policy to achieve solely macroeconomic goals such as price stability and sustainable economic growth while applying its regulatory, supervisory and lender-of-last resort powers to help ensure financial stability (Bernanke (2002)). Others advocate for a monetary-policy response to cope with asset price bubbles. Specifically, they have argued that central banks should take pre-emptive actions against the formation of the bubbles to contain their potentially adverse consequences (Bordo and Jeanne (2002), Borio and Lowe (2002), Cecchetti, Genberg, Lipsky and Wadhwani (2000), and International Monetary Fund (2000)).

Whether the building up of imbalances in the financial market should warrant monetary policy response hinges largely on the accurate and reliable identification of asset bubble at an early stage. Failure to do so would risk making inappropriate policy response that may exacerbate rather than mitigate market volatility. Using Hong Kong stock prices as an example, this paper addresses the identification issue by testing for the existence of speculative bubbles using three different approaches. Despite the popular impression that bubbles exist in the stock market, there are very few systematic studies that have applied formal empirical tests on the behaviour of equity prices in Hong Kong.[1] The results of these tests give insight regarding the identification of asset price bubbles and thus shed light on the related policy debate. Specifically, if the detection of bubbles is found to be sensitive to the approach used in the test, the implementation of pre-emptive measure against their formation in asset markets by policymakers during the boom period will be difficult if not impossible to justify.

Conceptually, a bubble is said to exist if asset price deviates significantly from its sustainable path which is empirically very difficult to determine. It is therefore unlikely that asset bubble can be identified based on a single aspect of the price behaviour. A more appropriate approach to handle the identification issue is to check for the existence of bubbles using alternative approaches which focus on different aspects of the phenomenon.

The remainder of the paper is organised as follows: Section II gives a summary of the various methods employed to test for the bubbles and their technical details are found in Appendix I; Section III discusses the data and reports major findings while detailed regression results are found in Appendix II. Conclusions are presented in the last section.

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[1] One study that we are aware of on testing bubbles in the Hong Kong stock market is Chan, McQueen and Thorley (1998). The work concluded that the characteristics of Hong Kong equity prices did not conform to the predictions of the rational speculative bubbles model.

II. METHODS FOR TESTING BUBBLES

It has long been recognised that prices of financial assets may evolve according to self-fulfilling expectations of speculators instead of fundamentals. When this happens, prices are said to be driven by a speculative bubble. A bubble cannot persist indefinitely -- a period of price rise would be followed by a crash. In this study, three different tests are applied for identifying bubbles in the Hong Kong equity market, including the specification test by West (1987), the co-integration test by Diba and Grossman (1988) and the duration dependence test by McQueen and Thorley (1994). These approaches are chosen because of their focusing on different aspects of asset price bubbles. Together, results of the three tests allow for an assessment of stock market movements in Hong Kong. The conceptual framework of these approaches is outlined in the ensuing paragraphs while the technical details of each approach are spelled out in Appendix I.

(1) The Specification Test by West (1987)

The basic idea underlying the specification test is to compare two sets of estimated coefficients derived from alternative methods that relate the stock price with the expected present discounted value (PDV) of a given stock's dividend stream. These two sets of estimates will be consistent if there is no bubble but inconsistent otherwise. Under this approach, the existence of bubbles is tested by checking whether the two sets of estimates are the same other than the effect of sampling error. If these estimates are significantly different, a bubble is said to exist in the equity market.

Specifically, the two sets of estimates are derived from the following system of equations:

Group 1:

$$p_t = \rho\,(p_{t+1} + d_{t+1}) + \mu_{t+1} \qquad (1)$$

$$\Delta d_{t+1} = u + \phi_1 \Delta d_t + \ldots + \phi_q \Delta d_{t-q+1} + \upsilon_{t+1} \qquad (2)$$

Group 2:

$$\Delta p_{t+1} = m + \delta_1 \Delta d_t + \ldots + \delta_q\, \Delta d_{t-q+1} + \omega_{t+1} \qquad (3)$$

where p_t is the real stock price at time t;

ρ is the constant ex ante real discount rate;

d_t is the real dividend paid at time t;

u, m, $\phi_1, \ldots \phi_q$, $\delta_1, \ldots \delta_q$ are parameters to be estimated.

Equation (1) is the arbitrage condition yielding the discount rate while equation (2) gives the stochastic process of the stock dividends. Combining these two equations, the price of the stock can be derived as a function of its lagged dividends. The coefficients estimated from this pair of equations are used to compare with those obtained by regressing the stock price on a suitable set of lagged dividends in equation (3).[2] The comparison will give a bubble test statistics which is asymptotically distributed as χ^2_{q+1} under the null hypothesis of no bubble.[3] The rejection of the null hypothesis by the data in the chi-square test indicates the presence of bubbles in the stock market.

As there are possible sources of model mis-specification such as expectational irrationality (Ackley (1983)) and time varying discount rates (Leroy (1984)), four diagnostic checks suggested by West are adopted in this study. These include:

a) the serial correlation of the residuals;
b) the orthogonality between the instrumental variables and the residuals;
c) the stability of the regression coefficients; and
d) the sensitivity of the model to the number of lags used for the dividend process.

It should be noted that the power of this bubble test is limited as it requires a detailed specification of an underlying equilibrium model for the stock price. Rejection of the no bubble hypothesis may be attributable to the imposition of a wrong model instead of the existence of bubbles. Although several diagnostic checks have been applied in the study, the risk of the underlying model being mis-specified still exists.

(2) Co-integration Test by Diba and Grossman (1988)

The co-integration test of Diba and Grossman (1988) does not require a detailed specification of an underlying equilibrium model. Instead, the test is based on a simple rational expectation assumption which sets the stock price as the sum of the present value of expected future price, dividends and a set of unobservable fundamental variables.[4]

As demonstrated in Appendix I, if the unobservable variables and the first difference of dividends are stationary and bubbles do not exist, stock prices and dividends will be co-integrated. In other words, if prices and dividends are not co-integrated, bubbles may exist provided other unobservable fundamentals are stationary. This

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[2] Both sets of estimates are based on a standard efficient market model in which the stock price is equal to the discounted value of the expected future price plus dividends.

[3] q is the number of lags in the dividend process.

[4] Since these unobservable fundamental variables are unknown, this approach does not require a specific underlying equilibrium model for the stock price.

approach essentially entails checking for the stationarity property of stock prices and fundamentals, and any co-integration relationship between them.

Despite the simple testing procedure, results of the co-integration test have to be interpreted with caution. When stock prices and dividends are found to be co-integrated, the null hypothesis of no bubble is accepted. However, the reverse is not necessarily true. For the alternative hypothesis to hold, the presence of bubbles can be one of the many possible reasons. Other factors include, for example, the non-stationary nature of the unobservable variables.

(3) Duration Dependence Test of McQueen and Thorley (1994)

In a rational speculative bubble framework, stock prices may deviate from their fundamental value as long as rational investors believe that the bubble will continue to expand with a certain probability such that its return can compensate for the potentially larger and larger crashes. As a result, the pattern of excess returns in the market can be used to detect the existence of bubbles. Specifically, a long period of positive excess returns suggests the presence of a bubble. If a bubble already exists in the market, the probability that a run of positive abnormal returns will end should decline with the length of the run.[5] Based on this idea, McQueen and Thorley (1994) developed the duration dependence test for rational speculative bubbles.

In this test, positive or negative excess monthly returns are defined relative to the sample mean, while the excess weekly returns are defined as the residuals of an AR(4) model.[6] The duration dependence test divides excess returns into series of run length *I*. For a data set containing *T* observations, the log likelihood function is:

$$L = \sum_{i=1}^{\infty} \left[N_i \ln h_i + M_i \ \ln(1 - h_i) \right] \tag{4}$$

where N_i is the count of completed runs of length i in the sample;

M_i is the count of runs with a length greater than i;

$h_i = (1 + e^{-(\alpha + \beta \ln(i))})^{-1}$, is the conditional probability of a run ending at i given that it lasts at least until i.[7]

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[5] A run is defined as a sequence of excess returns of the same sign.

[6] The AR(4) model is used to capture some autoregressive effects of the weekly data.

[7] This log-logistic functional form bases on the idea of McDonald, McQueen and Thorley (1993). The function transforms the unbounded range of $\alpha + \beta \ln(i)$ into a (0, 1) probability space.

Under the null hypothesis of no bubble, the abnormal returns are random and there is no duration dependence. That is, $\beta = 0$. If a bubble exists, the probability of a positive run coming to an end should decrease with the run length, that is, $\beta < 0$ for positive run. Tests are performed by maximising the log likelihood function with respect to α and β. The likelihood ratio test of $\beta = 0$ is asymptotically distributed as χ_1^2.

As in the case for the two methods discussed earlier, there are some limitations in applying the duration dependence test for detecting the bubbles. First, there is a question as to how the abnormal return is identified from the data. As there is no well-accepted benchmark in this issue, alternative specifications of abnormal returns are examined in the study. Secondly, the conclusion of the test may be sensitive to the selection of the hazard function. The log-logistic functional form is adopted here because it is commonly used in transforming the unbounded range of a parameter into a (0,1) probability space.

III. Data and Results

a. DATA

Monthly data of Hang Seng Index and the dividend ratio from July 1974 to May 2002 are used in the West test and Diba and Grossman's test. Both series are deflated by the Consumer Price Index and shown in Chart 2. For the duration dependence test, both monthly and weekly Hang Seng Index in the same period are used. The basic summary statistics of the data used in the test is reported in Table 1.

Chart 2. Real Monthly Hang Seng Index and Dividend

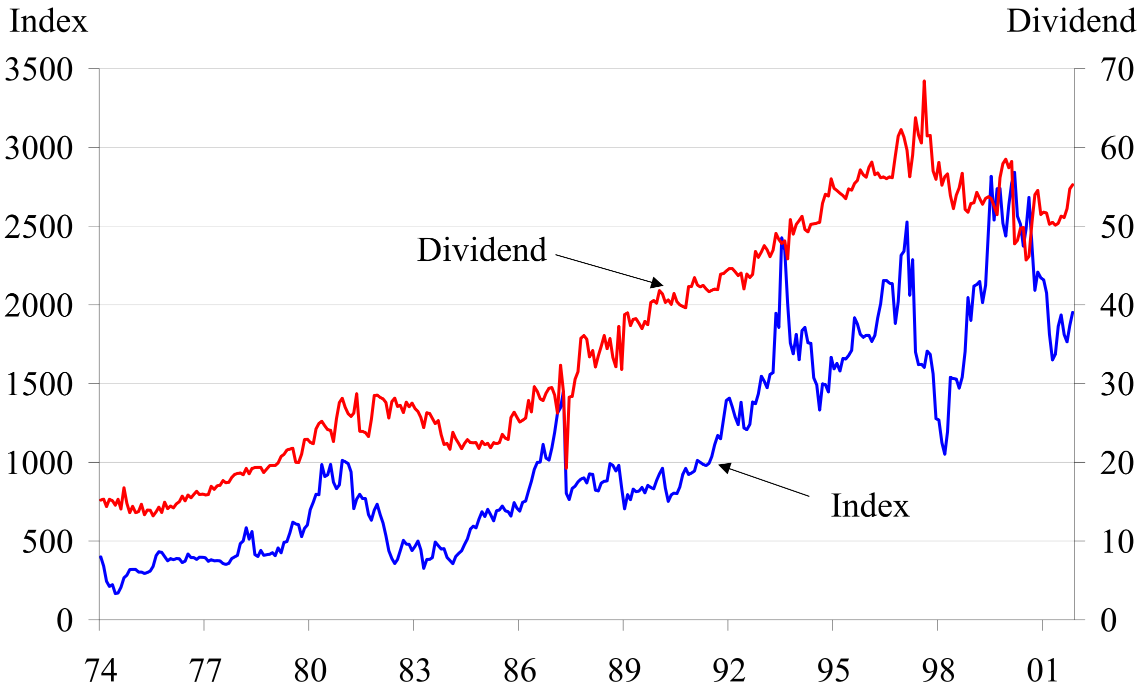

Table 1. Summary Statistics of Returns of Hang Seng Index

	Monthly	Weekly
Period	Jul 1974 – May 2002	3 Jul 1974 - 29 May 2002
Mean	1.87%	0.39%
Standard Deviation	0.0942	0.0399
Skewness	0.1767	-0.2421
Kurtosis	1.8908	2.9160

Note: All returns are annualised by continuous compounding. Weekly nominal returns are calculated from the Wednesday closing levels. In the case of a holiday or non-trading day on Wednesday, the Tuesday close is used. If Tuesday data are also unavailable, the Monday close used.

b. RESULTS

The results from regressions and diagnostic checks under the West approach are reported in the tables in Appendix II. The estimated discount rate (ρ) from equation (A4) is found to be at a reasonable level and remains robust when different assumptions regarding the dividend process are applied. Parameter estimates of the coefficients in equation (A5) are fairly stable under various dividend specifications. As for the diagnostic checks, the hypothesis of no serial correlation in the residuals is accepted in most cases. Moreover, the instrumental variables used in the estimation are found to be orthogonal to the residuals for most of the cases. Finally, to test for the stability of the regression coefficients, the full sample is split into two sets of sub-samples based on: 1) the mid-point of the sample period (June 1988), and 2) the structural break as a result of the change in exchange rate arrangement in October 1983. The null hypothesis of stability of the coefficients is not rejected in any of these sub-samples. In brief, these diagnostic checks do not suggest any significant mis-specification of the equations in the test.

Results of the bubble test statistics based on West's specification approach are reported in Table 2, including the test results for the sub-samples before and after the 1987 stock market crash. The null hypothesis of no bubble is strongly rejected in both sample periods under various assumptions regarding the dividend process. That is, **West's test rejects the hypothesis of no bubble in the Hang Seng Index.**

Table 2. Bubble Test Statistics of Monthly Hang Seng Index given by West's Test

Number of lags (q)	Full Sample (1974:07 - 2002:05)	Sub-sample I (1974:07 – 1987:10)	Sub-sample II (1987:11 - 2002:05)
1	20.8591 (0.0000)	33.4414 (0.0000)	4.6808 (0.0346)
2	30.5946 (0.0000)	40.5850 (0.0000)	21.3567 (0.0000)
3	55.9970 (0.0000)	61.9187 (0.0000)	36.1397 (0.0000)
4	60.8079 (0.0000)	63.2459 (0.0000)	37.9346 (0.0000)

Note: The bubble statistic follows $\chi^2(q+1)$ distribution. The p-values are given in brackets.

The results of the co-integration test based on Diba and Grossman's approach are given in Table 3. Stock prices and dividends are neither co-integrated in the full sample nor in the sub-samples. **The lack of significant co-integration results under this approach suggests that bubbles may exist in the Hong Kong equity market.**

Table 3. Co-integration Test of Diba and Grossman's Approach

Data	Full Sample (1974:07 - 2002:05)	Sub-sample I (1974:07 - 1987:10)	Sub-sample II (1987:11 - 2002:05)
Trace Statistic	9.8594 (0.3032)	8.1206 (0.4514)	11.6821 (0.1729)
Number of Observations	330	155	175

Note: The p-value in brackets is derived from Osterwald-Leunum (1992).

Table 4 reports the results of the duration dependence test based on McQueen and Thorley's approach. The maximum likelihood estimates of the log-likelihood function parameters α and β for the positive run of test are reported. Although the estimated β for the full sample and the first sub-sample of monthly excess returns are negative, which is an indication of the existence of bubbles, results of their likelihood ratio tests are insignificant. Different from the previous two approaches, **the null hypothesis of no bubble in the Hong Kong stock prices cannot be rejected.**[8]

Table 4. Test for Duration Dependence

		Number of Returns	α	β	Likelihood Ratio (p-value)
Monthly	Full Sample (1974:07 - 2002:05)	78	0.0811	-0.0306	0.0110 (0.9474)
	Sub-sample I (1974:07 – 1987:10)	36	0.2364	-0.3930	0.9817 (0.7200)
	Sub-sample II (1987:11 – 2002:05)	44	0.0287	0.5157	1.1547 (0.6795)
Weekly	Full Sample (3 Jul 74 – 29 May 02)	359	-0.1133	0.1138	0.7773 (0.7679)

Note: The likelihood ratio test statistic follows the $\chi^2(1)$ distribution. The p-values are given in the brackets.

[8] Based on a different study period, Chan, McQueen and Thorley (1998) finds similar conclusion for the Hong Kong stock market.

IV. CONCLUSION

Despite the importance of the asset price volatility on the real economy, there are very few systematic studies on testing for bubbles in the Hong Kong equity market. This paper attempts to fill the gap by examining this issue using three alternative approaches in the rational speculative bubbles literature. The results from the specification test by West and the co-integration test by Diba and Grossman are similar and confirm the existence of asset price bubble in the Hong Kong stock market. In contrast, the results of the duration dependence test developed by McQueen and Thorley do not reject the null hypothesis of no bubble.

The mixed results from these tests shed light on the long-standing debate of whether monetary policy should be applied to prick asset price bubbles at an early stage. Given the difficulty in the identification of bubbles in the asset market, any pre-emptive monetary action to deter bubble formation will entail risks of accentuating market swings. It follows that the lack of an autonomous interest rate policy in Hong Kong to cope with asset bubbles is not a limitation. Instead of relying on monetary tools to deal with the building up of imbalances in the financial markets, policy should focus on prudential measures to strengthen the resilience of financial institutions and the development of financial infrastructure such as the payment systems to reduce the adverse impact of any bubble phenomena on the financial systems. Efforts to enhance the transparency of the equity market to reduce information asymmetry would also help deter asset bubble formation.

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Appendix I

Technical Details of the Three Approaches Used in Testing for Bubbles

(1) The Specification Test by West (1987)

According to a standard efficient market model, stock prices should be equal to the expected future price plus dividends, discounted at the required return of investors (Brealey and Myers (1981)):

$$p_t = \rho\, E(p_{t+1} + d_{t+1}) \,\big|\, I_t \qquad \text{(A1)}$$

where p_t is the real stock price at time t;

ρ is the constant ex ante real discount rate;

d_{t+1} is the real dividend paid at time $t+1$;

I_t is the information common to traders at time t.

The stock price given in equation (A1) may be solved forward recursively and rewritten as:

$$p_t = \sum_{i=1}^{n} \rho^i E(d_{t+i}) \big| I_t + \rho^n E(p_{t+n}) \big| I_t \equiv p^* + \rho^n E(p_{t+n}) \big| I_t \qquad \text{(A2)}$$

If the transversality condition (i.e. $\lim_{n\to\infty} \rho^n E(p_{t+n}) \big| I_t = 0$) fails, there is a family of solutions to equation (A1) (Blanchard and Watson (1982)). In fact, any p_t that satisfies

$$p_t = p_t^* + B_t, \qquad and \qquad E(B_t | I_{t-1}) = \rho^{-1} B_{t-1} \qquad \text{(A3)}$$

is also a solution to equation (A1) where B_t is by definition a speculative bubble. The check for the no bubble null hypothesis in the West's method is carried out by testing $p_t = p_t^*$ versus $p_t = p_t^* + B_t$.

Specifically, if the Δd_t follows an AR(q) process, West (1987) shows that the two sets of parameters that are needed to calculate the expected present discounted value of the stock's dividend stream can be represented in the following system of equations.[9]

$$p_t = \rho\,(p_{t+1} + d_{t+1}) + \mu_{t+1} \qquad \text{(A4)}$$

$$\Delta d_{t+1} = u + \phi_1 \Delta d_t + \ldots + \phi_q \Delta d_{t-q+1} + \upsilon_{t+1} \qquad \text{(A5)}$$

$$\Delta p_{t+1} = m + \delta_1 \Delta d_t + \ldots + \delta_q \Delta d_{t-q+1} + \omega_{t+1} \qquad \text{(A6)}$$

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9 Two systems are originally proposed for testing for bubble: one is for the case whereby d_t follows an AR(q) process and the other is applied when Δd_t follows AR(q). As the data series of d_t is non-stationary in Hong Kong, only Δd_t is used in this study.

The parameters in this system are estimated by the GMM method, with the variables on the right-hand side of the dividend equation (A5) as instruments.

Based on the cross-equations restrictions from the formulas of Hansen and Sargent (1981) regarding rational expectation, the relationship between two sets of parameter estimates can be derived. That is, the corresponding constraints for the parameters of the above system, $R(\theta)$, are given as follows:

$$0 = m - [\rho(1-\rho)^{-1}\ \Phi(\rho)^{-1} + \Phi(\rho)^{-1} - 1]\ u$$

$$0 = \delta_j - \left\{\Phi(\rho)^{-1} \sum_{k=j+1}^{q} \rho^{k-j}\ \phi_k + [\Phi(\rho)^{-1} - 1]\phi_j\right\} \qquad j = 1, \ldots, q-1$$

$$0 = \delta_q - [\Phi(\rho)^{-1} - 1]\ \phi_q$$

where $\hat{\theta}$ is the estimated parameter vector, $\hat{\theta} = (\hat{\rho}, \hat{u}, \hat{\phi}_1, \ldots, \hat{\phi}_q, \hat{m}, \hat{\delta}_1, \ldots, \hat{\delta}_q)$,

and $\Phi(\rho)^{-1} = \left[1 - \sum_{i=1}^{q} \rho^i\ \phi_i\right]^{-1}$.

Under the null hypothesis of no bubble, $R(\theta) = 0$. A bubble test statistic (BTS) based on the variance-covariance matrix of the system, V, is derived as follows:

$$\text{Bubble test statistic (BTS)} = R(\hat{\theta})' \left[\left(\frac{\partial R}{\partial \hat{\theta}}\right) V \left(\frac{\partial R}{\partial \hat{\theta}}\right)'\right]^{-1} R(\hat{\theta}) \qquad \text{(A7)}$$

Under the null hypothesis, the statistic BTS is asymptotically distributed as χ^2_{q+1}. In other words, if the null hypothesis is rejected by the data by the chi-square test, bubbles are present in the stock market.

(2) The co-integration test by Diba and Grossman (1988)

The co-integration test of Diba and Grossman (1988) does not require a detailed specification of an underlying equilibrium model. Under rational expectation assumption, it is shown that:

$$p_t = (1+r)^{-1} E_t (p_{t+1} + \alpha\, d_{t+1} + \mu_{t+1}) = B_t + F_t \qquad \text{(A8)}$$

where r is the constant real interest rate;

α is a constant that reflects the investor's valuation of expected dividends relative to expected capital gains;

B_t is the bubble component;

μ_{t+1} is a variable that the investor observes but the researcher does not;

$F_t = \sum_{j=1}^{\infty}(1+r)^{-j} E_t(\alpha d_{t+j} + \mu_{t+j})$ is the market-fundamentals component.

Re-arranging the equation (A8) and solving it recursively forward will give:

$$p_t - \alpha r^{-1} d_t = B_t + \alpha r^{-1}\left[\sum_{j=1}^{\infty}(1+r)^{1-j} E_t \Delta d_{t+j}\right] + \sum_{j=1}^{\infty}(1+r)^{-j} E_t \mu_{t+j} \qquad \text{(A9)}$$

If Δd_t and the unobservable variables μ_{t+j} are stationary, and if bubbles do not exist, then stock prices and dividends are co-integrated with the co-integrating vector $(1, -\alpha r^{-1})$. This implies that if prices and dividends are not co-integrated, bubbles may exist provided other unobservable fundamentals are stationary.

(3) The duration Dependence Test of McQueen and Thorley (1994)

The duration dependence test divides excess returns into series of run length where a run is defined as a sequence of excess returns of the same sign with a random length *I*. Here, *I* is a positive discrete random variable generated by the density function f_i = Prob($I = i$) and its corresponding cumulative density function $F_i \equiv$ Prob($I < i$). If N_i is the count of completed runs in the sample, then the density version of the log likelihood is:

$$L(\theta \mid S_T) = \sum_{i=1}^{\infty} N_i \ln f_i$$

where θ is a vector of parameters, S_T is a data set which contains T observations on the random run length, *I*.

Instead of focusing on the unconditional probabilities given by the density function, a hazard function, h_i = Prob($I = i \mid I \geq i$), which is the probability that a run ends at i given that it lasts at least until i, is introduced. The conditional probabilities given by the hazard function are more appropriate for capturing the relationship between the probability that a run continues and the length of the run in the duration dependence test. From the definition, the hazard function is related to the density function by:

$$h_i = \frac{f_i}{(1 - F_i)} \quad \text{and} \quad f_i = h_i \prod_{j=1}^{i-1} (1 - h_j).$$

Using the above relationship, the hazard function version of the log likelihood is:

$$L = \sum_{i=1}^{\infty} \left[N_i \ln h_i + M_i \ln(1 - h_i) \right] \tag{A10}$$

where M_i is the number of runs with a length greater than i.

To perform the duration dependence test, a functional form must be chosen for the hazard function h_i. We follow McDonald, McQueen and Thorley (1993) and choose the log-logistic functional form for h_i, where $h_i = (1 + e^{-(\alpha + \beta \ln(i))})^{-1}$. The choice of this log-logistic function allows us to transform the unbounded range of $\alpha + \beta \ln(i)$ into a (0, 1) space of h_i, the conditional probability of ending a run.

Under the null hypothesis of no bubble, the abnormal returns are at random and there is no duration dependence. That is, $\beta = 0$. If bubbles exist, the probability of a positive run ending should decrease with the run length, that is, $\beta < 0$ for positive run. Tests are performed by maximising the log likelihood function with respect to α and β. The likelihood ratio test of $\beta = 0$ is asymptotically distributed as χ_1^2.

Appendix II

Regression Results of West's Specification Test

Table A1. Regression Results for Equation (A4)

Equation (A4)	$p_t = \rho\ (p_{t+1} + d_{t+1}) + \mu_{t+1}$			
	$q = 1$	$q = 2$	$q = 3$	$q = 4$
ρ	0.9640 (0.0000)	0.9637 (0.0000)	0.9633 (0.0000)	0.9633 (0.0000)
Serial Correlation	0.1486 (0.6998)	0.1491 (0.6994)	0.1490 (0.6995)	0.1490 (0.6994)
Orthogonality	2.2297 (0.4276)	6.5956 (0.0194)	16.8373 (0.0001)	17.1633 (0.0002)
Stability (mid-sample)	0.5339 (0.8249)	0.5749 (0.8778)	0.5978 (0.9203)	0.5903 (0.9623)
Stability (1983:10)	0.0649 (0.9348)	0.0900 (0.9519)	0.1103 (0.9888)	0.1066 (0.9998)

Note: The serial correlation test for residuals refers to the Breusch-Godfrey serial correlation LM test. The statistics for the orthogonality test between the instrumental variables and the residuals follows $\chi^2(q)$ distribution. The stability test is conducted to check whether the regression coefficients is stable (a) between the first half and the second half of the sample and (b) after the linked exchange rate was introduced in October 1983. In both cases, the statistics for the stability test follows $\chi^2(1)$ distribution. The p-value of each statistic is shown in the bracket. The value in "q" refers to the number of lags in other two equations in the tri-variate system. In general, the diagnostic tests show that there is no significant mis-specification problem in the model.

Table A2. Regression Results for Equation (A5)

Equation (A5)	$\Delta d_{t+1} = u + \phi_1 \Delta d_t + \ldots + \phi_q \Delta d_{t-q+1} + \upsilon_{t+1}$			
	$q = 1$	$q = 2$	$q = 3$	$q = 4$
u	0.1496 (0.1287)	0.1765 (0.0696)	0.1794 (0.0626)	0.1964 (0.0399)
ϕ_1	-0.2500 (0.0091)	-0.2921 (0.0016)	-0.2962 (0.0008)	-0.2988 (0.0006)
ϕ_2		-0.1679 (0.0421)	-0.1758 (0.0177)	-0.1935 (0.0074)
ϕ_3			-0.0272 (0.6789)	-0.0569 (0.4289)
ϕ_4				-0.0982 (0.2645)
Serial Correlation	2.4514 (0.1174)	6.1186 (0.0134)	7.8759 (0.0050)	5.6425 (0.0175)
Orthogonality	0.0269 (0.9437)	0.0326 (0.9840)	0.0268 (0.9999)	0.0195 (0.9998)
Stability (mid-sample)	0.0614 (0.9356)	1.9687 (0.6648)	2.0149 (0.7494)	2.6791 (0.7482)
Stability (1983:10)	0.1712 (0.9099)	0.3397 (0.9138)	0.7278 (0.9047)	1.9062 (0.8274)

Note: The serial correlation test for residuals refers to the Breusch-Godfrey serial correlation LM test. The statistics for the orthogonality test between the instrumental variables and the residuals follows χ^2(q) distribution. The stability test is conducted to check whether the regression coefficients is stable (a) between the first half and the second half of the sample and (b) after the linked exchange rate was introduced in October 1983. In both cases, the statistics for the stability test follows χ^2(q+1) distribution. The p-value of each statistic is shown in the bracket. In general, the diagnostic tests show that there is no significant mis-specification problem in the model.

Table A3. Regression Results for Equation (A6)

Equation (A6): $\Delta p_{t+1} = m + \delta_1 \Delta d_t + \ldots + \delta_q \Delta d_{t-q+1} + \omega_{t+1}$

	$q = 1$	$q = 2$	$q = 3$	$q = 4$
m	4.2677 (0.4985)	3.5507 (0.5695)	1.7603 (0.7658)	1.3147 (0.8241)
δ_1	4.7911 (0.0000)	6.6100 (0.0000)	8.5472 (0.0000)	8.6109 (0.0000)
δ_2		7.2559 (0.0000)	10.7086 (0.0000)	11.1611 (0.0000)
δ_3			11.6612 (0.0000)	12.4164 (0.0000)
δ_4				2.4940 (0.0945)
Serial Correlation	0.3426 (0.5583)	0.8801 (0.3482)	1.1289 (0.2880)	1.0017 (0.3169)
Orthogonality	0.7418 (0.7762)	0.6910 (0.8601)	0.6222 (0.9174)	0.6136 (0.9587)
Stability (mid-sample)	2.6303 (0.3338)	8.8304 (0.0147)	10.4123 (0.0206)	10.2512 (0.0419)
Stability (1983:10)	0.1696 (0.9103)	0.6021 (0.8737)	0.7228 (0.9053)	1.0802 (0.9121)

Note: The serial correlation test for residuals refers to the Breusch-Godfrey serial correlation LM test. The statistics for the orthogonality test between the instrumental variables and the residuals follows $\chi^2(q)$ distribution. The stability test is conducted to check whether the regression coefficients is stable (a) between the first half and the second half of the sample and (b) after the linked exchange rate was introduced in October 1983. In both cases, the statistics for the stability test follows $\chi^2(q+1)$ distribution. The p-value of each statistic is shown in the bracket. In general, the diagnostic tests show that there is no significant mis-specification problem in the model.