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EXTRACTION OF US DOLLAR INTEREST RATE EXPECTATIONS FROM DERIVATIVE PRICES

Key Points :

- *The Federal Reserve's monetary policy decisions play a major role in determining economic developments in Hong Kong under the Linked Exchange Rate system. It is therefore of interest to assess financial market sentiment of the likely future course of US monetary policy.*

- *Fed funds futures and Eurodollar futures options, the two most commonly traded interest rate derivatives, contain important information about market participants' expectation of US dollar interest rates.*

- *Market expectation about the probability of changes of the Fed funds target rate can be derived from Fed funds futures prices using a two-step procedure. This probability gives a simple estimate of market sentiment regarding possible changes in the target rate in the coming FOMC meeting.*

- *Estimates of the probability density function (PDF) obtained from Eurodollar futures options, whose underlying asset is the 3-month LIBOR, provides a more complete assessment of market expectation about possible US interest rate changes in the near future. The evidence in this paper shows that the changes in the PDF reflect the changes in the US interest rate outlook triggered by the release of economic news or Fed officials' remarks.*

- *The information derived from Fed funds futures and options on Eurodollar futures is complementary and hence should be used jointly to monitor market sentiment regarding future movements in US dollar interest rates.*

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I. INTRODUCTION

Under the Linked Exchange Rate system, Hong Kong dollar interest rates follow closely their US dollar counterparts. Monetary policy decisions by the Federal Reserve ("the Fed") consequently play a major role in determining economic developments in Hong Kong. This raises the issue of how the HKMA can gauge the future path of interest rates. Recently, a number of studies have attempted to extract interest rate expectations held by market participants from prices of interest rate derivatives.[1] This paper discusses and evaluates methodologies used to extract information from Fed funds futures and Eurodollar futures options, which are the two most commonly traded interest rate derivatives.

The rest of the paper is organised as follows: Section II reviews how to extract market expectations from Fed funds futures prices; Section III examines a method to compute the distribution of interest rate expectations from Eurodollar futures option prices. It is then followed by a discussion on the latest interest rate outlook inferred from these two derivatives. Conclusions are presented in the last section.

II. INFORMATION EMBEDDED IN FUTURES PRICES

a. Fed Funds Futures Market

The Fed funds target rate (TR) is the operating target of the US monetary policy. Through open market operations, the Fed influences the level of the effective Fed funds rate (FF) — the interest rate prevailing in the interbank market for overnight deposits among banks in the US — to ensure that the FF does not deviate much from the TR.[2,3] As the FF follows closely the TR, the Fed funds futures contract, the settlement price of which is based on the FF, has been widely used as a tool to gauge market expectation of future levels of TR.

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1 See Robertson and Thornton (1997), and Coutant, Jondeau and Rockinger (2001) and the references therein.

2 By law, US banks must hold reserves with the banks that make up the Federal Reserve Bank system. Since these reserves do not earn interest, banks have an incentive to lend any excess funds to other banks in need of reserves. These interbank transactions are collectively known as the fed funds market.

3 See Meulendyke (1998).

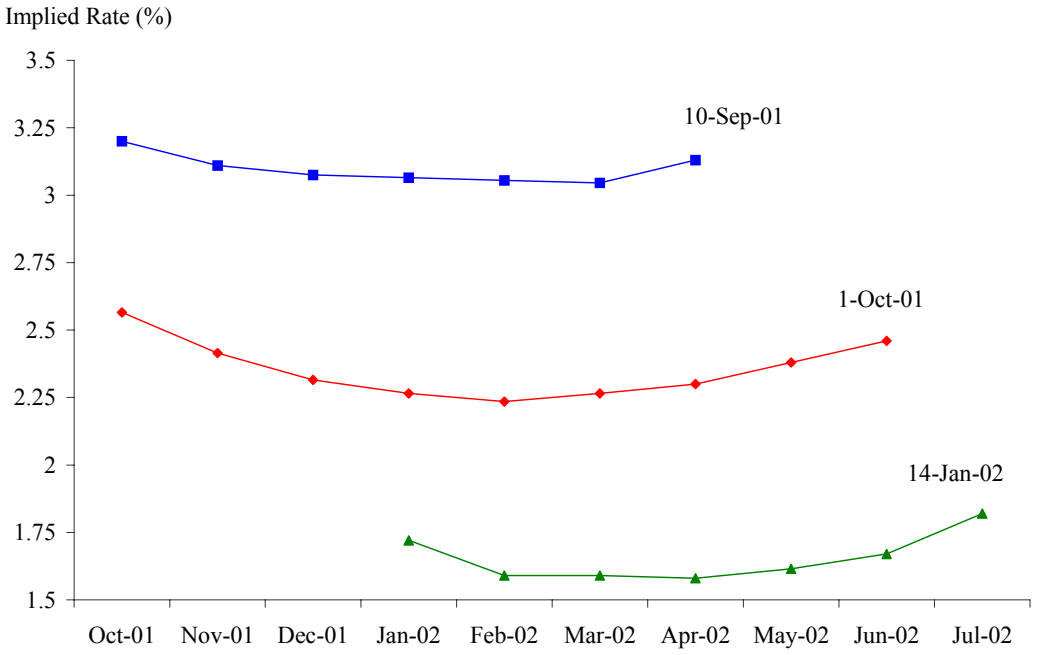
The Fed funds futures contract was first offered by the Chicago Board of Trade in October 1988 to provide a hedging instrument against US dollar interest rate risks. The settlement price of the contract is the simple average of the daily FF of each calendar month as reported by the Federal Reserve Bank of New York. Trading of the contract is fairly active with an average monthly turnover at around 300,000 contracts.

b. Methodology

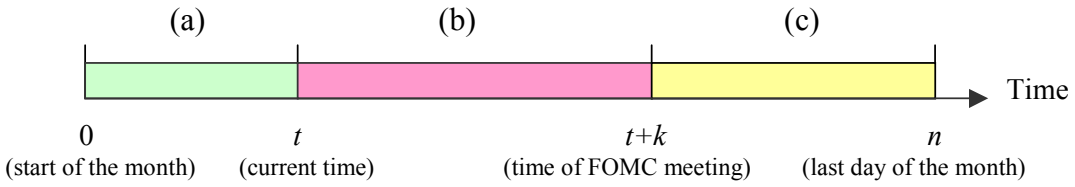
Chart 1 shows the path of Fed funds rate implied by the Fed funds futures prices on different dates.[4] The shape of any given curve provides an indicator of market expectation on a particular day of future movements of interest rates. Day-to-day changes in market sentiment are reflected by shifts of the curves. A comparison of the position and shape of the curves over time can thus be used to monitor changes in the interest rate outlook. As an illustration, Chart 1 depicts the situation before and after the September 11 incident in the US. On 10 September 2001, the implied rates from Fed funds futures reflected market anticipation of an end to the series of interest rate cuts initiated in early 2001 and that the FF would only be cut by another 25 basis points to around 3% towards year-end. After the terrorist attacks on 11 September, however, market expectations shifted dramatically. The implied Fed funds rate on 1 October 2001 showed that the general view was that notwithstanding the significant cuts in interest rates immediately after the September 11 incident, the aftermath effect would continue to weigh on the economy and thus interest rate would remain at a low level in the coming months. Confidence in the US economy continued to decline towards the close of the year and further cuts in interest rates to boost the economy were widely expected. For example, on 14 January 2002, the implied rate of the Fed funds futures contract reflected market expectations of a further drop in the FF to around 1.5% in the first quarter of 2002.

[4] The implied rate of the Fed funds futures contract is derived by 100 minus the Fed funds futures contract price.

Chart 1: Implied Interest Rates from Fed Funds Futures Contracts



Box 1: Derivation of the Expected post-FOMC Federal Funds Target Rate (ETR^{post}) From the Implied Fed Funds Rate



Implied Fed funds futures rate at day t (FFF_t)

$= \frac{1}{n}$ [realised FF_s (segment (a)) + prevailing TR at time t (segment (b))

+ ETR^{post} (segment (c))]

$$= \frac{1}{n}\left(\sum_{i=1}^{t} FF_i + k*(prevailing\ TR\ at\ time\ t) + (n-t-k)*ETR^{post} \right)$$

$$\text{i.e. } ETR^{post} = \frac{n*FFF_t - \sum_{i=1}^{t} FF_i - k*(prevailing\ TR\ at\ time\ t)}{n-t-k}$$

In sum, the gradual downward movement of the implied Fed funds rate curves in late 2001 and early 2002 shows market expectations of continued cuts in interest rates in the light of the weak US economic outlook.

On any given day, the Fed funds future prices can also be used to derive market expectations of the decision in the next FOMC meeting. The derivation can be done in two steps: (1) calculation of expected post-FOMC Fed TR using the current Fed funds futures prices and (2) derivation of the probability of Fed's action using a binomial model, which is defined below.

As shown in Box 1, the current implied Fed funds futures rate (at day t) is an average of the realised effective Fed funds rates between 0 and t (segment (a)) and the expected effective Fed funds rates after day t until the month-end (segments (b) and (c)).[5] As the FF moves closely with the TR, the unrealised FFs can be approximated by the corresponding TRs. While the TR is certain before the FOMC meeting, the post-FOMC TR is, however, unknown at time t. In other words, the pre-FOMC FF (segment (b)) is assumed to be equal to the current target rate while the post-FOMC FF (segment (c)) is assumed to be equal to the expected post-FOMC target rate which corresponds to the probability weighted decision of the FOMC on the Fed funds target rate. Based on these approximations and assumptions, the expected post-FOMC Fed funds target rate (ETR^{post}) at any day t can be calculated from the implied Fed funds futures, the historical FFs and the current TR.

After computing the expected post-FOMC TR, a binomial model is applied in step (2) to assess the probability of two possible outcomes in the next FOMC meeting. This model assumes that the FOMC will decide to adopt one of two possible interest rate levels, that is, an upper or a lower value, 25 basis points apart, compared to the expected level estimated in step (1).[6] Under the model, the expected post-FOMC TR can be expressed as the weighted average of the two possible outcomes. The likelihood of either of these outcomes can be computed after substituting the corresponding interest rates (see Appendix 1 for details).[7]

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[5] The settlement price of the Fed funds futures contract is the simple average of the daily FF of the calendar month.

[6] These upper and lower values are not set with reference to the prevailing target rate. For example, irrespective of the current target rate at 1.75%, the upper and lower values correspond to 2.25% and 2% respectively if the estimated level calculated from step (1) falls within 2% and 2.25%. Since the only two possible outcomes are higher than the current target rate (1.75%), it implies that the market is 100% confident that a rate increase will be effected and the probabilities computed refer to a 25bp increase vs a 50bp increase. Alternatively, by setting the lower value equal to the prevailing target rate and the upper rate being 50bp above it, the probabilities computed will refer to a 50bp increase vs no change in TR.

[7] This method suffers from limitation. First, in deriving the ETR^{post}, it is assumed that the pre-FOMC FFs are identical to the prevailing TR. This is not always true. Second, the Fed funds futures contracts may be used extensively by hedgers resulting in a positive hedging premium (Robertson and Thornton (1997)). Thus, the prices of the futures may not truly reflect the expectation of movement in TR.

The model assumes that market expectations take one of two possible values. In reality, market participants may expect interest rates to be changed to any of a number of values. To address this issue, we next show how to extract information about market expectations regarding US dollar interest rates using option prices.

III. INFORMATION CONTAINED IN THE EURODOLLAR FUTURES OPTIONS

The underlying asset of the Eurodollar futures options is the 3-month LIBOR prevailing on the contract's expiry day.[8] Since longer-term yields are usually taken as the average of current and future expected short-term interest rates, the movement in the 3-month LIBOR is mostly determined by expected changes in the daily Fed funds rate from now till 3 months after the contract's expiry day.[9] Any information about the movement in the 3-month LIBOR extracted from the Eurodollar futures options is therefore useful in inferring the evolution of the market sentiment regarding the future US dollar interest rates, in particular the distribution or the probability density functions (PDFs) of future interest rates.[10]

Various methods to extract the PDFs from options prices have been suggested in the literature. The method of a mixture of lognormal (MLN) is chosen in this study because past research suggested that it is the best one among several common techniques.[11,12] To illustrate how the PDF derived from the MLN method can throw light on the expected future interest rates, changes in PDF during the days around the FOMC meetings held on 6 November 2001 will be examined. The change in US$ interest rate outlook before and after Chairman Greenspan's testimony in US Congress on 24 January 2002 will also be discussed.

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[8] The underlying asset of the Eurodollar futures is the 3-month LIBOR determined by the quotes of a number of randomly selected banks on the settlement day. On the last trading day, the options will be settled against the futures, which represents a 3-month implied forward rate on the contract expiry date. The settlement date for the options contract is set at 11a.m. on the second London bank business day preceding the third Wednesday of the contract month for March, June, September and December. For other months, the settlement date will be three business days prior to the third Wednesday.

[9] Despite their difference in credit risk, the movements of the Fed funds rate and the 3-month LIBOR are highly correlated.

[10] The probability distribution derived assumes that investors are risk neutral. In the real world, investors are likely risk-averse. Despite this short-coming, the risk-neutral PDF is used in the literature because (1) it is easy to estimate and interpret, (2) the true market density function may not differ much from the risk-neutral one, at least for some markets (Rubinstein (1994)), and (3) with the assumption that risk-premium is relatively stable over time, changes in risk-neutral PDF from one day to the other reflect quite well about the changes in market's beliefs about the future outcomes of the underlying asset.

[11] McManus (1999) shows that the MLN is best method to extract the information content of interest rate futures options among common techniques, including jump diffusion, the 4-th order and 6-th order Hermite polynomial approximation and the maximum entropy methods.

[12] Using prices from Eurodollar futures options, Giamouridis and Tamvakis (2001) show that the mixture of lognormal method seems to be more affected by errors in recording and reporting the data. Despite this, the results from both methods are found to be robust.

a. Data

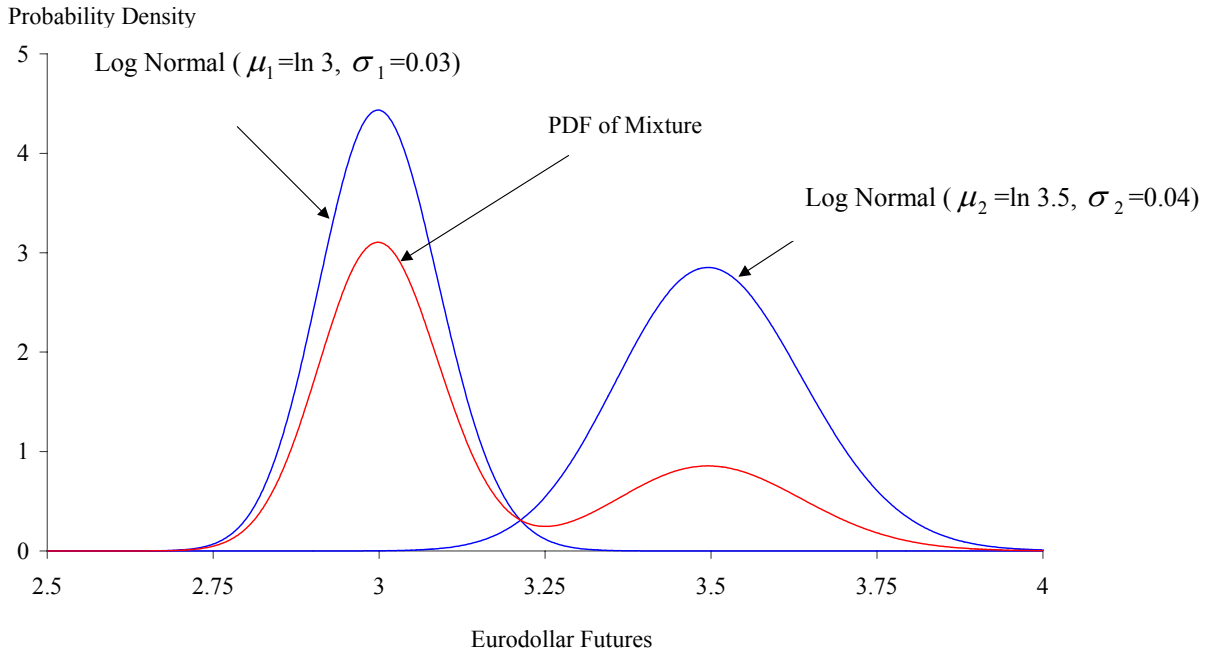
The Eurodollar futures contracts and the Eurodollar futures options contracts traded in Chicago Mercantile Exchange (CME) are used to infer market sentiment regarding US interest rates. These contracts are chosen because the futures are widely used by commercial banks and other financial institutions to hedge their short-term interest rate risk. According to the CME, the Eurodollar futures are "the most liquid exchange-traded contracts in the world when measured in terms of open interest". However, to avoid estimation problems due to insufficient data and potentially "stale" prices from less active months, this study focuses only at the contracts which expire in the quarter-end months. The risk-free interest rate used for discounting is constructed from the Eurodollar rates obtained from Bloomberg.

b. Methodology

The distribution of the standard Black's model for options on futures is assumed to be lognormal. As some of the assumptions in the Black's model (such as that the implied volatility is identical for all strikes) are inconsistent with the empirical evidence, it is suggested that any method to estimate the distribution of the underlying asset should be flexible enough to yield PDFs other than lognormal. By assuming that the PDF is a combination of different lognormal distributions, the MLN method is able to capture the effects of departures from the standard assumptions on the distribution. An example of how the MLN method can produce a distribution different from the standard lognormal is shown in Chart 2. In the chart, the distribution of the underlying is assumed to be a mixture of 70% from one lognormal PDF (with mean = ln 3 and standard deviation = 0.03) and 30% from the other (with mean = ln 3.5 and standard deviation = 0.04). In contrast to the usual unimodal bell-shape distribution assumed by Black's model, the shape of the PDF given by the MLN method is now bimodal, suggesting that the MLN method is capable of producing more realistic distribution when there exist any departures from the standard assumptions.[13]

[13] The shape of the PDF given by the MLN method may be bimodal or unimodal depending on the final estimates of the weight, mean and variance of individual distribution. It should be noted the standard Black's model is a special case of the MLN method. Therefore, it is not surprising if the unimodal distribution can be recovered from the MLN method in particular cases.

Chart 2: An Example of the PDF Derived from a Mixture of Two Lognormal Distributions



In this study, like most others using the MLN method, two lognormals are employed because of data limitation and computational constraints. Thus, the PDF ($q(\tilde{r}(T))$) of the interest rate futures, $\tilde{r}(T)$, on the maturity date is assumed to be a weighted mixture of two lognormal distributions given by:

$$q[\tilde{r}(T)] = \phi\, q_1[(\tilde{r}(T)] + (1-\phi)\, q_2[\tilde{r}(T)]$$

where $q_i[\tilde{r}(T)] = \dfrac{1}{\sqrt{2\pi}\,\sigma_i\,\tilde{r}(T)} e^{-\frac{1}{2}\left(\frac{\log(\tilde{r}(T)) - \mu_i}{\sigma_i}\right)^2}$ for $i = 1, 2.$

and μ_i and σ_i denote the mean and standard deviation of the distribution respectively, $0 \le \phi \le 1$ is the weight assigned to the first lognormal distribution and T is the time to maturity of the option.

Based on this PDF, theoretical prices for interest rates futures and options can be derived. By minimising the sum of squared errors associated with the theoretical and observed prices of options and interest rate futures, the set of parameters of the risk-neutral PDFs, i.e. $\theta = (\phi, \mu_1, \sigma_1, \mu_2, \sigma_2)$, for the future interest rates is estimated. A more detailed description of the estimation method is provided in Appendix 2.

After estimating the parameters of the distribution, the likelihood of a change in the interest rate can be inferred from the cumulative distribution function (CDF), which gives the probability that the futures rate will be less than a specified level when the option expires. The mathematical details for computing the CDF are found in Appendix 2.

c. Results

i) Changes around FOMC meeting

As the focus of this study is on the monitoring of market sentiment, it is instructive to study price movements of the Eurodollar futures options contracts around FOMC meetings. For liquidity reason, the contracts chosen are those to be settled in the quarter-end month immediately following the FOMC meeting. For example, for the FOMC meeting held on 6 November 2001, the option contracts used to derive market expectations expired in December 2001.

In late October 2001, a broad range of economic indicators signalled that the US economy had already been in recession even before the September 11 incident. With further disruption brought by the incident, it was widely believed that the 100 basis point reduction after 11 September was not enough to lift the already sluggish economy out of recession. Prior to the November meeting, the market had already anticipated a rate cut at the meeting although views on the size varied among participants. On 6 November 2001, the Fed finally decided to lower the TR from 2.5% to 2.0%.

Chart 3 gives the evolution of the PDF during 1-7 November 2001 for the December contracts. From the PDFs, the following observations are noted:

- The PDFs for the December contract are in general more disperse prior to the meeting than after, indicating that the market was more uncertain on the interest rate outlook before the FOMC meeting.

- After the meeting, the PDFs shifted slightly to the left with the mean at around 1.8%. There was growing consensus that there would only be a smaller rate cut, if any, in the near future.

Chart 3: PDFs for December 2001 Contract during 1-7 November 2001

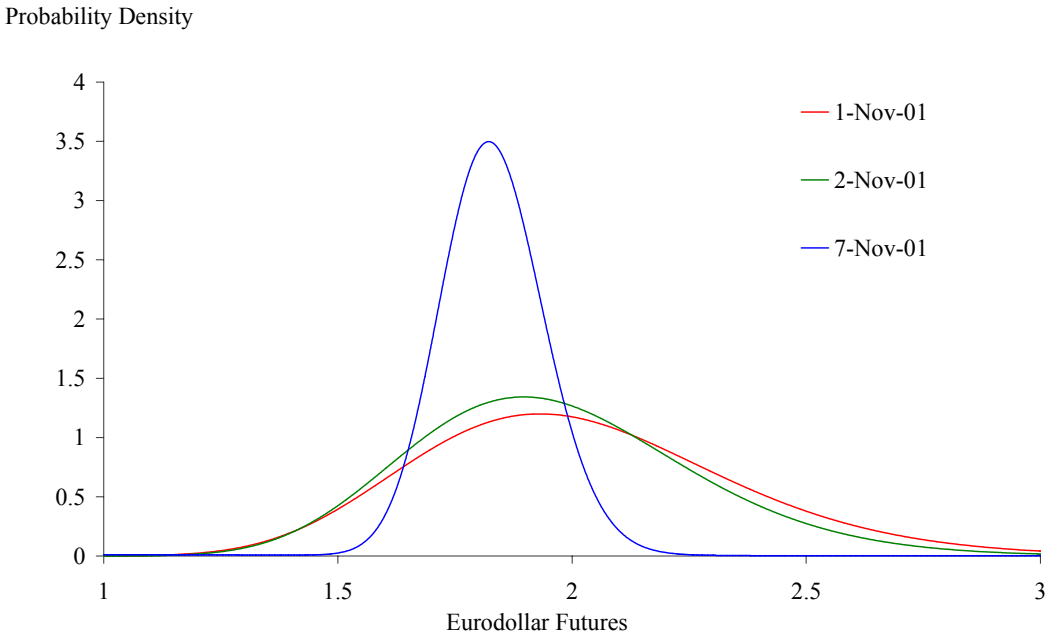
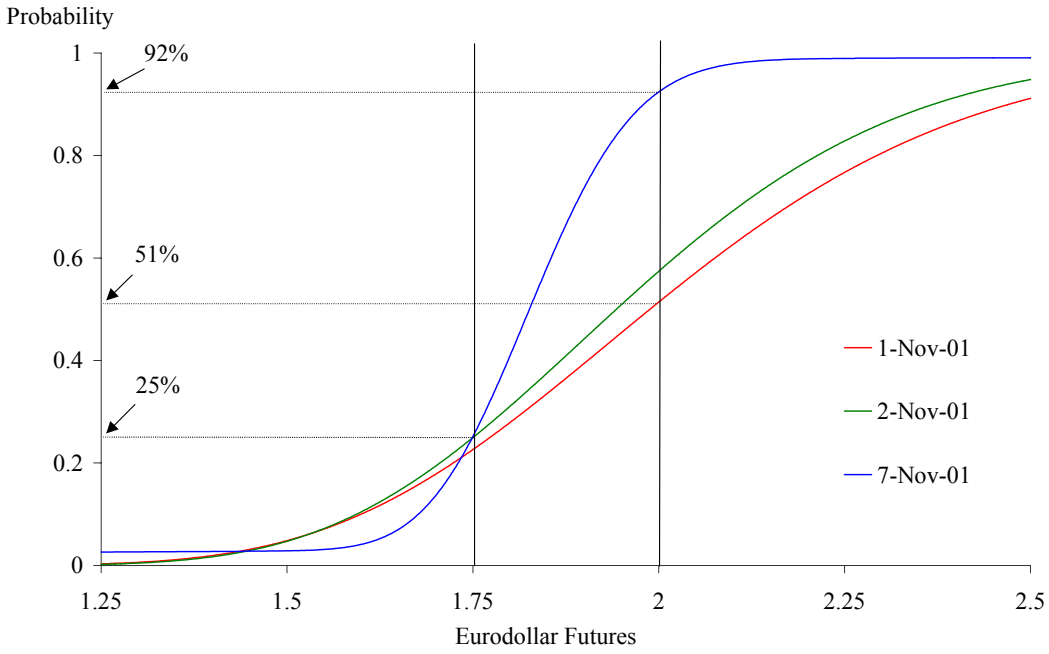


Chart 4 shows the changes in the CDF for December contracts during 1-7 November 2001. During the period, the 3-month Eurodollar rate dropped from 2.18% to 1.98%. Correspondingly, the probability of the interest rate at 2% or below rose from 51% to 92%. Despite this sharp change in sentiment, the probability of an interest rate at or below 1.75% remained largely unchanged at around 25% from 1 November to 7 November. This reflects that though the Fed cut the rate by 50 basis points during its 6 November meeting, the market expected that the interest rate easing cycle would probably be near to the end, as evidenced by little changes in the probability at 1.75% or below before and after the meeting.

Chart 4: CDFs for December 2001 Contract during 1-7 November 2001

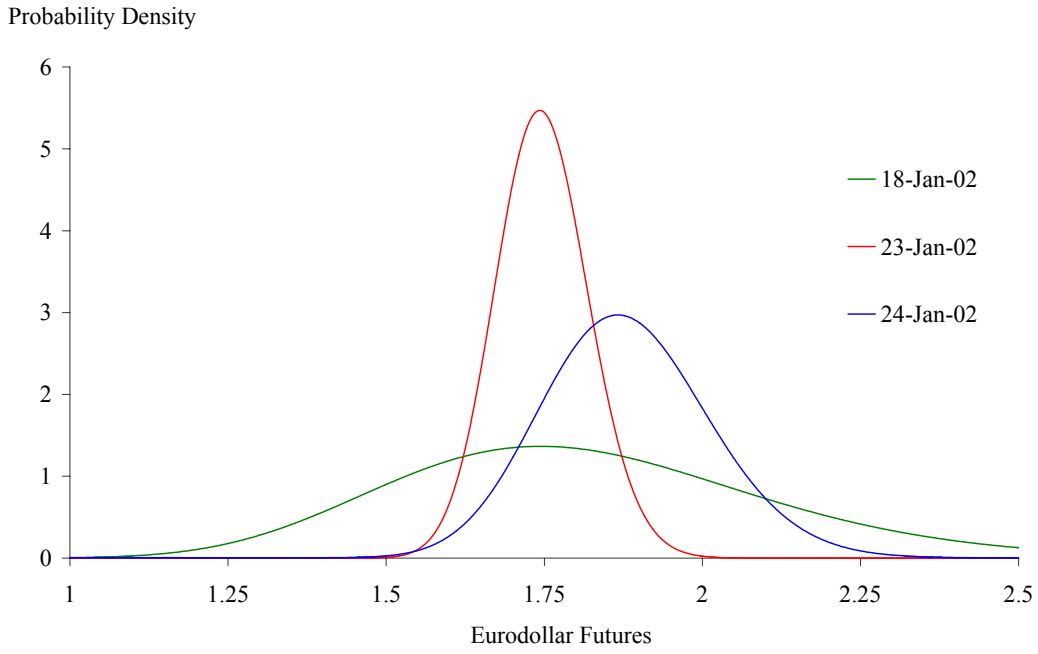


ii) Change in sentiment after Chairman Greenspan's Congressional testimony

On 24 January 2002, Chairman Greenspan testified in Congress and suggested that the US economy was stronger than he had implied in a speech in San Francisco a week earlier. This changed market sentiment on the US interest rate outlook towards tightening bias. Chart 5 shows the evolution of the PDF on selected days before and after Chairman Greenspan's testimony.

The mentioning of the downside risks to economic growth in Chairman Greenspan's speech on 11 January 2002 contrasted with the release of indicators suggesting the bottoming out in the US economy and brought uncertainty to the market. On 18 January 2002, for example, views on interest outlook were diverse and 44% expected the rate to drop further from the current 1.75%. By 23 January, the market emerged with a consensus view that the rate would stay at around 1.75% and zero probability was attached to 1.5% or below after reports on the market misinterpretation of Chairman Greenspan's remarks on the risks in the economy was circulated for a few days. On 24 January, increased likelihood of a rise in interest rate was reflected in the rightward shift of the PDF on the back of a more upbeat remark on the US economy by Chairman Greenspan. Moreover, the distribution also becomes broader, suggesting that some market participants expected rates to rise above 2% in March.

Chart 5: PDFs for March 2002 Contract during 18-24 January 2002



iii) Summary

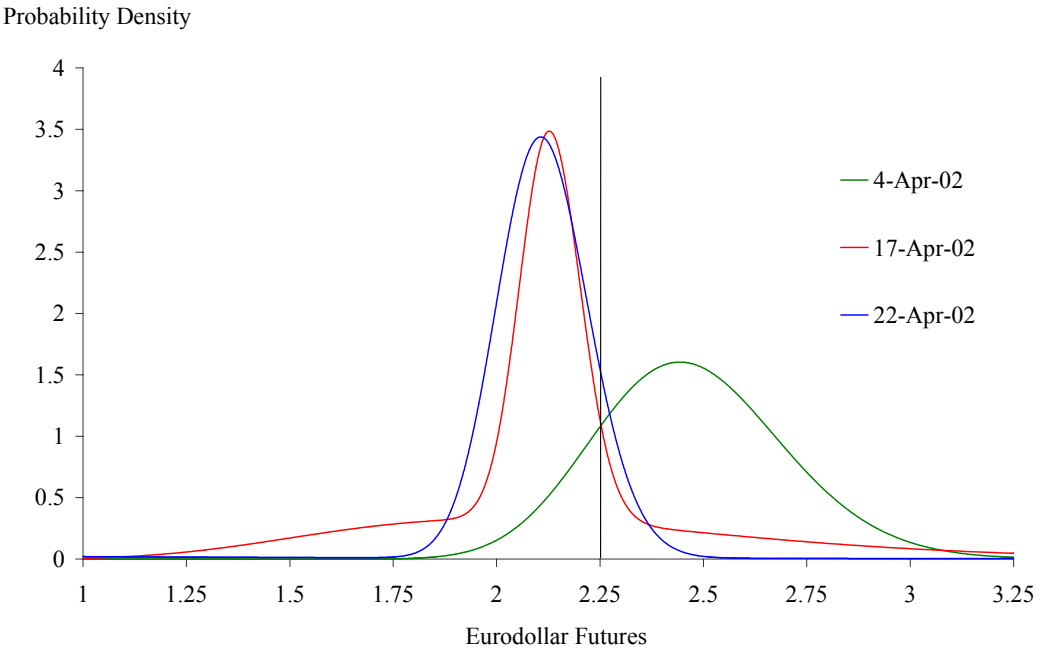
The above results suggest that the PDFs derived from Eurodollar futures options do provide useful information on market sentiment regarding future US dollar interest rates. As the interest rate concerned here is the 3-month Eurodollar futures instead of the TR, which is an overnight rate, the likelihood of a change in the level of the futures rate gives the probability of possible changes in the TRs from now till 3 months after contract's expiry day. Moreover, this method is capable of capturing day-to-day changes in market expectations and the influence of special events such as remarks of senior government officials.

IV. THE OUTLOOK FOR INTEREST RATES

In early April, the general market expectation was for the Fed to start tightening interest rate the latest by June given signs of continuing improvement in the economy. On April 4, for example, the binomial model based on the Fed funds futures prices discussed in Section II gives a probability of 39% for a 25bp rate increase in the May FOMC meeting and a 100% for a rate increase of the same magnitude in the June meeting. On April 17, Chairman Greenspan's testimony to Congress highlighted the

uncertainty about the US economy and the absence of immediate inflationary pressure. The market immediately ruled out the possibility of a rate increase in May. In fact, the probability of a 25bp rate increase in June given by the Fed funds futures prices dropped to 50% on April 18. Similar results are also observed in the June contract of Eurodollar futures options. Chart 6 gives the PDFs estimated by the method in Section III on April 4, 17 and 22. From the chart, it is clear that the interest rate outlook of the market has recently become more bearish when compared to in early April. The probability of a 3-month LIBOR at 2.25% or above in June dropped substantially from 76% on April 4 to 12% on April 22 while that of 2% or above decreased slightly from 90% to 82%. The downward adjustment in the probability so derived indicates that the market has embraced the expectation of a more gradual rate hike in the coming months.

Chart 6: PDFs for June 2002 Contract during 4-22 April 2002



V. CONCLUSION

The above analysis shows that derivatives prices contain information useful for gauging market sentiment regarding US dollar interest rates. However, it is difficult to judge which method is superior in assessing market sentiment as these two studies draw on two different instruments and two different approaches. On the one hand, the Fed funds futures is the closest proxy for the TR. On the other hand, the use of Eurodollar futures options prices may be more informative as it gives the whole distribution of future interest rates. Therefore, these approaches should arguably be used together for monitoring market sentiments regarding future movements in US dollar interest rate.

Appendix 1: Estimation of the Likelihood of the Fed's Action with a Binomial Model

Under the binomial model, the expected post-FOMC TR (ETR^{post}) obtained from the Fed funds futures can be expressed as the weighted average of the two possible outcomes:

$$ETR^{post} = pTR_d + (1-p)TR_u \qquad (A1)$$

where TR_d is the lower bound of the market expectation of the TR^{post};

TR_u is the upper bound of the market expectation of the TR^{post};

p is the probability of the TR being TR_d in the upcoming FOMC meeting.

After rearranging terms in equation (A1), the probability for the TR being TR_d is given as follows:

$$p = \frac{ETR^{post} - TR_u}{TR_d - TR_u}$$

As an example, on 31 May 2001, the expected post-FOMC TR derived from Fed funds futures was 3.80% and the then TR was 4%. Under the binomial model, it is assumed that market views were concentrated in two alternatives:

- TR remains unchanged at 4%.
- A cut in the TR by 25 basis points (bps) to 3.75%.

Thus, the probability that FOMC would cut TR by 25 bps in its 31 May 2001 meeting is estimated as:

$$p = \frac{3.80\% - 4.0\%}{3.75\% - 4.0\%}$$

$$p = 0.80$$

The result indicates a 80% likelihood of a 25 bps cut. As it turned out, the FOMC cut the interest rate by 25 bps in its meeting on 27 June 2001.

Appendix 2: Derivation of the Probability Density Functions from Eurodollar Futures Options Using the Method of Mixture of Lognormals

In this study, the risk-neutral PDF ($q(\tilde{r}(T))$) of the interest rate futures, $\tilde{r}(T)$, on the maturity date is assumed to be a weighted mixture of two lognormal distributions given by:

$$q[\tilde{r}(T)] = \phi\, q_1[(\tilde{r}(T)] + (1-\phi)\, q_2[\tilde{r}(T)] \qquad (A2)$$

where $q_i[\tilde{r}(T)] = \dfrac{1}{\sqrt{2\pi}\,\sigma_i\,\tilde{r}(T)} e^{-\frac{1}{2}\left(\frac{\log(\tilde{r}(T)) - \mu_i}{\sigma_i}\right)^2}$ for i = 1, 2.

μ_i and σ_i are the mean and standard deviation of the distribution respectively, $0 \le \phi \le 1$ is the weight assigned to the first lognormal distribution and T is the time to maturity of the option.

Based on (A2) and noting that the prices of options are equal to their discounted future payoffs, the theoretical prices of European call and put options can be written as:[14]

$$C_\theta(X) = e^{-r_f T}\left\{\phi\left[e^{\mu_1+\frac{1}{2}\sigma_1^2} N(d_1) - XN(d_2)\right] + (1-\phi)\left[e^{\mu_2+\frac{1}{2}\sigma_2^2} N(d_3) - XN(d_4)\right]\right\} \qquad (A3)$$

$$P_\theta(X) = e^{-r_f T}\left\{\phi\left[-e^{\mu_1+\frac{1}{2}\sigma_1^2} N(-d_1) + XN(-d_2)\right] + (1-\phi)\left[-e^{\mu_2+\frac{1}{2}\sigma_2^2} N(-d_3) + XN(-d_4)\right]\right\} \qquad (A4)$$

where N(.) is the cumulative distribution of a standard normal, X is the strike price, r_f is the risk-free interest rate for discounting, and

$$d_1 = \frac{1}{\sigma_1}[\mu_1 + \sigma_1^2 - \log(X)]\ ,\ d_2 = d_1 - \sigma_1$$

$$d_3 = \frac{1}{\sigma_2}[\mu_2 + \sigma_2^2 - \log(X)]\ ,\ d_4 = d_3 - \sigma_2$$

and the theoretical futures price is given by

$$F_\theta = \phi\, e^{\mu_1+\frac{1}{2}\sigma_1^2} + (1-\phi)\, e^{\mu_2+\frac{1}{2}\sigma_2^2} \qquad (A5)$$

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[14] Though the options traded at CME are American style, their theoretical prices are approximated as the European ones because (1) prices of European options are easier to calculate and (2) the difference between the American-style and European-style options is insignificant for Eurodollar futures options with a life less than 1 year (Cakici and Zhu (2001)).

The set of parameters of the risk-neutral PDFs, i.e. $\theta = (\phi, \mu_1, \sigma_1, \mu_2, \sigma_2)$, is estimated by minimising the sum of squared errors associated with the theoretical and observed prices of options and interest rate futures. In other words, the parameters of the risk-neutral PDFs are estimated by solving the following minimisation problem:[15]

$$\underset{\theta}{Min}\left[\sum_{i=1}^{n}[C(X_i)-C_\theta(X_i)]^2+\sum_{j=1}^{m}[P(X_j)-P_\theta(X_j)]^2+(F-F_\theta)^2\right] \qquad (A6)$$

where n is the number of call options;
m is the number of put options;
$C(X_i)$ are the observed prices of call options;
$P(X_j)$ are the observed prices of put options; and
F is the observed price of interest rate futures.

After estimating the parameters, the PDF can be derived by substituting the weight, means and variances in (A2). In addition to the PDF, the cumulative distribution function (CDF) at a particular interest rate R can be obtained as follows:

$$\text{Prob}\,[\tilde{r}(T)\le R]=\phi\, N\left(\frac{\log R-\mu_1}{\sigma_1}\right)+(1-\phi)\, N\left(\frac{\log R-\mu_2}{\sigma_2}\right) \qquad (A7)$$

From the CDF, the likelihood of a change in interest rate can be determined as it gives the probability that the futures rate will be less than a specified level when the option expires.

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[15] It should be noted that the set of parameters estimated in this minimisation problem is often not unique as under the current numerical algorithm, different initial values may yield different local minima instead of global solutions. Provided that these parameters do not violate the obvious assumptions ($0 \le \phi \le 1$ and σ_1, $\sigma_2 > 0$), the parameters with the smallest mean squared error are chosen where the mean squared error (MSE) is defined as:

$$MSE=\frac{1}{n+m-4}\left[\sum_{i=1}^{n}[C(X_i)-C_\theta(X_i)]^2+\sum_{j=1}^{m}[P(X_j)-P_\theta(X_j)]^2+(F-F_\theta)^2\right]$$

where n and m are the number of observed call and put prices.

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