Hong Kong Monetary Authority

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INFORMATION CONTENT OF MONETARY AGGREGATES: AN APPLICATION OF THE P-STAR MODEL

Key Points:

- *This paper examines the information content of monetary aggregates for future inflation in Hong Kong. To this end, we estimate a measure of real money gap (based on the p-star model), defined as the difference between the real money balance and its long-run equilibrium value, in order to test the hypothesis that an excess real money balance implies potential inflationary pressure in the future.*

- *Empirical estimates suggest that both narrow and broad real money gaps contain useful information about future inflation. Nevertheless, the real money gap is not superior to alternative models using the output gap and nominal money growth. In particular, non-nested model selection tests could not differentiate between these indicators in predicting future inflation.*

- *For forecasting purposes, nominal monetary aggregates have the advantage of being available in a more timely fashion. In addition, the output gap and the real money gap require measures of potential output and equilibrium velocity, estimates of which are subject to considerable uncertainties. However, the output gap can play a superior behavioural role in inflation modelling, as it measures demand pressure in the economy.*

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I. INTRODUCTION

Although monetary aggregates do not play a role in guiding policy under the Linked Exchange Rate system, they can still be useful as an economic indicator. This paper examines whether monetary aggregates in Hong Kong contain any information about future inflation. The concept of "real money gap", which refers to the deviation of real money balance from its equilibrium level, is employed for this purpose. There is considerable empirical support for using the real money gap (or equivalently, the price gap as explained later in the paper) in projecting inflation in a number of economies.[1] In particular, it was found to be superior than money growth and output gap in explaining future price changes in the Eurozone. It is of interest to explore whether the real money gap can be usefully applied to Hong Kong.

The rest of the paper is organised as follows. Section II explains the conceptual framework of the real money gap, which is equivalent to the price gap derived from the P-star model introduced by Hallman, Porter and Small (1991). Section III provides estimates of the real money gap based on narrow money (HK$M1) and broad money (HK$M3), and explores the bi-variate relationships between inflation and the real money gaps using correlation analysis and Granger-causality test. Section VI examines whether the real money gaps are useful in explaining future inflation using an empirical model of inflation for Hong Kong, and compare their explanatory power and forecasting performance with those of nominal money growth and the output gap. The last Section concludes.

II. MODEL SPECIFICATION

The P-Star model, introduced by Hallman, Porter and Small (1991), starts by constructing a measure of the long-run equilibrium of prices (P^*) based on the equation for the quantity theory of money. The price gap is defined as the deviation of the actual price level (P) from P^*. This is equivalent to a real money gap, which is the deviation of real money balance from its equilibrium, i.e. a long-run level consistent with the equilibrium velocity and potential output. The derivation of the real money gap and its

[1] See, for example, IMF (1996), Bank of Canada (1996), Gerlach and Svensson (2001).

equivalence to the price gap is shown in Annex A. A positive money gap signifies excess real money balances, suggesting potential inflationary pressure. To investigate the significance of the gap in explaining future price changes, inflation is modelled as a function of the real money gap, in addition to lagged values of inflation rates and inflation differential between Hong Kong and the US. Specifically,

$$(1) \qquad \pi_t = \beta_1 + \sum \delta_j \pi_{t-j} + \beta_2 \left(\pi_{t-1} - \pi_t^{us}\right) + \beta_3 \left(\tilde{m}_{t-1} - \tilde{m}_{t-1}^{*}\right) + \varepsilon_t$$

where

π_t = inflation in Hong Kong (measured as annualised quarter-on-quarter percent change in the composite consumer price index);

$(\pi_{t-1} - \pi_t^{us})$ = inflation differential between Hong Kong and the US;

$(\tilde{m}_{t-1} - \tilde{m}_{t-1}^{*})$ = the real money gap, where $\tilde{m}^{*}$ denotes equilibrium value of the real money balance (in logarithm).

The inflation differential between Hong Kong and the US $(\pi_{t-1} - \pi_t^{us})$ is added as an explanatory variable because inflation rate in Hong Kong should be tied to the US inflation rate in the long run under the Linked Exchange Rate system.[2] The coefficient of the inflation differential term should be negative.

The empirical question for the information content of monetary aggregates is whether β_3 is positive and significant. If so, the real money gap is said to contain useful information about future inflation on top of that provided by past inflation and inflation differential between Hong Kong and the US.

III. ESTIMATES OF REAL MONEY GAPS AND SOME INITIAL STATISTICAL EVIDENCE

Quarterly data, which span from the first quarter of 1985 to the fourth quarter of 2001, are used in the estimation. The composite consumer price index (CCPI) is used as a measure of prices.[3] The output variable is GDP at 1990 prices. Real money is measured by nominal money deflated by the CCPI. All data are de-seasonalised.

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[2] Because of lags in price adjustments, we can write $\pi_t = (1-\alpha_1)\pi_{t-1} + \alpha_1 \pi_t^{us} = \pi_{t-1} - \alpha_1(\pi_{t-1} - \pi_t^{us})$.

[3] Figures have been adjusted for one-off effects such as Rates rebates.

The construction of the real money gap requires estimation of the equilibrium values of velocity (v^*) and output (y^*). Hodrick-Prescott filter is used to this end. Charts 1 and 2 plot the inflation rate (4-quarter growth in the CCPI) and two measures of real money gaps. It appears that real money gaps led the inflation rate in the latter part of the sample period, particularly in the case of broad money. We also plot nominal money growth against inflation, and observe that nominal money growth tended to lead inflation in the recent years (Charts 3 and 4).

The standard Augmented Dickey Fuller (ADF) tests suggest that prices, nominal money, and output (in logarithms) are I(1), and that the real money, output and velocity gaps are stationary. To assess the bivariate relationships, the correlograms between inflation and the real money gaps are computed (Chart 5). For comparison purposes, we also calculate the correlations of inflation with output gap and nominal money growth. Several observations are worth noting. First, in terms of relationship with inflation, it seems that nominal broad money growth has the highest correlation, followed by the output gap. Second, the real broad money gap and the output gap both have a slightly higher correlation at their first lag with inflation than that in other lags, suggesting their quality as a leading indicator. Finally, the output gap and the real money gap are strongly contemporaneously correlated, which is perhaps not surprising as they are related by construction.

Chart 1
Inflation and Real Broad Money Gap

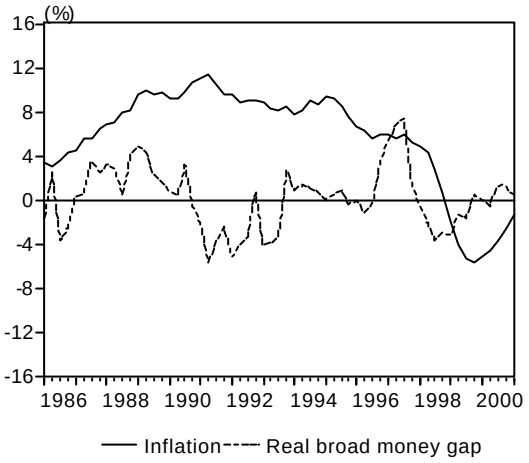


Chart 2
Inflation and Real Narrow Money Gap

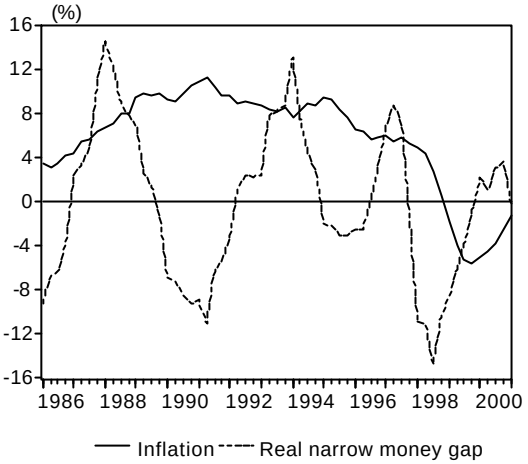


Chart 3
Inflation and Broad Money Growth
(4-quarter % changes)

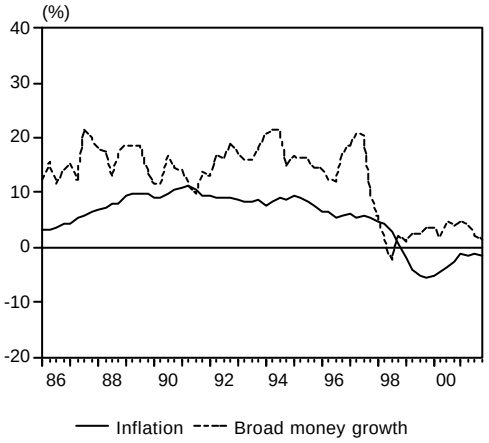


Chart 4
Inflation and Narrow Money Growth
(4-quarter % changes)

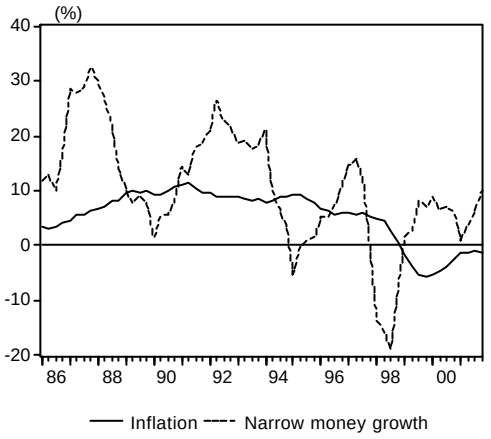
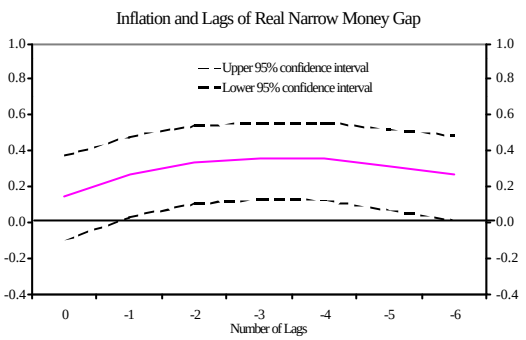
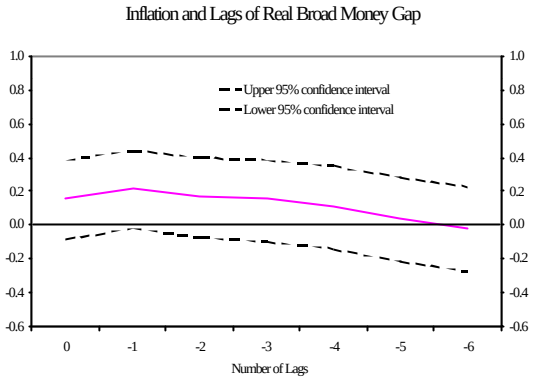


Chart 5. Cross Correlations

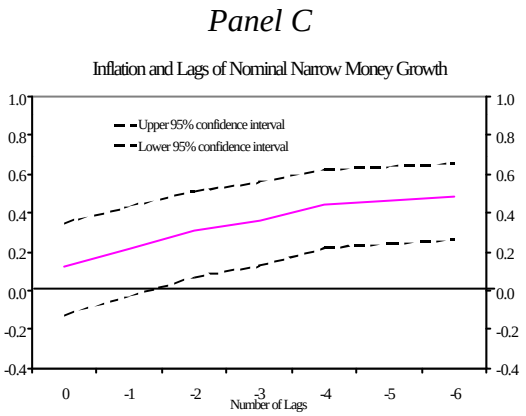
Panel A



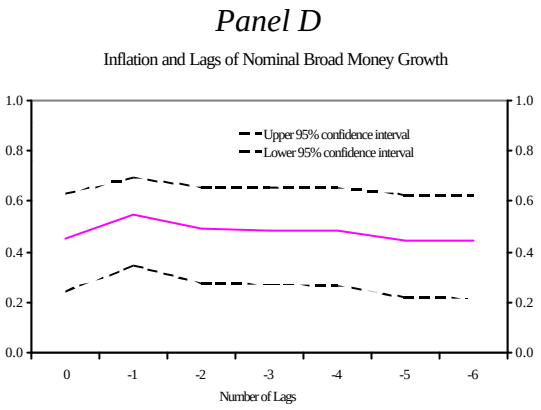
Panel B



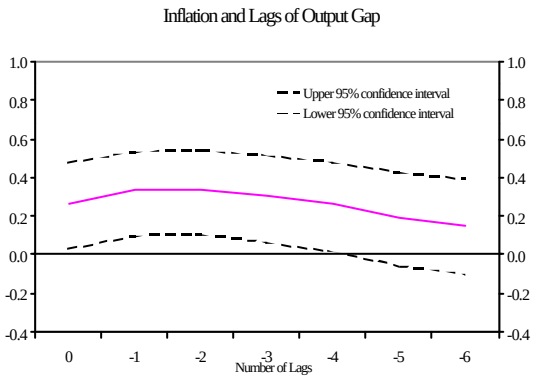
Panel C



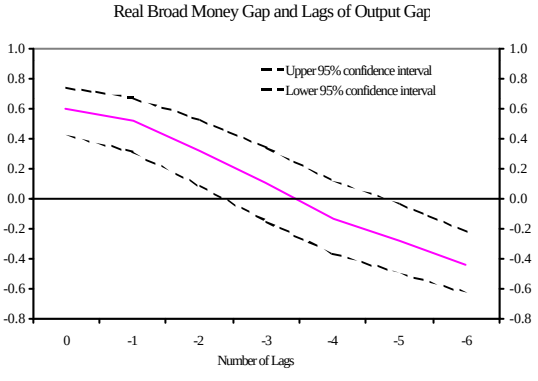
Panel D



Panel E



Panel F



Granger-causality tests were also conducted to ascertain the information content of the various variables. The results suggest that the output gap, nominal money growth and the two measures of real money gap all "granger-cause" inflation (Table 1). In other words, they all provide information about future inflation. However, the bi-variate analysis does not capture effects of other variables, which may have separate and additional information about inflation. The next section provides a more in-depth comparison between these potential indicators of inflation using the empirical framework as set out in equation (1).

Table 1. Granger-causality Tests
(1985:1 – 2001:4)

Null Hypothesis	Obs	Lags	F-statistics	P-values
Output gap does not Granger-cause inflation	61	6	2.85	0.02
Real money gap of HK$M1 does not Granger-cause inflation [(1)]	64	3	3.20	0.03
Real money gap of HK$M3 does not Granger-cause inflation	61	6	2.70	0.02
Nominal broad money does not Granger-cause inflation	61	6	2.51	0.03

[(1)] The null hypothesis that narrow money gap does not Granger-cause inflation was rejected for up to 3 lags.

IV. EVALUATION USING AN EMPIRICAL INFLATION MODEL

Following a "general-to-specific" approach, a parsimonious form of equation (1) was derived for real narrow and broad money gaps respectively (Table 2). A dummy variable is added as test statistics suggest a structural break in the third quarter of 1997, likely due to the effects of the Asian financial crisis. In addition, the test results suggest that the hypothesis that the sum of coefficients of lagged inflation equals to 1 cannot be rejected.

Table 2. Estimation Results
(Period: Q1/1985 – Q4/2001)

$$\pi_t = b_1 + \partial_1 \pi_{t-1} + (1-\partial_1)\ \pi_{t-2} + b_2\left(\pi_{t-1} - \pi_t^{us}\right) + b_3\left(\tilde{m}_{t-1} - \tilde{m}^*_{t-1}\right) + b_4 DM\ + \varepsilon_t$$

Variable	Equation (2)	t-ratios	Variable	Equation (3)	t-ratios
Constant	0.011	2.95*	*Constant*	0.012	3.49*
π_{t-1}	0.698	6.53*	π_{t-1}	0.702	6.68*
π_{t-2}	0.302	6.53*	π_{t-2}	0.298	6.68*
$(\pi_{t-1} - \pi_t^{US})$	-0.242	-3.91*	$(\pi_{t-1} - \pi_t^{US})$	-0.270	-4.38*
$(\tilde{m}1_{t-1} - \tilde{m}1^*_{t-1})$	0.083	2.83*	$(\tilde{m}3_{t-1} - \tilde{m}3^*_{t-1})$	0.215	3.15*
DM	-0.022	-3.40*	*DM*	-0.027	-4.17*
Adjusted R^2	0.905		Adjusted R^2	0.908	
P-value of Q-(2) statistic	0.65		P-value of Q-(2) statistic	0.93	

Notes: * denotes significance at the 1% level.
DM takes the value of 0 for the period before the third quarter of 1997 and 1 thereafter.

All the coefficients are of correct signs and statistically significant. There is no evidence of serial correlation in the residuals, as indicated by the P-value of Q(2) statistics. The estimation results suggest that the real money gap, whether based on narrow money and broad money, contains useful information about future inflation. The goodness-of-fit, as measured by adjusted R^2, is close for the two regressions. The non-nested model selection test (J-test) due to Davidson-MacKinnon (1993) was employed to choose between the two models, and the results suggest that it is not possible to distinguish between them.[4]

[4] The idea of the J-test is that if model A is the correct model, the fitted values from model B should not have explanatory power when included in model A, and vice versa. In our case, the p-values for model (2) against (3) and that for model (3) against model (2) are both around 0.2. As a result, the tests fail to differentiate between the two models.

Further analysis is conducted to investigate whether the real money gap is superior in explaining future inflation than the output gap and nominal money growth. For this purpose, we focus on the real broad money gap as the above results do not prefer one money measure to the other. Specifically, equation (3) is re-formulated by replacing the real money gap with the output gap and money growth alternatively. The results are reported in columns 2 and 3 of Table 3, i.e. (4b) and (4c). They suggest that the output gap and broad money growth are individually significant in explaining future inflation.[5] While adjusted R^2's are slightly higher for the real money gap and the output gap than that for money growth, the J-test does not favour any of the three models against another one.

An alternative way to examine which variable is more informative is to include all of them in a single equation as shown in (4d). Interestingly, only the output gap is found to be marginally significant. When including the real money gap and the output gap in a single equation (4e), neither is found to be significant. This probably reflects the high correlation between the two variables, as noted in the previous section.

[5] It is noted that narrow money growth is not statistically significant when introduced as a regressor in place of the real money gap.

Table 3. Estimation Results
(Period: Q1/1985 – Q4/2001)

Equation	(4a)	(4b)	(4c)	(4d)	(4e)
Constant	0.012 (3.49)***	0.012 (3.54)***	0.001 (0.20)	0.006 (1.11)	0.013 (3.64)***
p_{t-1}	0.702 (6.68)***	0.659 (6.12)***	0.745 (7.03)***	0.672 (6.37)***	0.664 (6.24)***
p_{t-2}	0.298 (6.68)***	0.341 (6.12)***	0.255 (7.03)***	0.328 (6.37)***	0.336 (6.24)***
$(p_{t-1} - p^{us}_{t})$	-0.270 (-4.38)***	-0.281 (-4.51)***	-0.238 (-3.82)***	-0.274 (-4.47)***	-0.283 (-4.61)***
$(\tilde{m}3_{t-1} - \tilde{m}3^{*}_{t-1})$	0.215 (3.15)***			0.065 (0.68)	0.132 (1.55)
$(y_{t-1} - y^{*}_{t-1})$		0.223 (3.18)***		0.150 (1.72)*	0.140 (1.59)
$\Delta m3_{t-1}$			0.062 (2.72)***	0.038 (1.47)	
DM	-0.027 (-4.17)***	-0.026 (-4.06)***	-0.016 (-2.25)**	-0.021 (-2.86)***	-0.027 (-4.24)***
Adjusted R^2	0.908	0.908	0.904	0.912	0.910
P-value of Q-(2) statistic	0.93	0.56	0.87	0.74	0.78

Notes: Δ m3$_t$ denotes annualised rate of quarterly percent change in broad money (in nominal terms).
t-ratios in parenthesis.
***, ** and * denote significance at the 1%, 5%, and 10% levels respectively.

The various models were also compared in terms of the out-of-sample forecasting performance. Specifications (4a) to (4c) were re-estimated for the period Q1/1985-Q4/1999, and one-step-ahead forecasts were made for the period Q1/2000-Q4/2001. The prediction errors, measured by root mean square error (RMSE) and mean absolute error (MAE), are shown in Table 4.[6] The prediction using the output gap seems to outperform with the smallest errors, but the differences among the three are small. In particular, the forecast results based on specifications (4a) and (4b) are displayed in

[6] These two forecast error statistics are used as relative measures to compare forecasts for the same series across different models. The smaller the error, the better the forecasting ability of the model.

Charts 6 and 7 respectively. It seems that both models have similar out-of-sample prediction performance with both actual and forecast values lying within the 95% confidence intervals except for a small outlier in the second quarter of 2001.

Table 4. Forecasting Errors
Estimation: 1985:1-1999:4
Forecast: 2000:1-2001:4

Equation Estimation	(4a)	(4b)	(4c)
Variable used	$(\tilde{m}3_{t-1} - \tilde{m}3^{*}_{t-1})$	$(y_{t-1} - y^{*}_{t-1})$	$\Delta m3_{t-1}$
RMSE	0.021	0.017	0.022
MAE	0.020	0.014	0.021

Note: RMSE = Root Mean Square Error and MAE = Mean Absolute Error.

Chart 6
Out-of-sample Forecast of Inflation
(Based on Model 4a,
with 95% confidence bands)

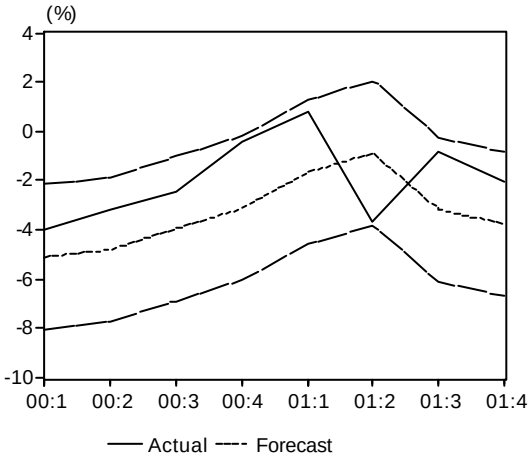
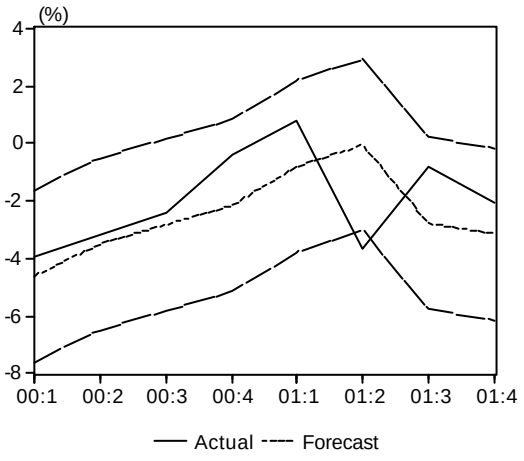


Chart 7
Out-of-sample Forecast of Inflation
(Based on Model 4b,
with 95% confidence bands)



Note: Inflation refers to quarter-on-quarter annualised rate.

In sum, the empirical estimates suggest that the real money gap contains useful information about future inflation, in additional to that provided by past inflation rates and inflation differential between Hong Kong and the US. Nevertheless, its informational value is comparable to that of the output gap and nominal money growth.

V. CONCLUSION

In this paper, we derive a measure of real money gap based on the P-star model. Empirical results suggest that both narrow and broad real money gaps contain useful information about future inflation in Hong Kong. Nevertheless, when compared with alternative models using the output gap or nominal money growth, there is no evidence of a superior performance by the real money gap. Indeed, non-nested model selection tests could not differentiate between money growth, the real money gap and the output gap in explaining future inflation.

For prediction purposes, nominal monetary aggregates have the advantage of being available on a more timely fashion. In addition, the output gap and the real money gap require measures of potential output and equilibrium velocity, estimates of which are subject to considerable uncertainties. Nevertheless, as the output gap measures the effect of demand pressure on inflation, its use would give a behavioural interpretation to the determination of inflation, rather than being purely as an indicator. Thus, for the purpose of analysing relationship between major macro-economic variables, including macro-modelling, the output gap would be the preferred choice.

Annex A

CONSTRUCTION OF REAL MONEY GAP

The P-Star model, introduced by Hallman, Porter and Small (1991), starts by constructing a measure of the long-run equilibrium of prices (P^*) based on the equation for the quantity theory of money. The price gap is defined as the deviation of the actual price level (P) from P^*. Specifically,

(A1) $MV = PY$ i.e. $P = \frac{MV}{Y}$

where M denotes the money stock, V velocity and Y real output.

P^*, the long run equilibrium price level, is defined as

(A2) $$P^* = \frac{MV^*}{Y^*}$$

where V^* denotes the equilibrium velocity and Y^* the potential output. P^* thus refers to a price level that would prevail if the velocity is at its equilibrium level and output at potential. Writing the equation in logarithmic forms (throughout this paper lowercase variables are the logarithms of their upper-case counterparts), the price gap can be decomposed into two components: the velocity gap, $(v - v^*)$, and the output gap, $(y - y^*)$.

(A3) $p - p^* = (v - v^*) - (y - y^*)$

In the original P-star model developed by Hallman, Porter and Small, inflation is related to the price gap in an error-correction fashion:

(A4) $$\pi_t = \pi_t^e + \alpha_1 \left(p_{t-1} - p_{t-1}^*\right) + \alpha_2 z_t + \varepsilon_t$$

where π_t^e denotes inflationary expectations, and z_t other possible exogenous variables. The coefficient of the price gap (α_1) is negative, implying that other things being equal, inflation should accelerate when prices fall below equilibrium.

The price gap can be equivalently expressed as a real money gap, which is the deviation of real money balance from its equilibrium, i.e. a long-run level consistent with equilibrium velocity and potential output. The derivation of the real money gap and its equivalence to the price gap is shown below:

(A5i) $m^* = p^* + y^* - v^*$

(A5ii) $m = p + y - v$

(A5iii) $(\tilde{m} - \tilde{m}^*) = -(v - v^*) + (y - y^*) = -(p - p^*)$

where $\tilde{m}$ denotes real money balances $(m - p)$ and $\tilde{m}^*$ its equilibrium level $(m^* - p^*)$. In other words, a positive money gap is equivalent to a negative price gap, implying an acceleration in inflation. This is in line with the conventional view that excess real money balances are a sign of potential inflationary pressure. Thus equation (A4) can be written as

(A6) $$\pi_t = \pi_t^e + \beta_1 \left(\tilde{m}_{t-1} - \tilde{m}_{t-1}^*\right) + \beta_2 z_t + \varepsilon_t$$

where $\beta_1 = -\alpha_1 > 0$

For empirical investigation, equation (A6) is elaborated to a more general form of equation (1) in the main text.

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