CHAPTER FIFTEEN

Value at Risk

In the very first chapter of this book, we touched on the concept of value at risk.

Several years ago, the concept of value at risk was still relatively new. Since then, it has gained a lot of publicity on both the market and the regulatory fronts.

From the participants' point of view, value at risk is an extremely useful tool for measuring market risk. It summarises the market risk exposure of all financial instruments in a bank's trading portfolio into a single number. Nowadays, if a dealer bank is not using value at risk or other similar methodologies to measure market risk for its trading activities, it will be perceived to be lagging behind the best practice standard.

The regulators also consider value at risk a useful tool in measuring market risk. The Basle Committee has issued an amendment to the 1988 Capital Accord to incorporate market risk in which value at risk is an acceptable and preferred method for determining the required capital level for a bank's trading risk.

In the first chapter, we defined value at risk as "the expected loss from an adverse market movement with a specified probability over a period of time". We also made a simple illustration to demonstrate the concept of value at risk. Although the concept of value at risk is quite simple and easy to understand, the implementation of value at risk is not an easy job. In this chapter, we will introduce some of the most commonly used value at risk approaches and discuss the advantages and shortcomings of using value at risk.

The variance-covariance method

The trading portfolio of a bank usually includes more than one product and currency. It is therefore important to address the correlation factor. A commonly used method is the variance-covariance method.

Under the variance-covariance method, we need to collect the historical volatility data plus one more – the correlation between each pair of assets.

Assume that we have a two-asset portfolio this time, and some of the information about these two assets are as follows:

	Asset A	**Asset B**
Current market price per unit	\$50	\$100
Number of units held	100	100
Total market value	\$5,000	\$10,000
Historical volatility	1.0% (one day)	2.0% (one day)

Assume that we are calculating the market risk capital charge for these two assets using Basle Committee's Quantitative criteria – 99% confidence level (or 2.33 standard deviations for one day) and a 10-day holding period.

Value at risk of A = $5,000 x 2.33 x 1.0% x $\sqrt{10}$ = $368.41

Value at risk of B = $10,000 x 2.33 x 2.0% x $\sqrt{10}$ = $1,473.62

The number 2.33 is the number of standard deviation which corresponds with 99% confidence level. The number $\sqrt{10}$ is the multiplication factor of the required holding period which is 10 days in this case. If the holding period is one year, the multiplication factor is square root of 252 because there are 252 trading days in one year.

The next step is to consider the correlation between these two assets. Again, we need to collect and analyze the historical correlation between each pair of assets. In order to make this analysis meaningful, we need a lot of observations - at least one year's data as required by the Basle Committee. And we need to use the familiar correlation formula:

$$\text{Risk}_{A+B} = \sqrt{R_A^{\ 2} + R_B^{\ 2} + 2C\,R_A R_B}$$

where R_A and R_B are the value at risk of A and B respectively, and C is the correlation between A and B

All correlation numbers lie between –1 and 1. Within this range, there are three critical correlation numbers: 1, 0 and -1. If the prices of these two assets always move perfectly together, we say the correlation is 1. We can simply add the two value at risk numbers together and derive the portfolio value at risk which is 1,842.03.

If the correlation is 0 between these two assets, which means that these two assets are completely independent of each other, the portfolio value at risk is:

$$\sqrt{368.41^2 + 1{,}473.62^2 + 0} = 1{,}518.97$$

If the correlation is -1, which means that they offset each other, the portfolio value at risk is:

$$\sqrt{368.41^2 + 1{,}473.62^2 + 2 \text{ x } (-1) \text{ x } 368.41 \text{ x } 1{,}473.62} = 1{,}105.21$$

Or you can simply subtract $368.41 from $1,473.62.

As we can see, the concept of the variance-covariance method is quite simple. But again, the implementation is quite complicated. We need to measure each asset's volatility and the correlations between each pair of assets based on historical data. For a two-asset portfolio, it is not very difficult. But for a portfolio which has thousands of positions, this is a big job.

The above example is a simple form of variance-covariance method. Each bank would have its own version in applying this method. A commonly used variance-covariance method in the market usually involves several steps:

1. Unbundle different types of risk from each asset or position based on market factors. For example, a foreign bond would be separated into two cash-flow components – principal and interest payment; and each of these two components will be further unbundled by different market factors such as interest rate and exchange rate factors. Therefore, each "cell" would have a cash flow exposed to only one type of market factor.

2. Group the cells with the same (or similar) characteristics together.

3. Calculate the value at risk for each cell group. Some make an assumption that the distribution is normal.

4. Calculate the portfolio value at risk using the variance-covariance matrix.

The historical simulation method

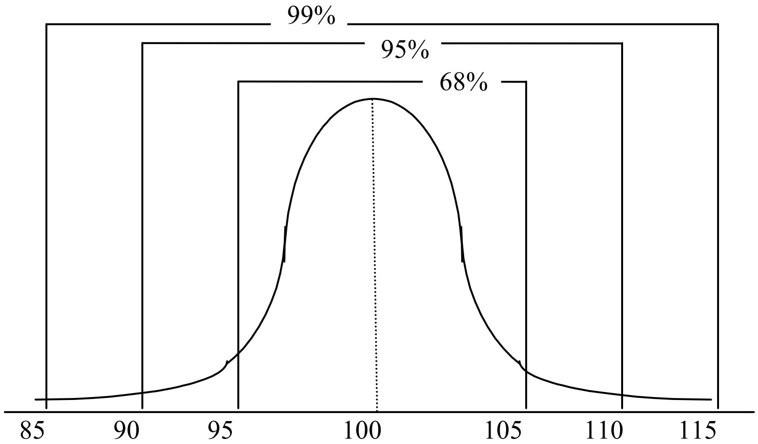
The historical simulation method is based on the assumption that history will repeat itself.

Assume that we have a portfolio of two assets and their current market values are $70 and $30 respectively. Next, we need to "re-value" these assets and come up with some "alternative values" based on their historical price movements. Assume further that we need 100 sets of these alternative values (some people call them observation points) to make the result meaningful.

We observe the market prices of these two assets for the past 100 trading days. Based on these observation points, we can establish a list of alternative values, denoted as V_n and W_n, as follows:

Observed market values V_n and W_n, where n = 0,...,100	Observed portfolio value P_n, where n = 0,...., 100	Δ in observed market value $\Delta P = P_n - P_{n-1}$, where n = 0,...., 100	Alternative value $AV = P_0 + \Delta P$
V_0 = $70, W_0 = $30	P_0 = $100		
V_1 = $68, W_1 = $22	P_1 = $90	ΔP_1 = -$10	$90
V_2 = $70, W_2 = $25	P_2 = $95	ΔP_2 = +$5	$105
V_3 = $73, W_3 = $27	P_3 = $100	ΔP_3 = +$5	$105
V_4 = $71, W_4 = $30	P_4 = $101	ΔP_4 = +$1	$101
etc.	etc.	etc.	etc.
V_{99} = $68, W_{99} = $25	P_{99} = $93		
V_{100} = $67, W_{100} = $30	P_{100} = $97	ΔP_{100} = +$4	$104

Assume that the portfolio's average rate of return and standard deviation are 0% and 5% respectively, and the 100 alternative values are distributed normally as follows:



The distribution indicates that a lot of the alternative values (approximately 68 percent of them) fall in the range between 95 and 105. If we expand the range a little wider, say between 90 and 110, there are approximately 95 percent of the alternative prices falling in this range. If we expand this range wider again between 85 and 115, it covers almost all the alternative prices, 99.7 percent of them.

For value at risk, we are only concerned with the adverse price movement, or the "bad" side of the price range. Therefore, in our example and assuming normal distribution, the one-day one-standard deviation value at risk is $5, one-day two-standard deviation value at risk is $10, and one-day three-standard deviation value at risk is $15. These one-tailed probability numbers correspond to 84%, 97.7% and 99.86% confidence levels respectively.

With the historical simulation approach, the assumption of normal distribution can in fact be relaxed. The 100 sets of daily change in the portfolio value, ΔP_n, can be ranked from the day with the worst performance to the day with the best performance.

Daily profit and loss ranked in ascending order:

Rank	1	2	3	4	5	6	....	96	97	98	99	100
Performance	-$15	-$13	-$12	-$9.5	-$9	-$8.8	....	+$7	+$9	+$9.3	+$13	+$14

A line can then be drawn at the 95th percentile to find the value at risk with 95% confidence level, one tailed. In our example, it would be the fifth worst performance or -$9.

Another advantage of using historical simulation is that since all data is already available, there is no need to care about correlation as they are already embedded in the historical data.

In order to make the calculation meaningful, we need a significant number of observation points or historical data. The Basle Committee requires a minimum data of one year for the market risk capital charge calculation. Sophisticated market participants usually use longer period (three to five years) of historical data. Therefore, the system capacity is also an issue.

The Monte Carlo method

In forecasting the future, there are two distinct approaches. Models that assume a fixed relationship between the inputs and that the inputs lead to an unambiguous result are called deterministic. Models that depend on random or uncertain inputs which provide a distribution of probable results are called stochastic. This may sound a bit abstract. Mark Kritzman gave an excellent example to explain the difference between deterministic process and stochastic process: "a model that predicts an eclipse, for example, is deterministic, because it relies on known fixed laws governing the motions of the earth, the moon and the sun. You are unlikely to hear an astronomer say that there is a 30 % chance of an eclipse next Wednesday. A model that predicts tomorrow's weather, however, is stochastic, because many uncertain elements influence the weather."[1] The observatory is sometimes blamed for telling people that there is only a slight chance of rain but it turns out pouring right at lunch time. The observatory may not have done a poor job. The reason is that the uncertain elements, which the observatory cannot forecast, dominate the result of the forecast in this case.

Monte Carlo simulation involves a stochastic process.

The concept of Monte Carlo method is quite simple. For value at risk in a Monte Carlo simulation, the researcher needs to obtain a series of values from changes in market factors. These values of changes in market factors are then added to the current market value, and thus a series of alternative values are derived just like the historical simulation method. However, the main difference between these two methods lies in the way of obtaining the series of changes in the market factors.

Unlike the historical simulation method, the Monte Carlo simulation method does not rely on past price movements in forecasting future prices. The stochastic process says that the forecast of future prices should be expressed in terms of probability distribution and unaffected by the price one week ago or one month ago. In other words, a company's share price of $241 a week ago has no more meaning than the fact that its price a week ago was $241.

Although we say that the Monte Carlo process does not rely on past price experience to predict the future, it does require the researcher to define certain parameters based on past experience, such as volatility and correlations between market factors.

The Monte Carlo method requires the researcher to derive a value or values through the use of a sequence of random paths which are selected from each of the market factor's *normal distribution*. The condition of normal distribution means that the width of the distribution is based on variance of the market factor. Through this method, the simulation can be done many times and the results of each of the

---

[1] *Mark Kritzman, "About Monte Carlo Simulation", Financial Analysts Journal, November/December 1993.*

simulation processes are unrelated to each other. That is, they follow the independence condition.

The theoretical foundation for stochastic process is that if "we sum or average a group of independent random variables, which themselves are not normally distributed, the sum or average will be normally distributed if the group is sufficiently large."[2]

In layman's term, a researcher simulates many times through a random process to obtain a series of values from changes in market factors in a stochastic process. Although we use the term random, the process actually goes through certain precise mathematical processes. To simplify our discussion, we could think that the research simulates the behavior of the current market price through a random process. In this random process, the current market price goes through many random paths independent of each other under certain defined parameters. By the end, the researcher obtains a distribution of values.

How many times a researcher should repeat the simulation process or how many paths he should use depends on how accurate he wants the outcome to be and how powerful his computer is. Researchers usually use at least one thousand paths in a Monte Carlo simulation process.

But this is just the value at risk of one instrument in the portfolio. If there are 10,000 instruments in the portfolio, the research has to repeat the above process 10,000 times. If 1,000 paths are being used for each instrument, the researcher has to run the simulation 10,000,000 times. Therefore, running a Monte Carlo simulation for value at risk purpose is quite time-consuming and is seldom run on a full portfolio basis. It is usually run on option 1related instruments and on a less frequent basis for management information purpose.

Some pros and cons about value at risk

The financial world is moving towards a direction of increasingly relying on quantitative methodology to manage risk. The value at risk method is based on sound mathematical foundation. Its result can be explained by statistical theories and is more accurate than other traditional risk measurement methodologies.

The most significant benefit of using value at risk to measure a portfolio's risk exposure is that all the exposures in the portfolio can be summarised into one single meaningful number and this number is tied directly to the P/L. Value at risk expresses the risk exposure of a portfolio in terms of how frequent a specific level of potential loss will be exceeded. Management only needs to review the bank's daily value at risk number to understand the risk exposure of the portfolio.

Nevertheless, there are some shortcomings in the various approaches of the implementation of value at risk. Here are some of them:

---

[2] *Mark Kritzman, "About Monte Carlo Simulation", Financial Analysts Journal, November/December 1993.*

1. The variance-covariance models assume portfolio values are normally distributed, but usually they are not. Therefore, using standard deviation to measure potential loss would likely provide an inaccurate result.

2. The variance-covariance method usually applies the delta-gamma method to calculate the value at risk for portfolios which contain options. Delta is a linear measurement and is good only for a small change in the price of the underlying (or market factors). For large changes, delta is not accurate. Ironically, measuring large changes is what value at risk attempts to do. Even with enhanced models which incorporate gamma, "since the change in slope or curvature of the option value curve is not constant for different prices of the underlying, the risk measure including gamma will again provide an inaccurate measure of market risk for larger changes in the price of the underlying."[3] Therefore, if a portfolio has a lot of options, it is more suitable to use Monte Carlo method.

3. Most value at risk models use historical correlations to measure portfolio value at risk and assume that historical correlations are stable. However, historical correlations are not stable among market factors and financial instruments, especially during stress periods.

4. All value at risk models require a very large data base. To handle the calculation, there will be a great demand on the computer system capability, especially for Monte Carlo models. Updating data also requires human resources.

5. The use of different data could also generate different value at risk results. For example, Bank A uses one year of historical data and Bank B uses five years of historical data. Even if they try to measure the same portfolio, the results will be different.

6. Most value at risk models use a one-day holding period to measure the amount of value at risk because market participants expect that financial markets are liquid enough to enable positions to be closed out in one day. This is acceptable in normal situations. But in abnormal situations, a holding period of one day may be too optimistic. In other words, many value at risk models may understate the amount of value at risk. The Basle Committee requires a holding period of 10 days for the purpose of calculating market risk capital. The most convenient way for banks to comply with the Basle requirement is to apply the rule of square root of time - the resultant amount of value at risk times the square root of 10 or 3.1623. But this also has a problem because it implies that during this 10-day holding period, the bank will do nothing to manage this portfolio of positions. In addition, multiplying the square root of time may be acceptable for linear products but it will provide a distorted result for non-linear products such as options.

[3] *Charles Smithson, "Value-at-risk", Risk, February 1996.*

7. Both the historical simulation method and variance-covariance method rely heavily on historical data to forecast future prices. As we all know, the past cannot predict the future, especially under a stress situation.

Any risk measurement methodology is just a tool for management to better measure, understand, control and manage risk exposures. It depends on how management intelligently interprets the results and puts them in proper use. Some market participants even warn that if value at risk is not used with proper stress testing, it will certainly give management a false picture about the institution's true risk exposure because the value at risk method is more suitable for measuring risk exposure in normal circumstances. Also, every risk measurement methodology has its advantages and disadvantages and value at risk is no exception. Therefore, value at risk should be used with other traditional risk measurement methodologies to provide management with a complete set of tools for measuring risk.