CHAPTER 3
Pricing of a Forward Contract and the Yield Curve[1]

Pricing Of A Forward Contract

We are now going to look at the spot and forward price relationship. There are at least two ways to purchase an asset at a future date. One way is to purchase the asset now and store it until the target future date. The other way is to enter into a forward contract that calls for the purchase of the asset on that target future date. In theory, these two methods should lead to the same result. If the results are different, someone (the arbitrageurs) can make a risk-free profit by selling the asset in one market and buying in another simultaneously. This arbitrage activity will continue until the spot and forward prices go back to their respective theoretical levels.

For example, you plan to host a Christmas party one year from now for all your colleagues and you need 100 turkeys. To have all the turkeys you need, you can either:

<u>STRATEGY A</u>

Buy 100 turkeys and pay the market price of turkey (assuming it to be $100 each) in cash today for $10,000 and put them in the refrigerator and store them for one year (assuming there is no storage cost).

<u>STRATEGY B</u>

Enter into a one-year forward contract today to buy 100 turkeys at a total price of S_F and invest some money to make the total principal and interest just enough to pay for the forward contract one year from now. Now the question is: how much should S_F be in order to prevent any arbitrage activity?

In Strategy A, you pay $10,000 to obtain 100 turkeys today. Your $10,000 are gone and you cannot use them again to do anything. But in Strategy B, you do not pay the $10,000 for the turkeys today. You pay one year later. All you have to do now is enter into a forward contract. So what are you going to do with the $10,000? The most logical way is to invest them in some risk-free assets, such as US Treasury Bills.

Assuming that the current risk-free rate of return is 5 percent, compounding annually.

$$S_F = \$10,000 \times (1 + 5\%)$$
$$= \$10,500$$

[1] Part of this chapter was written by Mr Chau Ka-lok and Mr Cedric Wong.

And $10,500 is the forward price which is what you should pay for the 100 turkeys one year from today.

What happens if the forward price is now quoted at $11,000? An arbitrageur can sell the forward contract to you at $11,000, borrow $10,000 from a bank at a rate of 5 percent (assuming he can obtain this risk-free borrowing rate), buy 100 turkeys and store them for one year (again assuming there is no storage cost). After one year, the arbitrageur can make a risk-free profit of $500 by repaying the bank $10,500, deliver 100 turkeys to you and get your $11,000. And if the arbitrageurs keep selling these one-year turkey forwards, the forward price will eventually decline to say, $10,900. At $10,900, the arbitrageurs can still make a risk-free profit of $400 and they will continue to sell these forward contracts. This process continues until the forward price reaches its theoretical level of $10,500.

This is the simplest way to explain the price relationship between spot and forward. Pricing forwards actually is more complicated because storage is usually not free. For arbitrageurs, risk-free borrowing rates are usually unattainable.

Cost Of Carry

In the above example, it was shown that an "equilibrium" will be reached so that the forward price of an asset can be determined. In that example, it is assumed that no cost will be incurred for storing the turkeys for one year. Now the question is: what should the three-month forward price be if it costs $1.00 to store each turkey for one year?

We can use the strategy above and change the forward period from one year to three months. We borrow $10,000 today from the bank at an interest rate of 5% per annum to buy 100 turkeys. For a three-month period we have to pay 5/4 = 1.25% interest. Therefore we have to pay $10,000 X (1+0.0125)=$10,125 back to the bank. At the same time we have to pay the warehouse $100 X 1/4 = $25 for the storage cost. That means it would cost us $10,150 in total. In order to prevent arbitrage opportunities, the forward price would come down until it converges to the price given by this strategy, which is $10,150/100 = $101.50 per turkey.

If the forward price at two years is required, then the amount paid back to the bank has to be calculated with compound interest, that is $10,000 X $(1+0.05)^2$ = $11,025. Adding a storage cost of $1 X 100 X 2 = $200, the forward price is thus $(11,025+200)/100 = $112.25.

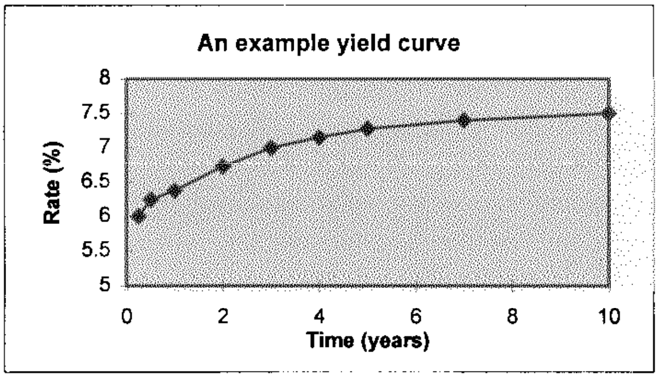
The above illustrates the relationship between the forward price and the spot price (the price of the asset today), and it can be summarised in terms of what is known as the cost of carry. This measures the storage costs plus the interest that is paid to finance the asset less the income earned on the asset. Storage costs may include transportation costs and other costs that are incurred during the period (t). Usually the storage costs and income are expressed in terms of amount per asset (as in the definition of the risk free interest rate r). For a commodity with a storage cost u that is measured as a percentage of the price, the cost of carry is $r + u$. For a dividend paying stock, it is $r - q$ where q is the dividend. The forward price F is then related to the spot price S by

$$F = S(1 + r + u - q)^t$$

The Yield Curve

In the above sections, you have seen the pricing of simple forward contracts relating to physical assets. The same principle can be applied to the pricing of equity related forwards. However, the pricing of interest rate and foreign exchange related contracts is slightly more complicated. Before we move on to interest rate related derivatives, it is important to understand some basics about the markets. The most important concept is the yield curve. Many people working in investment banks are paid "telephone-number-like" salaries just for guessing and playing with the "shapes" of the yield curves and putting in trades in order to benefit from the movements of these curves, so you would appreciate the importance of this.

For a simple definition of a yield curve, though it is arguable, most people would use one of the following: it is a plot of 1) yield of government securities or 2) annual compounded interest rate (zero coupon yield, or the yield of a zero coupon bond) against time. For most currencies, you will see interest rates for maturities of five years or more. An example is given in the graph below.



This graph is sometimes referred to as the "Term Structure of Interest Rates".

In many markets, zero coupon rates of less than one year in maturity are quoted by different banks and brokers. However, rates for longer than one-year maturity are not quoted. They are usually obtained from the prices of other traded instruments, for example interest rate futures and interest rate swap rates. From these prices, the equivalent zero coupon bond yields of the different maturities are calculated. This process is known as "stripping" the yield curve.

How could we know what the level of the rates should be in a few years time? We need to have some reference interest rates to start with. In most big countries, the government would issue some debt instruments to borrow money from the public. These instruments have fixed coupons and have a wide range of maturities. As expected, the US government is the biggest issuer of debt instruments, and the Treasury bills and bonds have maturites ranging from one week to thirty years. Other government usually issue instruments of shorter maturities. The Hong Kong Monetary Authority (HKMA) has recently issued some bonds with maturity in ten years, which give indication of the interest rates up to ten years. From these bond yields, the market would establish the rates of the different traded instruments, usually at a "spread" above the bond yields.

Theories Of Term Structure Of Interest Rates

Most of you would have come across time deposit accounts. The big banks usually offer fixed term deposits from one week up to one year and offer different interest rates for different fixed periods. After deregulation, these interest rates are affected by the market rates, so this is very close to the actual yield curve. (In reality, the deposit rates offered by banks are slightly higher than the Hong Kong dollar yield curve. This reflects the credit risk of the banks.) It is common knowledge that in most circumstances the longer the fixed period, the higher the interest rate. In fact, historically, yield curves are mostly upward sloping, which means that long-term interest rates are higher than short-term interest rates. People have been trying to explain this phenomenon and come up with three popular theories:

Unbiased Expectations Theory

This theory proposes that the forward rate represents the average opinion of the expected future spot rate (or short-term interest rate) for the period in question. For example, if today's one-year interest rate is 6% and it is expected that this one-year interest rate will rise to 8% in one year's time, then this situation would be reflected in today's two-year rate. There should be no difference between a) we invest the money for a fixed one-year period and re-invest this amount with interest for another year; and b) invest the money for a fixed two-year period. Given the interest rates above, the two-year rate today could be calculated by:

$$(1+r) \times (1+r) = (1+0.06) \times (1+0.08)$$

which gives $r = 6.995\%$. In other words, this theory suggests that, if today's two-year rate is 6.995%, it implies that the marketplace (that is, the general opinion of the investors) believes that the one-year rate would rise to 8% in one year's time.

Market Segmentation Theory

Under the theory, different investors and borrowers are supposed to be restricted by law, preference, or custom to certain maturities and they do not switch maturities. There need be no relationship between short-, medium-, and long-term interest rates. The short-term interest rate is determined by supply and demand in the short-term bond market, the medium-term interest rate is determined by supply and demand in the medium-term bond market, and so on. However, this theory is not as popular as the other two theories as it does not directly explain why yield curves are upward sloping more often then downward sloping.

Liquidity Preference theory

In some ways this is the most appealing theory. It argues that forward rates should always be higher than expected future short-term interest rates. The basic assumption is that, as an investor, you would probably like to put money in a short-term fixed period account (if the interest rate offered is the same as that of a longer-term fixed account) because it would not tie up the money for too long in case you need it. Borrowers, on the other hand, usually prefer to borrow at fixed rates for longer periods of time. For example, if the bank offers a relatively low mortgage rate today, you would probably want to arrange a fixed rate mortgage for say the next 10 years so that you would be certain that you get a good offer for a long period of time. In the absence of any incentive to do otherwise, this is how people would behave, i.e. investors would deposit their money for short time periods, and borrowers would choose to borrow for longer periods. Banks would then find themselves financing substantial amounts of longer-term fixed rate loans with short-term deposits. This would involve a high degree of interest rate risk. In practice, in order to match depositors with borrowers and avoid the risk, banks raise long-term interest rates relative to expected future short-term interest rates. This reduces the demand for long-term fixed rate borrowing and encourages investors to deposit their money for long terms.

This theory leads to a situation that long rates are higher than the expected future short-term rates. It is consistent with the empirical result that yield curves tend to be upward sloping more often than they are downward sloping.

These three theories or the combination thereof provide the fundamental framework explaining the term structure of interest rates.