



## ON THE ESTIMATION OF THE OUTPUT GAP OF HONG KONG

Prepared by Michael Cheng, Lorraine Chung and Ip-Wing Yu<sup>1</sup>  
Research Department

### Abstract

Estimation of the output gap is important in assessing the domestic inflationary pressures. In this study, we assess several output gap estimation methods for the Hong Kong economy, including: (1) the production function approach; (2) the Hodrick-Prescott (HP) filter; (3) the Kalman filter; and (4) the IMF multivariate filter, based on a number of criteria, including the robustness to revisions, and the predictability of future inflationary pressures. Overall, the IMF filter is found to perform relatively well on most criteria, particularly on the robustness to revisions. Nevertheless, the edge of the IMF filter in forecasting inflationary pressures is still unclear when compared with other approaches. Thus, it may still be worthwhile to supplement the IMF filter concurrently with other methods, to facilitate our monitoring of inflationary pressures in Hong Kong.

JEL classification: D24, E31, E32, O53

Keywords: Potential Output, Output Gap, Inflation

Authors' Email Addresses: [mkscheng@hkma.gov.hk](mailto:mkscheng@hkma.gov.hk); [lstchung@hkma.gov.hk](mailto:lstchung@hkma.gov.hk);  
[ip-wing\\_yu@hkma.gov.hk](mailto:ip-wing_yu@hkma.gov.hk)

The views and analysis expressed in this paper are those of the authors, and do not necessarily represent the views of the Hong Kong Monetary Authority.

<sup>1</sup> The authors would like to thank Dong He and Lillian Cheung for their valuable comments. All remaining errors are the authors' own.

*EXECUTIVE SUMMARY:*

- *The HKMA currently uses the production function approach to estimate the potential output and the output gap in our monitoring of inflationary pressures in Hong Kong. With such approach in use for more than ten years, there is a need to review it, and to consider whether other methods developed in the recent literature could improve our estimates.*
- *This paper considers several estimation methods of the potential output and the output gap for the Hong Kong economy, including: (1) the production function approach; (2) the HP filter; (3) the Kalman filter; and (4) the IMF multivariate filter. We assess these output gap estimates based on whether they possess the following desirable features: (1) consistency with economic priors; (2) transparency; (3) capability of providing information regarding the uncertainty surrounding the estimate; (4) robustness to revisions over time; and (5) informative about future inflationary pressures.*
- *The IMF filter compared favourably against other methods on most criteria, particularly on the robustness to revisions. Nevertheless, other methods, such as the Kalman filter, can yield an output gap estimate that is more informative about the upcoming inflationary pressures. As the edge of the IMF filter is not so clear when it comes to forecasting inflationary pressures, it is still desirable to supplement the IMF filter with other methods, to facilitate our monitoring of inflationary pressures in Hong Kong.*
- *With that said, all approaches indicate that there is a sharp turn in both the sign and the size of the output gap during the Asian Financial Crisis and the Global Financial Crisis, while the gap was also estimated to be sizably negative during the SARS period as well. Finally, most approaches indicate that the output gap had turned positive since 2010 Q4, and had picked up to a moderate level in the first half of 2011.*

## I. INTRODUCTION

The HKMA currently uses the production function approach to estimate the potential output and the output gap of Hong Kong (see Ha and Leung, 2000). Under such approach, the growth in the potential output is driven by that of the factor inputs within an accounting framework. An advantage of such approach is that it can identify the source of medium to long-term growth, which can help to inform debates on economic policies. The estimated output gap was also helpful in assisting our monitoring of Hong Kong's inflationary pressures in the past.

Nevertheless, the current approach has already been in use for more than ten years. With the recent development of numerous methods to estimate the output gap in the literature, it is about time to conduct a thorough review of different techniques in potential output estimation and identify possible areas of improvement on HKMA's current approach.

This paper considers and assesses several estimation methods of the potential output and the output gap for the Hong Kong economy, including: (1) the production function approach; (2) the Hodrick-Prescott (HP) filter; (3) the Kalman filter; and (4) the IMF multivariate filter.<sup>2</sup> We evaluate these methods against a number of criteria, including the robustness to revisions, in the sense that the output gap estimate at each point in time would be less subject to revision later on when more data become available over time, and the information content of the output gap estimate about upcoming inflationary pressures.

Our assessments suggested that the IMF filter is relatively advantageous comparing with other methods. In particular, the IMF filter can provide a relatively consistent estimate of the output gap, with the magnitude subject to revisions over time being comparatively smaller. Nevertheless, other methods, particularly the Kalman filter approach, can yield an output gap estimate that can be more informative about future inflationary pressures. Thus, it may be desirable to supplement the IMF filter with other methods, in order to grasp more accurately the inflation development in Hong Kong.

The rest of this paper is organised as follows. Section II reviews the various estimation methods, including their rationales and possible weaknesses. Section III discusses estimates of the potential output and the output gap under different methods. Section IV assesses different methods, including their reliabilities and information contents. Section V concludes.

---

<sup>2</sup> More elaborative method would use Dynamic Stochastic General Equilibrium (DSGE) model (e.g. Coenen, Smets and Vetlov, 2009). We will examine this method in future studies.

## II. METHODOLOGY

We now briefly review the four methods that are used in this paper to estimate the potential output, namely (1) the production function approach; (2) the HP filter; (3) the Kalman filter; and (4) the IMF multivariate filter.<sup>3</sup> & <sup>4</sup>

### (1) Production function approach

The HKMA currently uses the production function approach to estimate the potential output of Hong Kong, with the production function assuming to be in Cobb-Douglas form:

$$Y_t = A_t K_t^{1-\alpha} E_t^\alpha \quad (1)$$

where  $Y_t$  is the real output;  $A_t$  is the total factor productivity;  $K_t$  is the capital stock<sup>5</sup>,  $E_t$  is total employment; and the parameter  $\alpha$  is the labour share, setting to be 0.65.<sup>6</sup> The potential output is then estimated as:

$$\bar{Y}_t = \bar{A}_t K_t^{1-\alpha} \bar{E}_t^\alpha \quad (2)$$

and

$$\bar{E}_t = \bar{L}_t (1 - NAIRU_t) \quad (3)$$

where  $\bar{Y}_t$  is the potential output;  $\bar{A}_t$  is the HP-filtered total factor productivity;  $\bar{E}_t$  is the trend employment;  $\bar{L}_t$  is the HP-filtered labour force; and  $NAIRU_t$  is the non-accelerating inflation rate of unemployment (NAIRU), estimated using the methodology of Peng, Cheung and Fan (2001). As the data used are in annual frequency, the resulting estimate of the potential output in annual frequency is interpolated to quarterly frequency using the quadratic match-sum method of EViews.

---

<sup>3</sup> The source of our data set is summarised in Annex I.

<sup>4</sup> El-Ganainy and Weber (2010) also apply these methods to estimate the potential output of Armenia.

<sup>5</sup> Data on the capital stock are obtained by the perpetual inventory method, with annual depreciation rates of machinery and equipment, and building and construction setting to be 0.07 and 0.03 respectively. For more details, see Ha and Leung (2000).

<sup>6</sup> We note that the labour share is commonly set as the value of compensation of employees as a percentage of GDP. In the case of Hong Kong, this would yield a value of 0.49 for the labour share. This estimate, however, may require further adjustments, as part of the income of those self-employed are also labour income (e.g. Krueger, 1999 and Sarel, 1997), while the contribution of the public sector also needed to be taken care of, as the labour and capital shares only make sense with regard to the market sector of the economy (see Batini, Jackson and Nickell, 2000). Making the suggested adjustments would yield estimates of the labour share in the range of 0.46 to 0.59. With all that said, the estimate of potential output growth and output gap turned out not to be particularly sensitive to the value of the labour share, and so we choose to maintain the value of 0.65 in this study.

While this approach has been in use for more than 10 years, and has fared well as an indicator of the impending inflationary pressures in the past, we believe that there is still room for further improvement. For instance, a fluctuation in the labour force can be driven by either a fluctuation in the working age population, or the labour force participation rate. However, this distinction is completely neglected in the current approach. Moreover, the current approach assumes that all fluctuations in the labour input are on the “extensive” margin, rather than on the “intensive” margin. In other words, the fluctuation in the labour input is solely due to individuals’ change of employment status, with changes in individuals’ hours of work playing no role at all, which is too simplistic an assumption. Thus, we augment the current production function approach with information from the labour force participation rate and hours of work per employee. It is important to emphasise that such augmentation is entirely conventional in the literature, and have been implemented in studies of other countries.<sup>7</sup> With that said, the “new” production function that we propose can be represented as<sup>8</sup>:

$$Y_t = A_t K_t^{1-\alpha} (E_t H_t)^\alpha \quad (4)$$

which comparing with equation (1), has an extra term  $H_t$ , the hours of work per week. The potential output is now estimated as:

$$\bar{Y}_t = \bar{A}_t \bar{K}_t^{1-\alpha} (\bar{E}_t \bar{H}_t)^\alpha \quad (5)$$

and

$$\bar{E}_t = \bar{N}_t \bar{P}_t (1 - NAIRU_t) \quad (6)$$

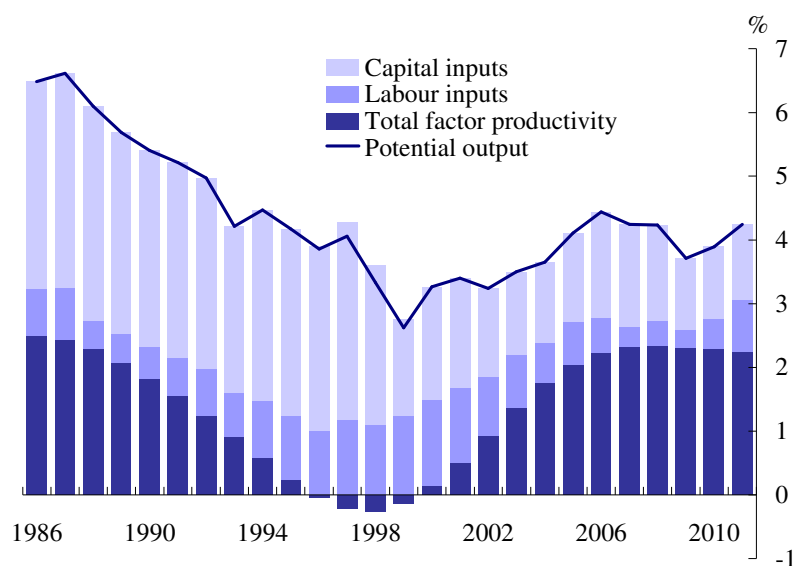
where  $\bar{N}_t$  is the working age population,  $\bar{P}_t$  is the HP-filtered labour force participation rate and  $\bar{H}_t$  is the HP-filtered hours of work. The resulting growth decompositions are presented in Chart 1:

---

<sup>7</sup> For example, see Sun (2010).

<sup>8</sup> It is also common in the literature to include the manufacturing capacity utilisation rate in the production function, to represent variations in the intensity of using capital stock in the manufacturing sector. Data on the manufacturing capacity utilisation rate are however not available in Hong Kong. That said, Hong Kong is a service-based economy, and the manufacturing sector accounted for only about 2% of GDP. Therefore, the capacity utilisation rate should not be an important determinant of the potential output.

**Chart 1: Decomposition of potential output growth<sup>9</sup>**



Source: Staff estimates

As shown in Chart 1, Hong Kong's potential output growth in the past was mainly driven by capital accumulation and productivity growth, while the contribution of labour inputs was comparatively moderate. That said, the contribution of capital accumulation showed signs of diminishing over time, probably reflecting the increasing service-oriented nature of the economy, which relies less and less on capital input. On the other hand, the contribution of productivity growth was somewhat pro-cyclical, moving closely with the output fluctuation over the business cycle.

An advantage of using the production function approach, as just demonstrated, is that it allows the potential growth to be decomposed into growth of factor inputs. This will be useful to an improved understanding of the sources of economic growth and to inform policy debates on options to increase potential output growth through targeting the factor inputs, for example, by raising the total factor productivity. Nevertheless, the production function approach has its own shortcomings. To derive the potential output, we have to rely on the HP filter to smooth the factor inputs. Thus, any pitfall of the HP filter will be carried forward to the production function approach. On the other hand, the method also depends on the estimate of the NAIRU. This is not ideal, as there may be some endogenous interactions between the potential output and the NAIRU, if they are not estimated simultaneously.

<sup>9</sup> The potential output growth for 2011 is a projected value, based on assumptions that the growth/level of the factor inputs in 2011 will converge towards their respective average over the past 10 years.

(2) **HP filter**

The HP filter has been applied extensively in the literature as a convenient method of estimating the potential output, and is based on the following algorithm:

$$\text{Min} \sum_{t=1}^T \{\ln Y_t - \ln \bar{Y}_t\}^2 + \lambda \sum_{t=2}^{T-1} \{[\ln \bar{Y}_{t+1} - \ln \bar{Y}_t] - [\ln \bar{Y}_t - \ln \bar{Y}_{t-1}]\}^2 \quad (7)$$

where  $Y_t$  and  $\bar{Y}_t$  are the real and potential output respectively, and  $\lambda$  is the smoothing parameter, setting to be 1,600 with quarterly data.

While being easy to implement as well as more transparent, the HP filter is a pure statistical technique, in which economic theory does not play any role at all in determining the potential output. Also the filter itself will become imprecise at the end of the sample, in the sense that estimates will be subject to non-negligible revisions, when new data become available. Practitioners can mitigate this end-point bias by using forecast data to extend the sample, but this simply transform the problem of the end-point bias to the problem of whether the forecast data are reliable.

(3) **Kalman filter**

The functional form of the Kalman filter that is used to estimate the potential output in this paper is based on Gerlach and Yiu (2004), which in turn extended the work of Clark (1989) and Watson (1986). As Gerlach and Yiu (2004) argue, this filter stems from a time series model, and so may perform well in many economies with potentially different time series behaviour of output.

Under Gerlach and Yiu (2004), the potential output  $\bar{Y}_t$  is assumed to follow a random walk process with a drift term  $\mu_{t-1}$ :

$$\ln \bar{Y}_t = \mu_{t-1} + \ln \bar{Y}_{t-1} + \varepsilon_t \quad \text{with} \quad \varepsilon_t \sim N(0, \sigma_\varepsilon^2) \quad (8)$$

The drift term is also assumed to follow a random walk process, as setting the drift term to be a constant may be too restrictive:

$$\mu_t = \mu_{t-1} + \varepsilon_t^\mu \quad \text{with} \quad \varepsilon_t^\mu \sim N(0, \sigma_\mu^2) \quad (9)$$

On the other hand, the output gap  $y_t = \ln(Y_t / \bar{Y}_t)$  is assumed to follow an AR(2) process, as Gerlach and Yiu (2004) argue that higher order terms are likely to be insignificant, and it would be difficult to estimate the model in the presence of superfluous parameters:

$$y_t = \varphi_1 y_{t-1} + \varphi_2 y_{t-2} + \varepsilon_t^y \quad \text{with} \quad \varepsilon_t^y \sim N(0, \sigma_y^2) \quad (10)$$

This set of equations is then casted in a state-space form, and is estimated using the maximum likelihood method. The results are summarised in Table 1.<sup>10</sup>

**Table 1: Maximum likelihood estimation results<sup>11</sup>**

| Parameter              | Values          |
|------------------------|-----------------|
| $\varphi_1$            | 1.61<br>[0.00]  |
| $\varphi_2$            | -0.78<br>[0.00] |
| $\sigma_\varepsilon^2$ | 1.32<br>[0.05]  |
| $\sigma_\mu^2$         | 0.01<br>[0.00]  |
| $\sigma_y^2$           | 0.33<br>[0.07]  |

Source: Staff estimates.  
Note: P-values in brackets.

Unlike the production function approach and the HP filter, the Kalman filter is capable of providing information on the uncertainty surrounding the estimate of the potential output and the output gap. On the other hand, similar to the HP filter, the Kalman filter is a pure statistical technique, with no economic relationship embedded. Therefore, information from other variables, for example, inflation and unemployment, are not utilised, which could be unsatisfactory.

#### (4) **IMF multivariate filter**

The IMF filter is a technique recently developed by the IMF to estimate the potential output and the output gap. Unlike the HP filter and the Kalman filter, this filter is built on economic theories and relationships, and so will utilise information in inflation and unemployment to estimate the potential output. As shown by Benes et al. (2010), this filter is also more robust to end-point revision than the HP filter. On the other hand, this filter, unlike the production function approach, will simultaneously estimate both the potential output and the NAIRU. Moreover, this filter can also provide information on the uncertainty surrounding the estimate of the output gap. Thus, a priori, the IMF filter compared favourably against other methods in this paper.

<sup>10</sup> Data on the real output are scaled up by 100 to facilitate estimation.

<sup>11</sup> Gerlach and Yiu (2004) have applied the Kalman filter to estimate the potential output and the output gap in some Asian economies, including Hong Kong. Some differences are found between their estimation results and ours, which can be attributed to the difference in the sample period covered, as well as data revisions.



The IMF filter is based on the following set of equations<sup>12</sup>:

The output gap is defined as:

$$y_t = 100 * \ln(Y_t / \bar{Y}_t) \quad (11)$$

and is assumed to depend on its own lag, and the difference between the year-on-year core inflation rate (excluding rent)  $\pi_{4,t-1}$ , and the long-term inflation expectation  $\pi_{4,t-1}^{LTE}$ .

$$y_t = \rho_1 y_{t-1} - \frac{\rho_2}{100} (\pi_{4,t-1} - \pi_{4,t-1}^{LTE}) + \varepsilon_t^y \quad (12)$$

The rationale of this equation is that any inflation in excess of the inflation expectation will lead to the erosion of competitiveness, which will consequently dampen output.

The inflation rate  $\pi_{4,t}$  is assumed to depend on its own lag, the level and the change in the output gap:

$$\pi_{4,t} = \pi_{4,t-1} + \beta y_t + \Omega (y_t - y_{t-1}) + \varepsilon_t^{\pi_4} \quad (13)$$

The lagged inflation term is used to proxy the inflation expectation. The level of the output gap in equation (13) represents the trade-off of an increased output gap leading to an increased inflation. The change in the output gap is there to capture speed-limit effects due to capacity constraints.

The long-term inflation expectation is assumed to follow a random walk process:

$$\pi_{4,t}^{LTE} = \pi_{4,t-1}^{LTE} + \varepsilon_t^{\pi_{4,t}^{LTE}} \quad (14)$$

The unemployment gap  $u_t$  is defined as the difference between the NAIRU  $\bar{U}_t$  and the actual unemployment rate  $U_t$ :

$$u_t = \bar{U}_t - U_t \quad (15)$$

and the unemployment gap is assumed to be related to the output gap in a fashion similar to the Okun's law:

---

<sup>12</sup> This set of equations incorporates relevant empirical relationships between actual and potential output, unemployment and core inflation within the framework of a small macroeconomic model. We drop the part on manufacturing capacity utilisation from the original model of Benes et al. (2010). This is due to the fact that the manufacturing sector accounts for only about 2% of Hong Kong's GDP. But even including this into the model will not affect much the resulting estimate of the potential output and the output gap.

$$u_t = \phi_1 u_{t-1} + \phi_2 y_t + \varepsilon_t^u \quad (16)$$

The NAIRU is assumed to depend on its own lag, a persistent shock  $G_t^{\bar{U}}$ , the output gap, and the difference between its own lag and the steady state unemployment rate  $U^{SS}$ . The presence of the output gap in the process is to represent a possible hysteresis effect<sup>13</sup>:

$$\bar{U}_t = \bar{U}_{t-1} + G_t^{\bar{U}} - \frac{\omega}{100} y_{t-1} - \frac{\lambda}{100} (\bar{U}_{t-1} - U^{SS}) + \varepsilon_t^{\bar{U}} \quad (17)$$

The persistent shock  $G_t^{\bar{U}}$  is assumed to follow an autoregressive process:

$$G_t^{\bar{U}} = (1 - \alpha) G_{t-1}^{\bar{U}} + \varepsilon_t^{G^{\bar{U}}} \quad (18)$$

The potential output depends on its trend growth rate  $G_t^{\bar{Y}}$  and changes in the NAIRU:

$$\bar{Y}_t = \bar{Y}_{t-1} - \theta(\bar{U}_t - \bar{U}_{t-1}) - (1 - \theta)(\bar{U}_{t-1} - \bar{U}_{t-20})/19 + G_t^{\bar{Y}}/4 + \varepsilon_t^{\bar{Y}} \quad (19)$$

The first difference,  $\bar{U}_t - \bar{U}_{t-1}$ , represents the impact of changes in the NAIRU on the growth rate of potential output via a Cobb-Douglas production function, where  $\theta$  is the labour share. The 19-quarter difference,  $\bar{U}_{t-1} - \bar{U}_{t-20}$ , represents the effect of any induced change in the capital stock on the potential output<sup>14</sup>,

The trend growth rate  $G_t^{\bar{Y}}$  is assumed to converge to its own steady state  $G_{SS}^{\bar{Y}}$  gradually:

$$G_t^{\bar{Y}} = \tau G_{SS}^{\bar{Y}} + (1 - \tau) G_{t-1}^{\bar{Y}} + \varepsilon_t^{G^{\bar{Y}}} \quad (20)$$

The above set of equations is estimated by the Regularised Maximum Likelihood Method<sup>15</sup>, with assumptions on the priors being based largely on that in Benes et al. (2010). The estimation results can be found in Table 2.

<sup>13</sup> The hysteresis effect refers to the possibility that any temporary change in the unemployment rate may become structural and permanent. One explanation for the presence of such effect is that workers who lost their jobs during recession may have their skills becoming obsolete. Thus, workers may have difficulties in finding jobs even if the economy recovers later on.

<sup>14</sup> As set by Benes et al. (2010), the adjustment process of the capital stock would last for 19 quarters, probably reflecting the high persistence that is deemed reasonable for the process.

<sup>15</sup> See Annex II.

As a robustness check, we experiment with alternative assumptions on the prior, and found that the resulting estimates of the potential output and output gap were not significantly affected.<sup>16</sup> We also experiment with an augmented version of the model, which extends the above model by additionally modelling the working age population, the labour force participation rate and labour productivity.<sup>17</sup> We find that the resulting estimates of the potential output and the output gap were also not materially affected.

**Table 2: Estimation results under the IMF filter**

| Parameter                        | Prior |            | Posterior |            |
|----------------------------------|-------|------------|-----------|------------|
|                                  | Mode  | Dispersion | Mode      | Dispersion |
| $G_{SS}^{\bar{Y}}$               | 4.000 | 1.000      | 4.032     | 0.156      |
| $U^{SS}$                         | 4.200 | 1.000      | 4.188     | 0.158      |
| $\theta$                         | 0.650 | 0.100      | 0.651     | 0.016      |
| $\beta$                          | 0.400 | 0.300      | 0.233     | 0.040      |
| $\Omega$                         | 0.500 | 0.300      | 0.297     | 0.041      |
| $\phi_1$                         | 0.800 | 0.300      | 0.788     | 0.046      |
| $\phi_2$                         | 0.300 | 0.300      | 0.176     | 0.026      |
| $\omega$                         | 3.000 | 1.500      | 2.984     | 0.238      |
| $\lambda$                        | 3.000 | 3.000      | 2.898     | 0.465      |
| $\alpha$                         | 0.900 | 0.300      | 0.895     | 0.048      |
| $\tau$                           | 0.250 | 0.300      | 0.199     | 0.037      |
| $\rho_1$                         | 0.800 | 0.300      | 0.797     | 0.042      |
| $\rho_2$                         | 5.000 | 0.300      | 4.946     | 0.475      |
| $\sigma_{\varepsilon\pi 4}$      | 0.500 | 0.300      | 0.721     | 0.036      |
| $\sigma_{\varepsilon u}$         | 0.500 | 0.300      | 0.330     | 0.034      |
| $\sigma_{\varepsilon \bar{U}}$   | 0.100 | 0.150      | 0.126     | 0.023      |
| $\sigma_{\varepsilon G \bar{U}}$ | 0.100 | 0.150      | 0.130     | 0.022      |
| $\sigma_{\varepsilon \bar{Y}}$   | 0.250 | 0.100      | 0.341     | 0.017      |
| $\sigma_{\varepsilon G \bar{Y}}$ | 1.000 | 0.300      | 1.387     | 0.048      |
| $\sigma_{\varepsilon \pi 4 LTE}$ | 0.500 | 0.300      | 0.666     | 0.048      |
| $\sigma_{\varepsilon y}$         | 1.000 | 0.300      | 1.325     | 0.047      |

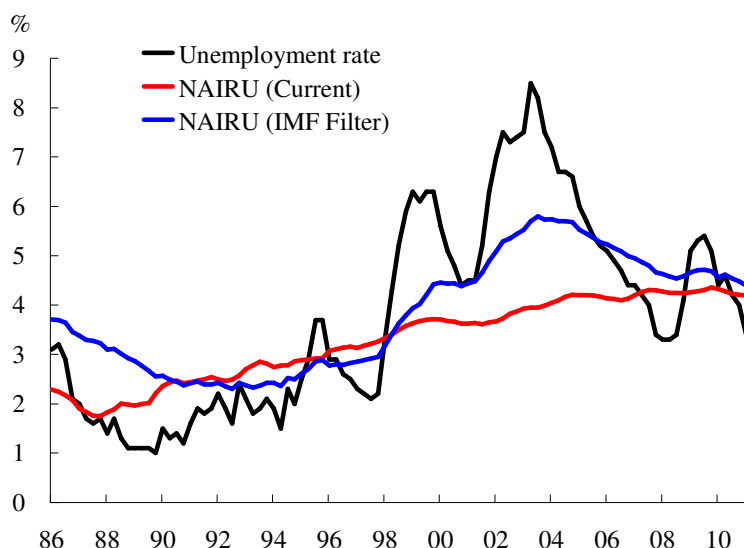
Source: Staff estimates.

<sup>16</sup> It is important to emphasise that the steady state values of the NAIRU and the potential output growth (i.e.  $U^{SS}$  and  $G_{SS}^{\bar{Y}}$ ) will influence the level of these two variables in the short run via the adjustment dynamics (see equations (17) and (20)), and the level at which these two variables will settle in the medium run. Given that the posterior estimate of the steady state for these two variables are close to the priors, it may be desirable to check whether the output gap estimate under the IMF filter are robust to changes in the prior of these two variables. As such, we alter the prior on the potential output growth from 4.0% in the baseline to either 4.5% or 5.0%, as well as the prior on the steady state unemployment rate from 4.2% in the baseline to 3.5% or 4.9%. We find that the resulting output gap estimates under these alternative priors do not differ much from that under the baseline. Thus, our output gap estimate obtained using the IMF filter should be robust.

<sup>17</sup> See Annex III for details about the extended version of the model, which is based on that presented at the EMD workshop of the IMF in April 2010.

On the other hand, as we mentioned earlier, we can also obtain an estimate of the NAIRU using the IMF filter, which we plot in Chart 2 together with the existing HKMA's estimate of the NAIRU, and the actual unemployment rate:

**Chart 2: Actual unemployment rate and the NAIRUs**



Sources: Census and Statistics Department (C&SD) and staff estimates

Comparing with the existing estimate, the “new” estimate of the NAIRU is higher for most of the time, and also displayed a somewhat greater volatility.<sup>18</sup> Nevertheless, both measures suggested that labour market slackness had disappeared since 2010 Q4.

That said, while the IMF filter has a strong economic foundation, it is possible that the structure embedded in the model may have a material influence on the resulting estimates of the potential output and the output gap, and this would require further examination in future studies.

### III. ESTIMATES OF THE POTENTIAL OUTPUT AND THE OUTPUT GAP

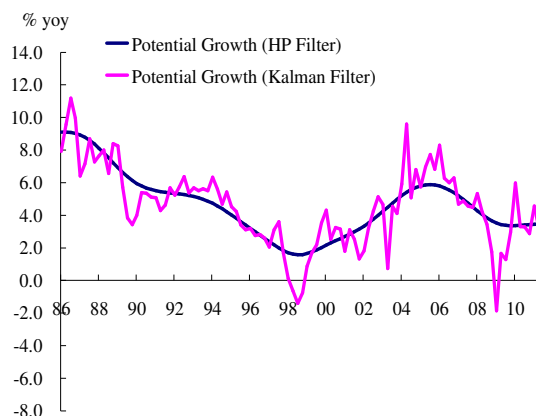
We now discuss our estimates of the potential output and the output gap under different approaches. At the first glance, the following broad trend in the potential output growth rate can be observed in most if not all approaches (Chart 3 and Chart 4): the potential growth rate decreased gradually during 1980s, and bottomed following the outbreak of the Asian Financial Crisis. The potential output growth rate then picked up

<sup>18</sup> The relatively high volatility of the “new” NAIRU estimate is possibly due to the fact that the NAIRU in the IMF filter is modelled as being directly affected by the output gap, and so can be very sensitive to cyclical positions.

again, disrupted briefly by the burst of the dot-com bubble in 2001 and the outbreak of SARS in 2003, before peaking in around 2005-06. Thereafter, potential growth decreased again, particularly so during the outbreak of the Global Financial Crisis in 2008-2009.

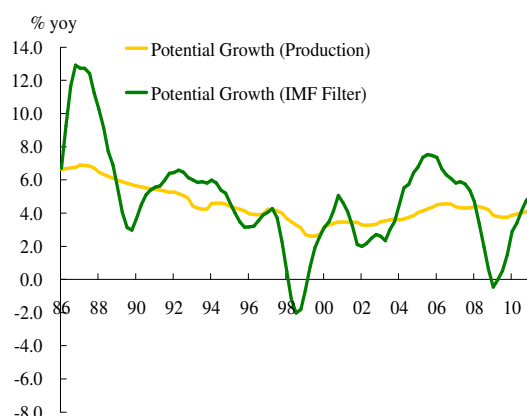
Regarding each approach, the estimated potential growth rate under the HP filter exhibits the smoothest trend, whereas that given by the Kalman filter is the most volatile. On the other hand, the Kalman filter and the IMF filter suggested that the potential output had contracted (on a year-on-year basis) during the Asian Financial Crisis and the Global Financial Crisis. This should not be surprising, as contractions of potential output during financial crisis were not uncommon, and had been documented in other countries (IMF, 2009). Our estimation results at hand suggested that Hong Kong is not an exception.

**Chart 3: Estimates of potential output growth**



Source: Staff estimates

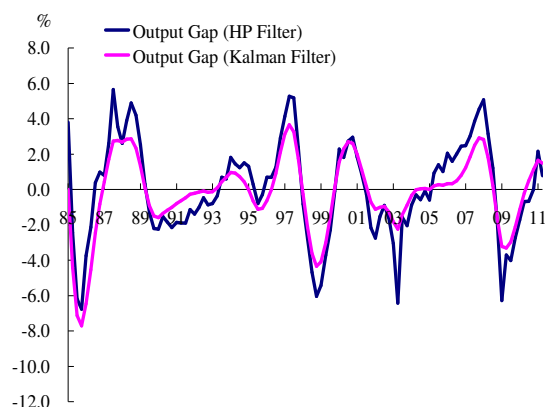
**Chart 4: Estimates of potential output growth**



Source: Staff estimates

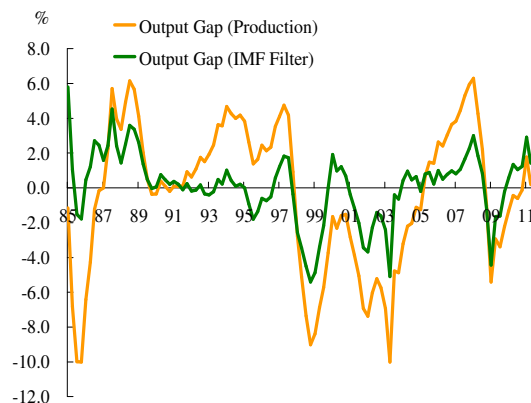
We then compute the output gap, measured as the log difference between the actual and the potential output (i.e.  $y_t = 100 * \ln(Y_t / \bar{Y}_t)$ ). All approaches show that there is a sharp turn in both the sign and the size of the output gap during the Asian Financial Crisis and the Global Financial Crisis (Chart 5 and Chart 6). In particular, the negative gap could have exceeded 6.0% during these crises. On the other hand, the output gap was also estimated to be sizably negative during the SARS period as well, and the magnitude was comparable to that observed during the two financial crises. These results were consistent with the observed sharp drop in the inflation rate during the corresponding periods. Finally, most approaches indicate that the output gap had turned positive since 2010 Q4, and had picked up to a moderate level in the first half of 2011.

**Chart 5: Estimates of the output gap**



Source: Staff estimates.

**Chart 6: Estimates of the output gap**



Source: Staff estimates.

#### IV. COMPARISON OF DIFFERENT METHODS

In this section, we summarise our assessment of the estimation methods, based on four criteria suggested by Cotis, Elmeskov and Mourougane (2003):

- (1) *Consistency with economic priors*: refers to the requirement that methods should be consistent with economic theory.

Both the HP and the Kalman filters are not meeting this criterion, as they are purely statistical-based, and do not have strong economic theory foundation, unlike the production function approach and the IMF filter.

- (2) *Transparency*: refers to the requirement that assumptions made during the estimation process are clearly identified and justified, and estimates should be easily replicated.

The HP filter is relatively transparent in this regard, in the sense that one can easily replicate the estimate, given that data on the real output are easily accessible. On the other hand, other methods would require either demanding estimations, or substantial data inputs, and so one might find it hard to replicate the estimates, meaning that the transparency of other methods may be arguable.

- (3) *Capability of providing information about the precision of the estimates*: refers to the requirement that methods should be able to provide information on the uncertainty surrounding the output gap estimate.

Both the Kalman and the IMF filters are capable of providing information about

the uncertainty (i.e. confidence bands) surrounding their output gap estimates, whereas the HP filter and the production function approach can only give point estimates. Thus, the Kalman and the IMF filters are more favourable in this regard.

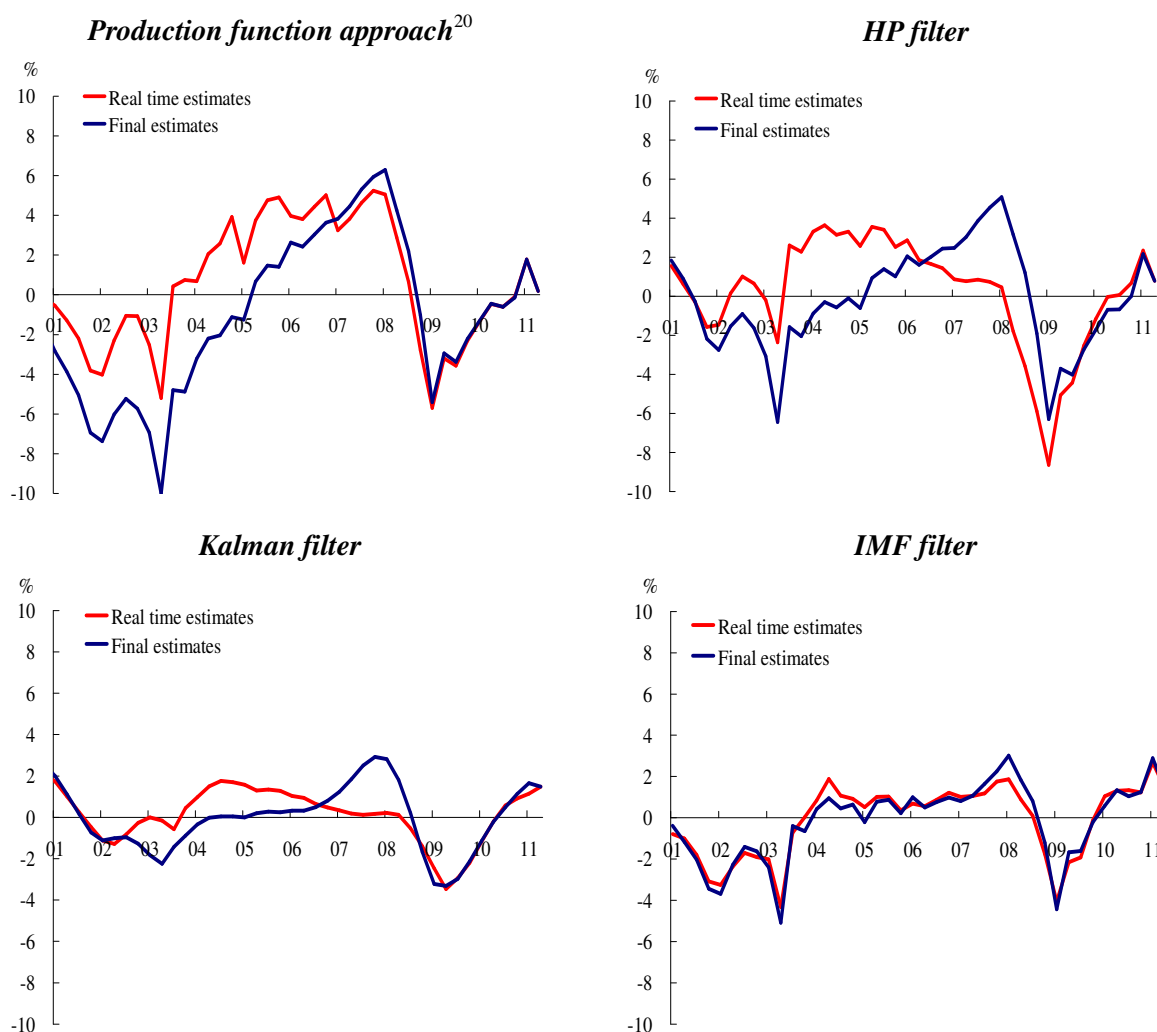
- (4) *Consistency over time (robustness to revisions)*: refers to the requirement that estimates should not be sensitive to the sample period, particularly to the last observation in the sample.

Assessing the methods against this criterion would require further statistical examination. We follow Benes et al. (2010) by comparing, under each method, the nowcast (the real-time estimate), with the final estimate of the output gap that is based on the full sample (Chart 7). To be more precise, we calculate the mean and median of the absolute value of the differences between the nowcast and the final estimate over the past ten years (i.e. 2001 Q1 – 2010 Q4) (Table 3). On the above basis, the IMF filter would require comparatively smaller revisions upon the arrival of new data, followed by the Kalman filter. The HP filter and the production function approach, on the other hand, would require non-negligible revisions, due to the end-point bias as mentioned earlier on.<sup>19</sup>

---

<sup>19</sup> Thus, our results have extended the findings of Benes et al. (2010) that, apart from the HP filter, the IMF filter is also more robust than the Kalman filter and the production function approach to the end point bias.

**Chart 7: Comparison of the real time and the final estimate of the output gap**



Source: Staff estimates.

**Table 3: Absolute revision of end-point estimate**

|                              | <i>Mean</i><br>(Percentage points) | <i>Median</i><br>(Percentage points) |
|------------------------------|------------------------------------|--------------------------------------|
| Production function approach | 2.31                               | 2.02                                 |
| HP filter                    | 2.17                               | 1.96                                 |
| Kalman filter                | 0.89                               | 0.76                                 |
| IMF filter                   | 0.38                               | 0.32                                 |

Source: Staff estimates.

<sup>20</sup> The figures for 2011 Q1 and Q2 are projected values, rather than estimated values.



Thus, on the basis of the four criteria, the IMF filter performs relatively well comparing with other methods (Table 4).

**Table 4: Comparison of different estimation methods**

| Method                       | <i>Desirable feature</i> <sup>21</sup>  |                     |                               |                              |
|------------------------------|---|---------------------|-------------------------------|------------------------------|
|                              | <i>Consistency with economic priors</i> | <i>Transparency</i> | <i>Precision of estimates</i> | <i>Consistency over time</i> |
| Production function approach | ✓                                       | ?                   | ✗                             | ✗                            |
| HP filter                    | ✗                                       | ✓                   | ✗                             | ✗                            |
| Kalman filter                | ✗                                       | ?                   | ✓                             | ✗                            |
| IMF filter                   | ✓                                       | ?                   | ✓                             | ✓                            |

Source: Staff estimates.

That said, any estimate of the output gap also has to be *informative about future inflationary pressures*, in order to be useful for policy analysis. Thus, as an additional criterion to assess the methods, we follow Coenen et al. (2009) and Fueki et al. (2010) in testing the forecasting power of different output gap estimates on inflation, in which we use a bivariate (BV) model with a rolling window of 40 quarters (with an initial sample of 1994 Q1 – 2004 Q4 and a final sample of 2001 Q2 – 2011 Q1).<sup>22</sup>

$$\pi_{t+h}^h = a + b(L)\pi_t + c(L)x_t + \varepsilon_{t+h}^h \quad (21)$$

Where  $\pi_{t+h}^h = 100 * \left( \left( \frac{CPI_{t+h}}{CPI_t} \right)^{\frac{4}{h}} - 1 \right)$  is the annualised h-period inflation rate;

$\pi_t = 400 * \left( \frac{CPI_t}{CPI_{t-1}} - 1 \right)$  is the annualised one-period inflation rate;  $x_t$  is the output gap

estimate under one of the methods;  $CPI_t$  is a price index; and  $b(L)$  and  $c(L)$  are lag polynomials, with lag lengths selected by the Schwartz Information Criteria (SIC). We choose the underlying CCPI (excluding rent) to be the price index for our analysis, as our observations suggest that the CCPI rental component is driven by market rentals of new leases, rather than the output gap.

<sup>21</sup> These desirable features are discussed in great details in Cotis, Elmeskov and Mourougane (2003).

<sup>22</sup> An implicit assumption underlying the BV model is that inflation is I(0). If inflation is instead modelled as I(1), then the BV model should be casted as:  $\pi_{t+h}^h - \pi_t = a + b(L)\Delta\pi_t + c(L)x_t + \varepsilon_{t+h}^h$ . Annex IV of this paper contains the forecasting power test results under this alternative specification of the BV model, which again show that the Kalman filter is relatively more informative about inflationary pressures comparing with other approaches.

Forecasts from the BV model are evaluated at the horizons of one, four and eight quarters ahead, against that from (a) an autoregressive (AR) model, where the lag lengths are again determined by the SIC:

$$\pi_{t+h}^h = a + b(L)\pi_t + \varepsilon_{t+h}^h \quad (22)$$

And (b) a random walk (RW) model

$$\pi_{t+h}^h = 100 * \left( \frac{CPI_t}{CPI_{t-4}} - 1 \right) \quad (23)$$

Forecasts from the models are evaluated by the mean squared forecast errors (MSFE), where the MSFE is defined as:

$$MSFE = \sigma^2 + bias^2 \quad (24)$$

and the variance and the bias terms are defined as:

$$bias = \frac{1}{T} \sum_{t=1}^T e_{t+h}^{h,M} \quad (25)$$

$$\sigma^2 = \frac{1}{T} \sum_{t=1}^T \left( e_{t+h}^{h,M} - \frac{1}{T} \sum_{t=1}^T e_{t+h}^{h,M} \right)^2 \quad (26)$$

and  $e_{t+h}^{h,M} = \pi_{t+h}^{h,M} - \pi_{t+h}^h$ , M stands for one of the models (i.e. BV, AR and RW) and T is the number of forecast points.

Table 5 reports our results, showing the MSFE of the BV model using different output gap estimates, relative to that of the AR and the RW models, with a ratio smaller than one (highlighted in red) indicating that the BV model is outperforming the AR and the RW models at the corresponding horizon:

**Table 5: BV model (output gap in level) - comparison of MSFE**

| <i>h</i> | <i>Relative to</i> | <i>Production function approach</i> | <i>HP filter</i> | <i>Kalman filter</i> | <i>IMF filter</i> |
|----------|--------------------|-------------------------------------|------------------|----------------------|-------------------|
| 1        | <i>AR</i>          | 0.9848                              | 1.0551           | 1.0170               | 1.0022            |
|          | <i>RW</i>          | 0.5924                              | 0.6347           | 0.6118               | 0.6029            |
| 4        | <i>AR</i>          | 1.7584                              | 1.7505           | 1.0385               | 1.1420            |
|          | <i>RW</i>          | 1.2802                              | 1.2744           | 0.7561               | 0.8314            |
| 8        | <i>AR</i>          | 2.8033                              | 1.8646           | 1.4182               | 1.6338            |
|          | <i>RW</i>          | 2.2210                              | 1.4733           | 1.1236               | 1.2945            |

Note: A ratio smaller than one means that the BV model is outperforming the corresponding benchmark.

Source: Staff estimates.

At the one-quarter horizon, the BV model under most approaches are more informative about upcoming inflationary pressures than the RW model, while only that under the production function approaches are more informative than the AR model. At the four-quarter horizon, the BV model under the Kalman filter and the IMF filter are more informative than the RW model, while no approach can beat the AR model. At the eight-quarter horizon, no approach can perform better than either the AR or the RW models. Overall, on the basis of these results, it is apparent that the Kalman filter is more informative about inflationary pressures than other methods, as the MSFE under the Kalman filter is somewhat smaller considering all horizons as a whole.

To check whether our results are robust, we augment equation (22) of the BV model with import price inflation, as external factors can have considerable influences over local inflationary process:

$$\pi_{t+h}^h = a + b(L)\pi_t + c(L)x_t + d(L)\pi_t^{pm} + \varepsilon_{t+h}^h \quad (27)$$

where  $d(L)$  is a lag polynomials, and  $\pi_t^{pm} = 400 * \left( \frac{pm_t}{pm_{t-1}} - 1 \right)$  is the annualised inflation rate of unit retained import value  $pm_t$ . The results under the augmented BV model are summarised in Table 6:

**Table 6: BV model (output gap in level and import price inflation included)  
- comparison of MSFE**

| <i>h</i> | <i>Relative to</i> | <i>Production<br/>function<br/>approach</i> | <i>HP filter</i> | <i>Kalman<br/>filter</i> | <i>IMF filter</i> |
|----------|--------------------|---|------------------|--------------------------|-------------------|
| 1        | <i>AR</i>          | 1.0741                                      | 1.0790           | 1.1783                   | 1.1044            |
|          | <i>RW</i>          | 0.6461                                      | 0.6491           | 0.7088                   | 0.6644            |
| 4        | <i>AR</i>          | 1.8475                                      | 1.8099           | 0.9989                   | 1.1932            |
|          | <i>RW</i>          | 1.3451                                      | 1.3177           | 0.7273                   | 0.8687            |
| 8        | <i>AR</i>          | 2.5740                                      | 2.5388           | 1.4339                   | 1.3405            |
|          | <i>RW</i>          | 2.0393                                      | 2.0114           | 1.1361                   | 1.0620            |

Note: A ratio smaller than one means that the BV model is outperforming the corresponding benchmark.

Source: Staff estimates.

The only notable change is that the Kalman filter is now able to beat the AR model at the four-quarter horizon. With that said, it remains comfortable to say that the Kalman filter is more informative about inflationary pressures than other methods. Thus, our results are robust to the inclusion of import price inflation into equation (22).

We also consider whether there would be any difference in performance by replacing the *level* of the output gap in equation (22) with its *first difference*. Our rationale for this is that, as many had argued, the speed at which the output gap is changing may be more relevant than the level of the gap itself in determining inflationary pressures (i.e. speed-limit effect). The results for the adjusted BV model, with and without import price inflation, are summarised in Table 7 and Table 8.

**Table 7: BV model (output gap in difference) - comparison of MSFE**

| <i>h</i> | <i>Relative to</i> | <i>Production<br/>function<br/>approach</i> | <i>HP filter</i> | <i>Kalman<br/>filter</i> | <i>IMF filter</i> |
|----------|--------------------|---|------------------|--------------------------|-------------------|
| 1        | <i>AR</i>          | 1.0323                                      | 1.0439           | 0.9825                   | 1.0190            |
|          | <i>RW</i>          | 0.6210                                      | 0.6280           | 0.5910                   | 0.6130            |
| 4        | <i>AR</i>          | 1.2617                                      | 1.0247           | 0.9916                   | 1.1504            |
|          | <i>RW</i>          | 0.9186                                      | 0.7460           | 0.7219                   | 0.8375            |
| 8        | <i>AR</i>          | 1.4915                                      | 1.1656           | 1.2610                   | 1.0992            |
|          | <i>RW</i>          | 1.1817                                      | 0.9235           | 0.9990                   | 0.8709            |

Note: A ratio smaller than one means that the BV model is outperforming the corresponding benchmark.

Source: Staff estimates.

**Table 8: BV model (output gap in difference and import price inflation included)  
- comparison of MSFE**

| <i>h</i> | <i>Relative to</i> | <i>Production function approach</i> | <i>HP filter</i> | <i>Kalman filter</i> | <i>IMF filter</i> |
|----------|--------------------|-------------------------------------|------------------|----------------------|-------------------|
| 1        | <i>AR</i>          | 1.0589                              | 1.0795           | 1.0605               | 1.0763            |
|          | <i>RW</i>          | 0.6370                              | 0.6494           | 0.6380               | 0.6474            |
| 4        | <i>AR</i>          | 1.2952                              | 1.0862           | 0.9926               | 1.2428            |
|          | <i>RW</i>          | 0.9430                              | 0.7908           | 0.7226               | 0.9048            |
| 8        | <i>AR</i>          | 1.4240                              | 1.2113           | 1.2532               | 1.2169            |
|          | <i>RW</i>          | 1.1282                              | 0.9597           | 0.9929               | 0.9641            |

Note: A ratio smaller than one means that the BV model is outperforming the corresponding benchmark.

Source: Staff estimates

Comparing with the baseline, using the first difference of the output gap rather than its level in equation (22) will lead to improvements in the forecasting power at the four-quarter and eight-quarter horizons. Moreover, the BV model under nearly all approaches can beat the RW model at all horizons. That said, the Kalman filter continued to perform relatively well among all approaches.

To sum up, on the basis of forecasting inflationary pressures, the output gap estimated by the Kalman filter compared favourably against other methods. On the other hand, despite its advantage in the other area, the edge of the IMF filter over other methods in forecasting inflation is not particularly distinct.

## V. CONCLUSION

In this paper, we assess several output gap estimation methods for the Hong Kong economy, including: (1) the production function approach; (2) the Hodrick-Prescott (HP) filter; (3) the Kalman filter; and (4) the IMF multivariate filter. Our assessments are based on a number of criteria, including: (1) consistency with economic priors; (2) transparency; (3) capability of providing information about the precision of estimates; (4) robustness to revisions; and (5) informative about inflationary pressures. On most criteria, the IMF filter performed relatively well compared with other methods, particularly on the robustness to revisions. Nevertheless, the output gap estimate obtained using the Kalman filter is more informative about future inflationary pressures, while the IMF filter shows no clear edge over the other methods. Thus, it may still be worthwhile to supplement the IMF filter with other methods, to facilitate our monitoring of the inflationary pressures in Hong Kong.

## REFERENCES

- Benes, J., Clinton, K., Garcia-Saltos, R., Johnson, M., Laxton, D., Manchev, P., and Matheson, T. (2010), “Estimating Potential Output with a Multivariate Filter”, IMF Working Paper WP/10/285.
- Batini, N., Jackson, B., and Nickell, S. (2000), “Inflation Dynamics and the Labour Share in the UK”, External MPC Unit Discussion Paper No. 2, Bank of England.
- Clark, P. (1989), “Trend Reversion in Real Output and Unemployment”, *Journal of Econometrics* 40, 15 - 32.
- Coenen, G., Smets, F., and Vetlov, I. (2009), “Estimation of the Euro Area Output Gap Using the NAWM”, Bank of Lithuania Working Paper 5.
- Cotis, J. P., Elmeskov, J., and Mourougane, A. (2003), “Estimates of Potential Output: Benefits and Pitfalls from a Policy Perspective”, OECD Working Paper.
- El-Ganainy, A., and Weber, A. (2010), “Estimates of the Output Gap in Armenia with Applications to Monetary and Fiscal Policy”, IMF Working Paper WP/10/197.
- Fueki, T., Fukunaga, I., Ichiue, H., and Shirota, T. (2010), “Measuring Potential Growth with an Estimated DSGE Model of Japan’s Economy”, Bank of Japan Working Paper No. 10 –E-13.
- Gerlach, S., and Yiu, M. (2004), “Estimating Output Gaps in Asia: A Cross-Country Study”, *Journal of the Japanese and International Economies* 18, 115 - 136.
- Ha, J., and Leung, C. (2000), “Estimating Hong Kong’s Output Gap and its Impact on Inflation”, HKMA Research Memorandum.
- International Monetary Fund, *World Economic Outlook*, October 2009.
- Krueger, A. (1999), “Measuring Labour’s Share”, NBER Working Paper No. 7006.
- Peng, W., Cheung, L., and Fan, K. (2001), “Sources of Unemployment: Recent Developments and Prospects”, HKMA Quarterly Bulletin, November.
- Sarel, M. (1997), “Growth and Productivity in ASEAN Countries”, IMF Working Paper WP/97/97.
- Sun, Y. (2010), “Potential Growth of Australia and New Zealand in the Aftermath of the Global Crisis”, IMF Working Paper WP/10/127.
- Watson, M. (1986), “Univariate Detrending Methods with Stochastic Trends”, *Journal of Monetary Economics* 18, 49 - 75.

**ANNEX I**

**DATA SOURCES**

| Variable  | Source(s)                | Definition   |
|-----------|--------------------------|--|
| $Y_t$     | C&SD                     | Real output  |
| $N_t$     | Staff estimates          | Working age population - Labour force divided by the labour force participation rate |
| $P_t$     | C&SD                     | Labour force participation rate  |
| $E_t$     | C&SD                     | Employment   |
| $H_t$     | C&SD                     | Median hours of work per week  |
| $NAIRU_t$ | Staff estimates          | Non-accelerating inflation rate of unemployment                                      |
| $I_t$     | C&SD                     | Private investment   |
| $K_t$     | C&SD and staff estimates | Private capital – calculated using the perpetual inventory method                    |
| $L_t$     | C&SD                     | Labour force   |
| $\pi^4_t$ | C&SD and staff estimates | Year-on-year core CCPI inflation rate (excluding rent)                               |
| $U_t$     | C&SD                     | 3-month moving average seasonally adjusted unemployment rate                         |
| $CPI_t$   | C&SD and staff estimates | The underlying CCPI (excluding rent)   |
| $pm_t$    | C&SD                     | Unit retained import value   |

Note: Any seasonal adjustment required is done by X.12.

**MAXIMUM REGULARISED LIKELIHOOD METHOD**

Under the Maximum Regularised Likelihood estimation method, the objective function is:

$$\max_{\theta} \log L(\theta; Y) - p \sum_i \frac{(\theta_i - \bar{\theta}_i)^2}{\sigma_{\theta_i}^2}$$

where  $\theta \in [\theta_i^L, \theta_i^U]$

Let  $\theta$  be the vector of parameters,  $Y$  be the set of data. The prior for each parameter is a normal distribution with mode  $\bar{\theta}_i$  and variance  $\frac{1}{p} \sigma_{\theta_i}^2$ , truncated at  $\theta_i^L$  from below and  $\theta_i^U$  from above. The parameter estimate can be seen as the mode of the posterior distribution. We follow Benes et al. (2010) in setting  $p=1$ , such that figures for the dispersion can be interpreted as the standard deviation of the priors.



**ANNEX III**

**THE AUGMENTED IMF MULTIVARIATE FILTER**

This Annex describes the extended version of the IMF multivariate filter, which is presented at the EMD workshop organised by the IMF in April 2010.

The extended version of the model drops equations (19) and (20) from the original model, but adds to the model several more equations on the labour productivity, the labour force participation rate and the working age population:

The labour productivity  $Q_t$  is defined as the (log) difference between output  $Y_t$  and employment  $E_t$  :

$$Q_t = Y_t - E_t \quad (\text{A.1})$$

with the following trend level  $\bar{Q}_t$  :

$$\bar{Q}_t = \bar{Q}_{t-1} + G_t^{\bar{Q}} / 4 + \varepsilon_t^{\bar{Q}} \quad (\text{A.2})$$

The term  $G_t^{\bar{Q}}$  is a short term growth trend evolving according to:

$$G_t^{\bar{Q}} = \pi G_{SS}^{\bar{Q}} + (1-\tau)G_{t-1}^{\bar{Q}} + \varepsilon_t^{G^{\bar{Q}}} \quad (\text{A.3})$$

and  $G_{SS}^{\bar{Q}}$  is the steady state of this trend.

Defining  $G_t^N$  as the growth in the working age population  $N_t$  :

$$G_t^N = 4(N_t - N_{t-1}) \quad (\text{A.4})$$

which will evolve according to:

$$G_t^N = \zeta G_{SS}^N + (1-\zeta)G_{t-1}^N + \varepsilon_t^{G^N} \quad (\text{A.5})$$

with  $G_{SS}^N$  as the steady state level.

The labour force participation rate gap  $p_t$  is defined as the difference between the labour force participation rate  $P_t$  and its trend level  $\bar{P}_t$ :

$$p_t = P_t - \bar{P}_t \quad (\text{A.6})$$

The trend  $\bar{P}_t$  will evolve according to the following:

$$\bar{P}_t = \eta 100 \ln\left(\frac{P_{SS}}{100}\right) + (1 - \eta)\bar{P}_{t-1} + \varepsilon_t^P \quad (\text{A.7})$$

where  $P_{SS}$  is the steady state level of the trend.

The labour force participation rate gap will be determined by the unemployment gap  $u_t$ :

$$p_t = \theta_1 p_{t-1} + \frac{100}{P_{SS}} \theta_2 u_t + \varepsilon_t^P \quad (\text{A.8})$$

The labour force identity:

$$L_t = N_t + P_t \quad (\text{A.9})$$

and the trend labour force  $\bar{L}_t$ :

$$\bar{L}_t = N_t + \bar{P}_t \quad (\text{A.10})$$

The unemployment trend identity:

$$\bar{U}_t - U^{SS} = -\left(1 - \frac{U^{SS}}{100}\right)(\bar{E}_t - \bar{L}_t - 100 \ln(1 - \frac{U^{SS}}{100})) \quad (\text{A.11})$$

The potential output will be determined by the following identity:

$$\bar{Q}_t = \bar{Y}_t - \bar{E}_t \quad (\text{A.12})$$

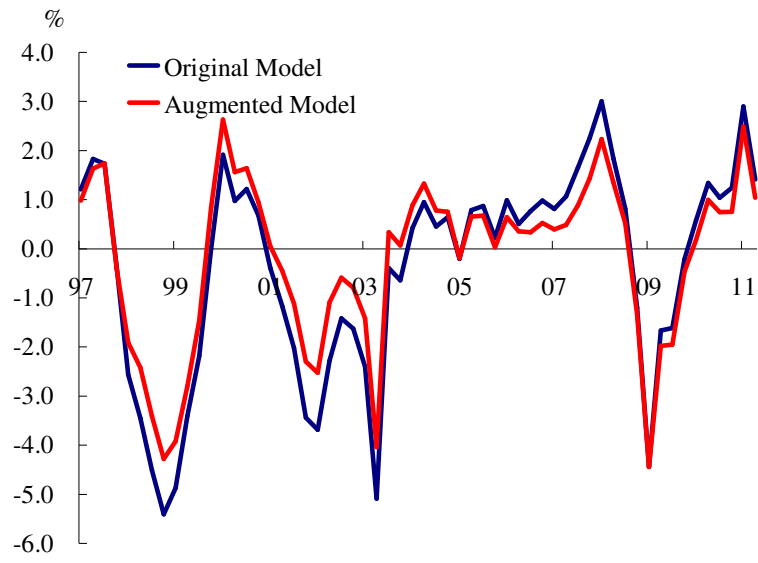
The estimation results under the extended version of the model are summarised in Table A.1. We find that the estimated output gap under the augmented model does not differ a lot from that under the original model (Chart A.1). Thus, for simplicity, we will stick to the original model in the future.

**Table A.1: Estimation results under the extended IMF filter**

| Parameter                            | Prior |            | Posterior |            |
|--------------------------------------|-------|------------|-----------|------------|
|                                      | Mode  | Dispersion | Mode      | Dispersion |
| $U^{SS}$                             | 4.200 | 1.000      | 4.218     | 0.137      |
| $\theta$                             | 0.650 | 0.100      | 0.651     | 0.014      |
| $\beta$                              | 0.400 | 0.300      | 0.283     | 0.037      |
| $\Omega$                             | 0.500 | 0.300      | 0.352     | 0.039      |
| $\phi_1$                             | 0.800 | 0.300      | 0.811     | 0.041      |
| $\phi_2$                             | 0.300 | 0.300      | 0.193     | 0.029      |
| $\omega$                             | 3.000 | 1.500      | 2.984     | 0.212      |
| $\lambda$                            | 3.000 | 3.000      | 2.914     | 0.410      |
| $\alpha$                             | 0.900 | 0.300      | 0.884     | 0.043      |
| $\tau$                               | 0.250 | 0.300      | 0.220     | 0.036      |
| $\rho_1$                             | 0.800 | 0.300      | 0.794     | 0.041      |
| $\rho_2$                             | 5.000 | 3.000      | 4.933     | 0.417      |
| $\zeta$                              | 0.100 | 0.150      | 0.059     | 0.015      |
| $\eta$                               | 0.100 | 0.150      | 0.103     | 0.021      |
| $\theta_1$                           | 0.900 | 0.150      | 0.926     | 0.028      |
| $\theta_2$                           | 0.100 | 0.150      | 0.098     | 0.018      |
| $\sigma_{\varepsilon\pi^4}$          | 0.500 | 0.300      | 0.768     | 0.034      |
| $\sigma_{\varepsilon^u}$             | 0.500 | 0.300      | 0.377     | 0.033      |
| $\sigma_{\varepsilon\bar{U}}$        | 0.100 | 0.150      | 0.164     | 0.021      |
| $\sigma_{\varepsilon\bar{G}\bar{U}}$ | 0.100 | 0.150      | 0.175     | 0.030      |
| $\sigma_{\varepsilon\pi^4LTE}$       | 0.500 | 0.300      | 0.779     | 0.042      |
| $\sigma_{\varepsilon^y}$             | 1.000 | 0.300      | 1.473     | 0.040      |
| $\sigma_{\varepsilon\bar{Q}}$        | 0.250 | 0.075      | 0.392     | 0.010      |
| $\sigma_{\varepsilon\bar{G}\bar{Q}}$ | 1.000 | 0.300      | 1.592     | 0.041      |
| $\sigma_{\varepsilon\bar{G}\bar{N}}$ | 0.150 | 0.075      | 0.245     | 0.010      |
| $\sigma_{\varepsilon\bar{P}}$        | 0.100 | 0.075      | 0.199     | 0.011      |
| $\sigma_{\varepsilon^p}$             | 0.150 | 0.075      | 0.303     | 0.009      |

Source: Staff estimates.

**Chart A.1: Estimates of the output gap under different versions of the IMF filter**



Source: Staff estimates.

**FORECASTING POWER TESTS UNDER THE ASSUMPTION THAT INFLATION IS I(1)**

The following Tables summarise the results of the forecasting power tests of different output gap measures, under the assumption that inflation in the BV model is modelled as I(1) rather than I(0):

$$\pi_{t+h}^h - \pi_t = a + b(L)\Delta\pi_t + c(L)x_t + \varepsilon_{t+h}^h \quad (\text{A.13})$$

The AR model is also modelled similarly as equation (A.13), but excluding the terms on the output gap.

As shown in the following Tables, our results again show that the Kalman filter is relatively more informative about inflationary pressures comparing with other approaches.

**Table A.2: BV model (output gap in level) - comparison of MSFE**

| <i>h</i> | <i>Relative to</i> | <i>Production function approach</i> | <i>HP filter</i> | <i>Kalman filter</i> | <i>IMF filter</i> |
|----------|--------------------|-------------------------------------|------------------|----------------------|-------------------|
| 1        | <i>AR</i>          | 1.0471                              | 0.9281           | 0.8170               | 0.9954            |
|          | <i>RW</i>          | 0.7878                              | 0.6983           | 0.6147               | 0.7489            |
| 4        | <i>AR</i>          | 1.5311                              | 1.4729           | 0.8095               | 1.4500            |
|          | <i>RW</i>          | 1.6496                              | 1.5869           | 0.8721               | 1.5622            |
| 8        | <i>AR</i>          | 1.7010                              | 1.5758           | 0.8764               | 2.1037            |
|          | <i>RW</i>          | 2.3821                              | 2.2068           | 1.2274               | 2.9460            |

Note: A ratio smaller than one means that the BV model is outperforming the corresponding benchmark.

Source: Staff estimates.

**Table A.3: BV model (output gap in level and import price inflation included) - comparison of MSFE**

| <i>h</i> | <i>Relative to</i> | <i>Production function approach</i> | <i>HP filter</i> | <i>Kalman filter</i> | <i>IMF filter</i> |
|----------|--------------------|-------------------------------------|------------------|----------------------|-------------------|
| 1        | <i>AR</i>          | 1.2097                              | 0.9920           | 0.7662               | 0.9635            |
|          | <i>RW</i>          | 0.9102                              | 0.7464           | 0.5765               | 0.7250            |
| 4        | <i>AR</i>          | 1.6621                              | 1.4341           | 0.6900               | 1.5160            |
|          | <i>RW</i>          | 1.7907                              | 1.5450           | 0.7434               | 1.6333            |
| 8        | <i>AR</i>          | 1.9401                              | 1.7806           | 0.9552               | 2.2490            |
|          | <i>RW</i>          | 2.7170                              | 2.4936           | 1.3377               | 3.1496            |

Note: A ratio smaller than one means that the BV model is outperforming the corresponding benchmark.

Source: Staff estimates.

**Table A.4: BV model (output gap in difference) - comparison of MSFE**

| <i>h</i> | <i>Relative to</i> | <i>Production function approach</i> | <i>HP filter</i> | <i>Kalman filter</i> | <i>IMF filter</i> |
|----------|--------------------|-------------------------------------|------------------|----------------------|-------------------|
| 1        | <i>AR</i>          | 1.0238                              | 1.0308           | 0.9248               | 1.1073            |
|          | <i>RW</i>          | 0.7703                              | 0.7756           | 0.6958               | 0.8331            |
| 4        | <i>AR</i>          | 1.0276                              | 0.9583           | 0.8658               | 1.1072            |
|          | <i>RW</i>          | 1.1071                              | 1.0325           | 0.9328               | 1.1929            |
| 8        | <i>AR</i>          | 1.0814                              | 1.1059           | 0.7862               | 1.1384            |
|          | <i>RW</i>          | 1.5144                              | 1.5487           | 1.1010               | 1.5943            |

Note: A ratio smaller than one means that the BV model is outperforming the corresponding benchmark.

Source: Staff estimates.

**Table A.5: BV model (output gap in difference and import price inflation included) - comparison of MSFE**

| <i>h</i> | <i>Relative to</i> | <i>Production function approach</i> | <i>HP filter</i> | <i>Kalman filter</i> | <i>IMF filter</i> |
|----------|--------------------|-------------------------------------|------------------|----------------------|-------------------|
| 1        | <i>AR</i>          | 1.0944                              | 1.0475           | 0.8840               | 1.1502            |
|          | <i>RW</i>          | 0.8234                              | 0.7881           | 0.6651               | 0.8654            |
| 4        | <i>AR</i>          | 1.0898                              | 1.0114           | 0.6704               | 1.0772            |
|          | <i>RW</i>          | 1.1741                              | 1.0897           | 0.7223               | 1.1606            |
| 8        | <i>AR</i>          | 1.1383                              | 1.2559           | 0.8034               | 1.3233            |
|          | <i>RW</i>          | 1.5941                              | 1.7589           | 1.1251               | 1.8532            |

Note: A ratio smaller than one means that the BV model is outperforming the corresponding benchmark.

Source: Staff estimates.